Promotion Tournaments  
and Individual Performance Pay  

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Forthcoming in the  
Journal of Economics & Management Strategy

Abstract  
We analyze the optimal combination of promotion tournaments and linear individual performance pay in an employment relationship. An agent’s effort is non-observable and he has private information about his suitability for promotion. Thus, the two incentive schemes need to be combined to serve both incentive and selection purposes. If harder working agents respond less to intensified effort incentives, we find that the principal puts less emphasis on individual performance pay when selection becomes more important. Thus, we provide a possible explanation as to why, in practice, individual performance pay is less prevalent than promotion-based incentives.

JEL Classification: D82, D86, M52.

Keywords: Promotion Tournaments, Piece Rates, Hidden Information, Hidden Action

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†We would like to thank two anonymous referees, whose comments have greatly helped us to improve the exposition of this paper. We also benefited from comments by Kristin Chlosta, Dominique Demougin, Mati Dubrovinsky, Oliver Fabel, Thomas Heßmann, Carsten Helm, Jenny Kragl, Matthias Kräkel, Dorothee Schneider, Chee-Wee Tan, and participants of the Society of Labor Economics Annual Meeting in New York, European Economic Association Congress in Budapest, Verein für Socialpolitik Annual Meeting in Munich, Canadian Economics Association Annual Meeting in Halifax, First Conference on Tournaments, Contests and Relative Performance Evaluation in Raleigh, and the WZB Young Researchers Workshop on Contests and Tournaments. Financial support by the Deutsche Forschungsgemeinschaft (DFG), grant SFB/TR 15, is gratefully acknowledged.

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1 Introduction

Firms are confronted with essentially two challenges to maximize the productivity of their workforce. First, firms need to design appropriate incentive schemes in order to motivate their employees to implement effort (motivation challenge). Second, firms have to utilize viable mechanisms to facilitate the assignment of employees to jobs they are most suitable for (selection challenge). In the economic literature, it is well-understood that pay-for-performance can both motivate a firm’s workforce and serve as a selection device.\(^1\) However, as Baker et al. \([1988]\) point out, most firms employ payment schemes that are largely independent of performance.\(^2\) Moreover, arguing that “promotions are used as the primary incentive device in most organizations”, Baker et al. \([1988, \text{pp. 600-1}]\) pose the question why promotion-based incentive systems are more prevalent than explicit pay-for-performance schemes. We provide a possible answer to this puzzle by analyzing the optimal combination of promotion tournaments and linear individual performance pay. We focus on these two particular incentive schemes because we believe that, to explain real-world phenomenons, it is natural to investigate the interaction of incentive and selection mechanisms that are most frequently observed in business practice. Indeed, we find that the optimal interplay of promotion tournaments and linear individual performance pay may involve only low-powered individual incentives when motivation and selection issues arise simultaneously.

We investigate the two lowest hierarchical tiers of a firm: production and lower management. To fill vacancies in management with suitable candidates, a firm can pursue two different strategies. First, managers can be recruited from the external labor market. In this case, their past experience and performance may be used as signals about their individual abilities,\(^3\) which in turn facilitates an efficient selection process. Clearly, recruiting from the external labor market can constitute the preferred hiring strategy when external candidates are sufficiently more suitable for the relevant position than current employees \([Tsoulouhas et al., 2007]\). For this to be the case it is necessary that firm-specific human capital is of little importance for the particular management position.

Alternatively, a firm can use its internal labor market and promote some production workers

\(^1\)See for instance Salanié \([2005]\) for the theoretical background. Empirically, Lazear \([2000]\) documents that the introduction of a simple piece rate scheme in a U.S. auto glass company increased output significantly and, at the same time, attracted more capable workers.

\(^2\)See also Parent \([2002]\) for empirical evidence.

\(^3\)The resulting effects on managerial incentives and optimal employment contracts are the subject of the career concerns literature, see, e.g., Holmström \([1999]\).
to management positions. Recruiting managers from the pool of current employees can be advantageous for firms due to several reasons. Firstly, because of their employment history in the respective firm, employees generally acquired firm-specific human capital and adapted to the corporate culture, which is potentially crucial for being a successful manager. Secondly, the prospect of promotion – and the associated benefits such as higher income, perks, status, and authority – can be a strong motivator for competing employees (e.g., Laezar and Rosen [1981], Nalebuff and Stiglitz [1983]). Thirdly, promotion tournaments can improve job assignments by facilitating the selection of more suitable employees for higher-level jobs (e.g., Rosen [1986], Clark and Riis [2001]).

Several empirical studies suggest that the advantages of internal promotions may dominate those of external recruitment, in particular with respect to positions in lower and middle management (e.g., Turner [1994] and Fellman [2003]). Put differently, the promotion of employees to management jobs is a common phenomenon observable in firms. A prominent example is United Parcel Service (UPS) with an explicit commitment to “a promote-from-within approach to management development” [UPS, 2009]. According to UPS, striking 85 percent of its full-time management employees in 2006 were promoted from non-management positions.

While promotion tournaments clearly constitute a frequently utilized incentive and selection device in firms, surprisingly little is known about how they interact with other incentive schemes such as individual performance pay. This paper therefore aims at shedding light on how firms can jointly use individual performance pay and internal promotions to cope with the two previously emphasized challenges: motivating effort and facilitating the efficient assignment of employees to various jobs. In doing so, we focus on a situation where a firm prefers internal promotions to external recruitment because prospective managers need to acquire sufficient firm-specific human capital to conduct their future tasks effectively.

We analyze a principal-agent relationship between the owner of a firm (principal) and two employees (agents). The agents share the same abilities in production but may differ in their skills for the management task. Initially, there is symmetric uncertainty about an agent’s management skills, i.e., no party can observe an agent’s suitability for the management job. Both

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4For instance, Hatch and Dyer [2004] provide empirical evidence that the investment in firm-specific human capital significantly increases firm performance.

5Baker, Gibbs, and Holmström [1994] conclude from their analysis of personnel data from a medium-sized U.S. firm that the performance of employees at lower hierarchy levels is used to learn about their abilities, which in turn facilitates promotion decisions and thus more efficient job assignments.

6We thank an anonymous referee for pointing out this example.
agents are first employed in production, allowing them to attain the level of firm-specific human capital required for the management position. Their respective effort as production worker is non-observable, but the principal receives contractible individual performances measures. Therefore, the principal can provide the agents with a piece rate scheme based upon their individual performance in order to motivate effort.\footnote{For example, at the auto glass company that Lazear [2000] investigates, installers receive a piece rate based on the number of glass units they installed.} Moreover, to fill the management position, the principal utilizes a typical promotion tournament: the better performing agent will be promoted and hence, becomes the new manager. While being employed as a production worker, each agent learns about his individual suitability for the management position in this particular firm. Consequently, agents become able to assess their individual valuation of being employed as a manager, which affects their incentives to compete for promotion.

A manager’s effort is also non-observable. Moreover, since lower-level managers generally perform difficult-to-measure tasks such as supervising subordinates or organizing the workflow in production, contractible performance measures are not available.\footnote{This assumption simplifies our analysis but is not crucial for the results. We discuss a potential extension of our model with incentive contracts on the management level in Section 5.} A manager therefore receives a fixed salary and exerts some minimum required effort level. The management compensation constitutes the prize in the promotion tournament. Hence, the firm’s compensation and promotion policy needs to serve two objectives: motivating production workers and, in case the recruited workers are heterogeneous, increasing the chances of promoting the better suited agent to the management level.

Our main result is that the introduction of a piece rate scheme may interfere with the selection of high-ability managers by means of a promotion tournament. A firm may therefore provide only low-powered individual incentives, or even refrain from using individual performance pay, when promoting the more suitable candidate is sufficiently important. The rationale behind this result is as follows. Because workers and managers perform different tasks, production output cannot serve \textit{per se} as a signal about a worker’s suitability for the management job. However, workers who perceive themselves as capable future managers have a higher valuation for being promoted. Because of this higher valuation, more capable candidates work harder when the firm selects the best-performing worker for promotion. Consequently, by implementing a promotion tournament, the firm can use a worker’s production output as a signal about his suitability for promotion. The provision of individual performance pay, however, can dilute the
informativeness of this signal. That is, if the recruited workers are heterogeneous, it becomes then less likely that the worker who is more suitable for promotion also has the higher output. The reason is that the other worker, who is less suited for promotion, may respond more strongly to intensified individual effort incentives on the production stage. Technically, such a situation occurs if workers’ marginal effort costs increase disproportionally. We argue in Section 3 that this characteristic is likely to be predominant in the production environment investigated in this study. If, however, the opposite is true, the result is reversed: a higher emphasis on individual performance pay then improves selection.

Our second result refers to how the employment contract, in balancing incentive and selection considerations, distorts agents’ effort choices. If both agents are high-skilled and therefore suitable for the management position, they work too hard as compared to the first-best effort level. By contrast, if both agents are low-skilled and thus less suited for the management position, they exert too little effort. The reason is that, under any given contract, high-skilled agents are more motivated in production as they gain relatively more from promotion. If agents are heterogeneous, inducing a large difference between effort of high-skilled and low-skilled agents improves selection. Taking this into account, the optimal combination of the piece rate scheme and the promotion tournament implies that the more able agent puts in too little effort, and the less able agent too much. Thus, the latter has an inefficiently high promotion probability. The principal is compelled to accept this inefficiency as a result of optimally trading off incentive and selection issues. This observation can be interpreted as a rationale for the occurrence of the Peter Principle, which states that employees are promoted to their level of incompetence [Peter and Hull, 1969].

Also pursuing the question of why promotion-based incentives are predominant in organizations, Fairburn and Malcomson [2001] focus on the impact of influence activities on the effectiveness of incentive schemes. In contrast to our framework, they analyze a situation where the performance of workers is non-verifiable, and managers allocate rewards according to their evaluations of workers’ achievements. Fairburn and Malcomson [2001] demonstrate that the use of promotion tournaments diminishes the responsiveness of managers to potential influence activities, that would render pay-for-performance schemes ineffective. Moreover, they show that the conflicting goals of incentive provision and risk allocation may also cause the Peter Principle. In their framework, however, this inefficiency occurs only if agents are risk-averse.
In our model, agents are risk-neutral.\footnote{Alternative explanations for the occurrence of the Peter Principle are provided by, e.g., Bernhardt [1995], Faria [2000], Lazear [2004], and Koch and Nafziger [2007]. For empirical evidence on the Peter Principle refer to Barmby, Eberth, and Ma [2007].}

There are alternative explanations in the literature as to why firms may be reluctant to adopt individual performance pay. Holmström and Milgrom [1991] show that it may be optimal for firms to refrain from providing individual incentives if effort has multiple dimensions, where some dimensions can be more easily measured than others. Moreover, according to Bernheim and Whinston [1998], contracting parties might want to leave some verifiable aspects of performance unspecified in contracts as this allows to punish undesired behavior. Another possible explanation originates from psychology: monetary incentive payments may crowd out intrinsic motivation [Deci, 1971]. Frey and Oberholzer-Gee [1997], Benabou and Tirole [2003], and Sliwka [2007] provide economic explanations for the occurrence of crowding-out.

More broadly, our paper contributes to the large literature on tournaments as an incentive device. The seminal papers by Lazear and Rosen [1981], Green and Stokey [1983], and Nalebuff and Stiglitz [1983] have identified several conditions under which relative incentive schemes dominate individual performance pay or vice versa. However, none of these papers considers the combination of both types of incentives. To the best of our knowledge, Kräkel and Schöttner [2008] is the only paper that also analyzes the optimal interaction of several incentive schemes, including promotion tournaments. However, they consider a quite different production environment where either the principal can observe workers’ abilities after the first employment period or there is always symmetric uncertainty about workers’ abilities.

Finally, our paper is also related to the literature on selection tournaments, which investigates relative reward schemes with the primary objective to facilitate the efficient assignment of employees to jobs. Such tournaments have been analyzed by Rosen [1986], Meyer [1991], Clark and Riis [2001], Hvide and Kristiansen [2003], Tsoulouhas, Knoeber, and Agrawal [2007] and, in the context of sabotage, by Lazear [1989], Chen [2003], and Münster [2007].\footnote{Gibbons and Waldman [1999, 2006] also investigate the optimal assignment of employees to different jobs, though in the absence of explicit tournament schemes in the usual fashion. In their framework, employees potentially differ in their respective abilities to perform tasks. Even though these abilities cannot directly be observed by firms, they can use employees’ respective output as biased signals about their individual abilities. This gradual learning process eventually facilitates the optimal assignment of employees to jobs their are best suited for.} In contrast to all of these authors, we focus on the selection effect of a promotion tournament in combination with a piece rate scheme. Furthermore, in our basic model, agents are heterogeneous in the tournament stage only because they differ in their respective valuation of the tournament
prize. In the aforementioned papers, however, agents’ heterogeneity is due to different abilities in the tournament stage.

From the studies mentioned above, Tsoulouhas et al. [2007] is closest to our paper. They consider a tournament between employees (insiders) and external candidates (outsiders) who differ in their suitability for becoming the new CEO. They explicitly focus on the trade-off between providing employees with efficient effort incentives and selecting the most suitable candidate among all contestants. Central to their study is the question whether handicapping outsiders can be efficient in order to strengthen internal effort incentives while jeopardizing the selection effect of the tournament. They show that handicapping outsiders can be optimal whenever insiders are not much worse than the external contestants in terms of their suitability for becoming the new CEO. Put differently, firms might be willing to sacrifice the efficiency of selection in order to reinforce internal effort incentives. Our study differs in two main aspects: first, we focus on a tournament between internal contestants who vary in their suitability for promotion. Second, we identify the optimal combination of a promotion tournament and individual performance pay as a means to balance selection and incentive effects appropriately.

The remainder of this paper is organized as follows. The model is introduced in Section 2. In Section 3, we derive agents’ effort levels at the production stage given the tournament prize and individual performance pay. The optimal combination of the tournament prize and the individual incentive scheme is characterized in Section 4. Section 5 discusses the impact of some of our assumptions on the results and considers some extensions. Section 6 concludes.

2 The Model

A risk-neutral principal owns a firm in which two types of tasks need to be performed: manufacturing tasks (production stage) and management tasks (management stage). The firm regularly recruits risk-neutral agents to carry out these tasks. There are more jobs in production than in management.

We focus on two representative periods in the firm’s life. At the beginning of the first period, the firm needs to hire two production workers. At the beginning of the second period, there is a vacant management position. We assume that the prospective manager requires a sufficient level of firm-specific human capital to conduct the corresponding tasks effectively. Therefore, the manager will be recruited from the internal pool of agents, i.e., from the two production
workers hired in the previous period.

There are two different types of agents in the labor market, denoted type \( A \) and type \( B \). They are equally skilled in the manufacturing task,\(^{11}\) but differ in their abilities for the management job. Agents of type \( A \) can conduct the management task more efficiently than agents of type \( B \). Prior to the contracting stage, neither the principal nor the agents observe their respective types. It is, however, common knowledge that an agent is of type \( A \) with probability \( p \), and of type \( B \) with probability \( 1 - p \), where \( 0 < p < 1 \). After accepting the contract offered by the principal and entering into the employment relationship, each agent becomes familiar with the tasks of a manager in this particular firm, and can thus assess his own suitability for the management position. Put simply, every agent learns his own type. Moreover, each agent also observes the type of his coworker, whereas the principal never observes the agents’ individual skills. This assumption captures the fact that employees who work closely together usually possess better information about one another’s talents and ambitions than the principal. Assuming that agents know one another’s type also greatly simplifies the analysis. For simplicity, an agent’s reservation utility is independent of his type and equals zero throughout the game.

At the production stage, agent \( i, i = 1, 2 \), chooses a non-observable effort level \( e_i \geq 0 \), leading to the verifiable output
\[
q_i = e_i + \mu_i, \tag{1}
\]
where \( \mu_1, \mu_2 \in \mathbb{R} \) are non-observable realizations of two identically and independently distributed random variables with zero mean.\(^{12}\) Implementing effort \( e_i \) imposes strictly convex increasing costs \( c(e_i) \). To ensure the existence of a pure-strategy equilibrium at the production stage, we further assume that \( \inf_{e>0} c''(e) > 0 \). Since effort is non-observable, the principal cannot specify a desired effort level in a court-enforceable contract. She can, however, offer an incentive contract based on the realized output \( q_i \). We restrict our attention to linear incentive schemes, assuming that the wage at the production stage consists of a piece rate \( r \) conditioned on \( q_i \) and a fixed payment \( w_1 \).

At the management stage, for the reasons discussed in the Introduction, there are no contractible performance measures available. A manager therefore exerts only a minimum required effort level (i.e., he performs his task in a way that is acceptable to the principal so that he will

\(^{11}\)That is, we assume that the manufacturing task is simple and therefore, does not require any particular skills (e.g., a job at an assembly line). Nonetheless, we discuss in Section 5 the case where agents possess different abilities in the production task.

\(^{12}\)This output function is frequently used in the tournament literature, see e.g. Lazear and Rosen [1981].
not be dismissed) and receives a fixed wage $w_2$ in return. Since agents of type $A$ have a higher ability for conducting the management task, their expected contribution to firm value is higher than that of type $B$ agents. Letting $\Pi_k$, $k \in \{A, B \}$, denote type $k$’s expected contribution to firm value on the management stage, it therefore holds that $\Pi_A > \Pi_B$. Moreover, we assume that – because of his better talent – type $A$ does not only perform the managerial task more effectively, but also needs less time to complete it. This implies that type $A$ also has lower costs for implementing the minimum effort level as compared to type $B$.\textsuperscript{13} Alternatively, type $A$ could enjoy certain attributes of the management job more than type $B$ (e.g., more interesting tasks, higher status), which lowers the former’s disutility of effort. To reduce the notational burden, we normalize type $B$’s effort costs for the management task to zero, while type $A$’s costs are $-\delta$, where $\delta > 0$. Thus, to ensure that one of the former production workers agrees to be employed as a manager, even if both workers are of type $B$, the principal needs to offer a non-negative management wage $w_2 \geq 0$. As a consequence, type $A$ obtains a higher utility from being employed as a manager than type $B$.

Timing is as follows. First, the principal offers two randomly chosen agents a contract consisting of a piece rate scheme $(r, w_1)$ for the production stage and a wage $w_2$ for the management stage. After accepting the contract, each agent learns his type and that of his coworker. Then, both agents are assigned to the manufacturing task and choose their respective effort levels $e_i$.\textsuperscript{14} Once the random variables and hence output levels $q_i$ are realized, both agents obtain their individual performance pay according to the stipulated piece rate scheme. Furthermore, the agent with the highest output level will be promoted to the management level and obtains the management wage $w_2$. The other agent leaves the firm and receives his reservation utility.\textsuperscript{15}

Two remarks on why we analyze this specific employment contract are in order. First, note that we do not allow for communication between the principal and the agents with respect to their types after the contracting stage.\textsuperscript{16} Instead, the principal stipulates a compensation scheme

\textsuperscript{13}In general, an agent’s talent will affect both the quality and the pace of finishing a task. For example, a better manager may find more effective solutions to organizational problems, and also come up faster with a viable solution than a less able manager. Thus, being of a superior type implies to have a higher productivity and lower effort costs, as in our framework. Usually, only one of these assumptions is made to model different abilities of agents. However, we require both of them because we only allow for a fixed wage at the management stage. Refer to Section 5 for a more detailed discussion of this point.

\textsuperscript{14}Agents thus observe their types after signing the contract but before choosing their effort levels. In practice, this information might be acquired during a training period, where workers already exert some effort. However, we assume that this period is relatively short, and can therefore be neglected.

\textsuperscript{15}Alternatively, one could assume that the losing agent stays with the firm and competes in the next period with a newly hired agent. However, such an extension complicates the analysis without offering any additional insights.

\textsuperscript{16}This assumption rules out the implementation of the first-best solution by asking both agents to report their
and a promotion policy that applies to all production workers independent of their type. This has the consequence that wages are attached to jobs rather than to individual types, which is typically the case in practice (see e.g. Baker et al. [1988], Eriksson [1999]). Establishing such simple employment rules facilitates recurrent recruitment and promotion procedures. For example, the owner of a large firm usually has to delegate the implementation of these procedures to a third party. Then, dictating employment rules that are not manipulable avoids agency problems such as influence activities or collusion, which may occur when payoff-relevant decisions are left to a third party’s discretion (see e.g. Tirole [1986] or Fairburn and Malcomson [2001]).

Second, the employment contract comprises a promotion tournament based on the first-period output for the following reasons. Since $\Pi_A > \Pi_B$, the principal prefers to select a type $A$ agent for the management position. With a management wage $w_2 \geq 0$, however, there will be no self-selection as both agents always prefer becoming a manager to leaving the firm. To increase the chances of employing a type $A$ agent as a manager, the principal can try to take advantage of the fact that type $A$ agents have a higher valuation of being managers. This objective can be achieved by a promotion tournament: as we will show in Section 3, type $A$ exerts higher effort than type $B$ whenever the randomly recruited agents are heterogeneous. Consequently, the better performing agent is more likely of type $A$. Therefore, under a promotion tournament, performance at the production stage serves as a signal about skills for the management job.

Finally, note that applying such a promotion tournament causes the following inefficiency: even though both types are equally skilled in production, they will exert different effort levels with respect to the manufacturing task. This can only be prevented by a purely random assignment of agents to the management position, which clearly comes at the cost of completely neglecting selection issues. We henceforth assume that the principal prefers utilizing a promotion tournament – thereby improving her information about agents’ types – to inducing efficient effort in the manufacturing task. Intuitively, this is the case whenever the promotion decision is sufficiently crucial for firm performance, i.e., $\Pi_A - \Pi_B$ is sufficiently high. We focus on such a

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17 Note that type $B$’s weak preference for becoming a manager can easily be turned into a strong one by introducing costs for moving to a new firm.

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situation because we are interested in studying the optimal combination of piece rates schemes and promotion tournaments when both incentive provision and selection are important.

3 Effort in the Production Stage

In this section, we derive agents’ effort choices in the production stage for a given employment contract. To do so, we need to account for three possible matches of agents: two homogeneous matches where both agents are either of type $A$ or of type $B$; and a heterogeneous match with a type $A$ and a type $B$ agent. For each match, we determine the combination of effort choices that constitutes a pure-strategy Nash-equilibrium.

By implementing effort in the production stage, agents do not only affect their incentive payments conditional on production output, but also their respective probabilities of being promoted to the management level. Agent $i$’s promotion probability is

$$\text{Prob}[q_i > q_j] = \text{Prob}[e_i - e_j > \mu_j - \mu_i] \equiv G(e_i - e_j), \quad i, j \in \{1, 2\}, \ i \neq j,$$  

where $G(\cdot)$ is the cumulative distribution function of the random variable $\mu_j - \mu_i$. Let $g(\cdot)$ denote the corresponding density function, which we assume to be differentiable and single-peaked at zero. Since $\mu_i$ and $\mu_j$ are identically distributed, $g(\cdot)$ is symmetric around zero.

We start our analysis by investigating the case of homogeneous agents. First, suppose that both randomly employed agents are of type $A$. Taking the effort of agent $j$ as given, agent $i$ chooses $e_i$ to maximize his expected payment

$$w_1 + G(e_i - e_j)(w_2 + \delta) + re_i - c(e_i).$$  

It is straightforward to verify that the Nash-equilibrium is unique and symmetric. The equilibrium effort, denoted $e_{AA}$, is implicitly defined by the first-order condition

$$g(0)(w_2 + \delta) + r = c'(e_{AA}).$$  

Similarly, for the case where both agents are of type $B$, equilibrium effort $e_{BB}$ is characterized by

$$g(0)w_2 + r = c'(e_{BB}).$$
To ensure that $e_{AA}$ and $e_{BB}$ indeed represent Nash-equilibria, it is sufficient to require that agents’ objective functions are concave. This is the case if

$$g'(e_i - e_j)(w_2 + \delta) - c''(e_i) < 0 \quad \text{for all } e_i, e_j \geq 0,$$

(6)

and

$$g'(e_i - e_j)w_2 - c''(e_i) < 0 \quad \text{for all } e_i, e_j \geq 0.$$

(7)

We assume that these conditions are satisfied for the highest $w_2$ that the principal is willing to offer the agents. Since $\inf_{e>0} c'' > 0$, this is the case whenever random influences on output are significant enough, i.e., $g(\cdot)$ is sufficiently ‘flat’.

Now we turn to the case of heterogeneous agents. Without loss of generality, assume that agent 1 is of type $A$ and agent 2 is of type $B$. Type $A$’s and type $B$’s respective optimization problems are:

$$\max_{e_1} w_1 + G(e_1 - e_2)(w_2 + \delta) + re_1 - c(e_1),$$

(8)

$$\max_{e_2} w_1 + [1 - G(e_1 - e_2)]w_2 + re_2 - c(e_2).$$

(9)

Type $A$’s and $B$’s equilibrium effort levels $e_A$ and $e_B$ are implicitly characterized by the following two first-order conditions:

$$g(e_A - e_B)(w_2 + \delta) + r = c'(e_A),$$

(10)

$$g(e_A - e_B)w_2 + r = c'(e_B).$$

(11)

The second-order conditions are identical to (6) and (7) and are thus satisfied.

From (10) and (11) it becomes clear that $\Delta e \equiv e_A - e_B > 0$. Because type $A$’s benefit from promotion is higher, he is motivated to work harder than type $B$ under any given incentive scheme. Hence, type $A$ has a higher probability of winning the promotion tournament, i.e., $G(\Delta e) > 0.5$.

We demonstrate in the Appendix (see Proof of Lemma 1) that $e_A$ and $e_B$ are increasing in the piece rate $r$ and the management wage $w_2$. Besides this incentive effect, increasing either $r$ or $w_2$ has also a selection effect. The latter arises from the fact that modifying $r$ or $w_2$ affects the effort difference $\Delta e$ and thus, agents’ promotion probabilities. The next lemma characterizes

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18Recall that agents’ effort costs are convex, whereas the principal’s expected profit will be concave in effort. Since the principal needs to compensate both agents for their disutility of effort to guarantee their participation, it cannot be optimal to induce arbitrarily high effort levels. Thus, there exists an upper bound for $w_2$. 

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this selection effect.

**Lemma 1** Suppose the randomly recruited agents are heterogeneous. If \( c''(e_A) > c''(e_B) \), then the type A agent responds less strongly to intensified effort incentives than the type B agent and, hence, type A’s probability of winning the promotion tournament is strictly decreasing in \( r \) and \( w_2 \). Otherwise, type A’s winning probability increases in \( r \) and \( w_2 \).

All proofs are given in the Appendix.

When the principal intensifies effort incentives by raising \( r \) or \( w_2 \), both types of agents are motivated to exert more effort. Whose effort level increases relatively more depends on the shape of the effort cost function. If marginal effort costs increase disproportionately (i.e., \( c''(e_A) > c''(e_B) \)), the harder working type A agent responds less strongly to intensified effort incentives than type B. In this case, providing higher incentives strictly lowers type A’s chances for promotion, and is therefore detrimental to selection. In contrast, if type A’s effort is more responsive to intensified incentives (i.e., \( c''(e_A) < c''(e_B) \)), his winning probability is strictly increasing in \( r \) and \( w_2 \). Finally, in the special case where \( c''(e_A) = c''(e_B) \), changing the contract parameters has no selection effect.

Our main result, emphasized by Proposition 2, is derived under the presumption that \( c''(e_A) > c''(e_B) \). This condition holds for all \( e_A \) and \( e_B \) if marginal cost of effort is increasing in \( e \), i.e., \( c'''(e) > 0 \). As discussed, this is equivalent to a situation where – under a tournament scheme – the effort choice of the harder working agent is less sensitive to intensified effort incentives. We believe that this is the more realistic case in a situation like ours, where two equally skilled production workers compete for promotion. A production worker who already puts in more effort than his co-worker should find it relatively more difficult to enhance his performance. Naturally, a worker’s physical capacity and potential working time per day is limited, and being closer to this limit implies that exerting more effort is increasingly burdensome. This is in particular the case in a situation where workers’ effort levels are already significant. Such a situation seems to be realistic for our analyzed production environment, where moderate performance could be ensured without explicit effort incentive schemes, e.g., by monitoring workers’ behavior in the workplace or threatening to dismiss low performers. In our model, we can interpret \( e = 0 \) as such a moderate performance level. Anticipating that his hard-working co-worker

\[19\] The sign of \( c''' \) is also crucial for the results in Ederer [2008], who analyzes the optimal feedback policy in tournaments. However, in contrast to our framework, this is due to dynamic incentive effects in a two-period model.
has less scope for significantly improving his performance, the less hard-working employee has an additional incentive to increase his effort in order to improve his chances of winning the promotion tournament. Thus, we can conjecture that the effort difference between workers in a heterogeneous match decreases, which is synonymous to assuming \( e''(e) > 0 \) for all \( e > 0 \).

4 The Principal’s Problem

In this section, we characterize the principal’s optimal choice of the management compensation as the tournament prize, and the piece rate scheme for the production task. To do so, we first introduce the principal’s optimization problem and discuss some basic properties of the corresponding solution. Afterwards, we characterize the induced effort levels. Finally, we derive our main result: we demonstrate how the principal adjusts the optimal piece rate scheme and the optimal management wage when selection becomes more important.

The principal’s problem is to choose the contract elements \( w_1, w_2, r \) which maximize her expected profit. The principal’s optimization problem can be stated as follows:

\[
\max_{w_1, w_2, r, e_A, e_B, e_{AA}, e_{BB}}
2p(1 - p)[(1 - r)(e_A + e_B) + G(\Delta e)\Pi_A + (1 - G(\Delta e))\Pi_B]
+ p^2[2(1 - r)e_{AA} + \Pi_A] + (1 - p)^2[2(1 - r)e_{BB} + \Pi_B] - 2w_1 - w_2
\]

s.t. (4), (5), (10), (11),

\[
w_1 + p(1 - p)[r e_A + G(\Delta e)(w_2 + \delta) - c(e_A)] + (1 - p)p[r e_B + (1 - G(\Delta e))w_2 - c(e_B)]
+ p^2[r e_{AA} + 0.5(w_2 + \delta) - c(e_{AA})] + (1 - p)^2[r e_{BB} + 0.5w_2 - c(e_{BB})] \geq 0,
\]

\[
w_2 \geq 0.
\]

Clearly, the principal’s objective function (12) consists of the different expected profits from each possible tournament match, weighted by their respective probabilities of occurrence. For example, the first line in (12) refers to the case where agents are heterogeneous, which occurs with probability \( 2p(1 - p) \). Then, at the production stage, the principal obtains the agents’ output minus the piece rate, \( (1 - r)(e_A + e_B) \). At the management stage, the principal receives \( \Pi_A \) if

\footnote{From a technical perspective, \( e''(e) > 0 \) ensures that \( e_{AA} \) and \( e_{BB} \) are concave in \( r \) and \( w_2 \), which supports concavity of the principal’s maximization problem (see also the Proof of Proposition 2).}
the agent of type $A$ is promoted, which occurs with probability $G(\Delta e)$, and $\Pi_B$ otherwise.

When maximizing her expected profit, the principal needs to account for the incentive compatibility constraints for each potential tournament match as well as for the agents’ ex-ante and interim participation constraints (13) and (14), respectively. The ex-ante participation constraint (13) requires that an agent’s expected utility from entering the employment relationship must be at least as high as his reservation utility, which is normalized to zero. In (13), the first term in square brackets refers to the case where the agent is of type $A$ while his co-worker is of type $B$. The second term in square brackets relates to the opposite case. The last two terms are associated with the homogeneous matches, where both agents implement identical effort levels in production and therefore, have the same promotion probability of 0.5.

To solve the principal’s problem, observe first that cost minimization requires the principal to choose $w_1$ such that (13) is binding. Consequently, we can eliminate $w_1$ from the principal’s optimization problem and hence, obtain the simplified problem:

$$\max_{r, w_2, e_A, e_B, e_{AA}, e_{BB}} \Pi := \pi_{AB} + \pi_{AA} + \pi_{BB} \quad \text{s.t. (4), (5), (10), (11), (14).}$$

(15)

The term $\pi_{kl}$, where $k, l \in \{A, B\}$, denotes the expected profit from a tournament match where one agent is of type $k$ and the other agent of type $l$, weighted by its probability of occurrence, i.e.,

$$\pi_{AB} \equiv 2p(1 - p)\left[e_A + e_B + G(\Delta e)(\Pi_A + \delta) + (1 - G(\Delta e))\Pi_B - c(e_A) - c(e_B)\right],$$

(16)

$$\pi_{AA} \equiv p^2\left[2e_{AA} + \Pi_A + \delta - 2c(e_{AA})\right],$$

(17)

$$\pi_{BB} \equiv (1 - p)^2\left[2e_{BB} + \Pi_B - 2c(e_{BB})\right].$$

(18)

From a closer inspection of (16)-(18) it becomes clear that the principal extracts the entire expected surplus from the employment relationship. For instance, in the heterogeneous match reflected by equation (16), the principal obtains the expected surplus from the management stage, in addition to the entire expected output from the production stage, $e_A + e_B$, net of agents’ effort costs. This is because agents do not possess private information prior to the contracting stage, and the principal can adjust the fixed wage $w_1$ such that both agents are just compensated for their expected costs of effort.

Let $e_A^*$, $e_B^*$, $e_{AA}^*$, and $e_{BB}^*$ denote the effort levels that solve the principal’s optimization problem (15). The incentive compatibility constraints for the different tournament matches al-
ready provide some information on these effort levels: the constraints for the homogeneous matches, (4) and (5), imply that effort in an AA-match always exceeds effort in a BB-match, i.e., \( e_{\text{AA}}^* > e_{\text{BB}}^* \). Furthermore, from the incentive compatibility constraints for the heterogeneous tournament, (10) and (11), we know that \( e_A^* > e_B^* \). Hence, it is clear that the induced effort levels in the different tournament matches cannot all be identical to the first-best effort level \( e^{FB} \), where

\[
e^{FB} = \arg\max_e e - c(e).
\]  

(19)

There are three potential reasons as to why the first-best effort is infeasible: asymmetric information on (i) effort, (ii) the agents’ types, and consequently, (iii) the nature of the tournament match (i.e., homogeneous or heterogeneous match). From all of these informational asymmetries, the impossibility to observe agents’ types is the one which is most detrimental to the principal: if the principal knew the agents’ types, she could simply assign the more able worker to the management job, and induce the first-best effort level on the production stage by choosing the piece rate appropriately (i.e., \( r = 1 \)). By contrast, observability of effort would be less helpful because, to improve selection, the principal would still need to employ a promotion tournament in order to induce the more able agent in a heterogenous tournament match to implement relatively more effort.

The next proposition emphasizes how the effort levels \( e_A^* \), \( e_B^* \), \( e_{\text{AA}}^* \), and \( e_{\text{BB}}^* \) are distorted relative to the first-best effort level \( e^{FB} \). After providing an intuition for the results, we will also analyze how the induced effort levels compare to those that would be chosen if the principal knew the nature of the tournament match, but does not observe the agent’s individual types.

**Proposition 1** Suppose the principal’s problem has an interior solution, i.e., all effort levels are positive and \( w_2^* > 0 \). Then, agents in a BB-match exert too little, and agents in an AA-match exert too much effort compared to the first-best effort level, i.e., \( e_{\text{BB}}^* < e^{FB} < e_{\text{AA}}^* \). In an AB-match, both types of agents implement less effort than in a homogeneous tournament match, i.e., \( e_A^* < e_{\text{AA}}^* \) and \( e_B^* < e_{\text{BB}}^* \). This implies that, in a heterogeneous match, type B’s effort is inefficiently low, i.e., \( e_B^* < e^{FB} \). However, type A’s effort \( e_A^* \) can be either too high or too low relative to the first-best effort level \( e^{FB} \).

To understand the intuition for the result in Proposition 1 with respect to the homogeneous matches, recall that the piece rate \( r \) and the management wage \( w_2 \) are substitutes with respect to the provision of incentives at the production stage. Accordingly, there exists an infinite num-
ber of combinations of \( r \) and \( w_2 \) that induce the desired effort levels in the \( AB \)-match at the same costs for the principal. Among these combinations, the principal selects the one that provides appropriate effort incentives in the homogeneous tournaments. If agents worked too little (too hard) in both homogeneous tournament matches compared to the first-best solution, the principal’s marginal benefit from increasing (decreasing) effort incentives would be positive. Consequently, under the optimal combination of a piece rate scheme and a promotion tournament, agents implement inefficiently high effort in \( AA \)-matches and inefficiently low effort in \( BB \)-matches.

Furthermore, for any arbitrary compensation scheme, an agent works harder if he faces an opponent with the same valuation for promotion, i.e., \( e^*_A < e^*_{AA} \) and \( e^*_B < e^*_{BB} \). Intuitively, different valuations for the management job imply different effort levels, which in turn gives the harder working agent an advantage in the competition for promotion. As a result, increasing effort is less beneficial for both types of agents in an \( AB \)-match. This implies that type \( B \)’s effort is even further below the first-best effort level when he competes with a type \( A \) agent for promotion. By contrast, whether type \( A \)’s effort level in the heterogenous tournament, \( e^*_A \), is closer to or further from the first-best effort level than his effort in the homogenous tournament, \( e^*_{AA} \), depends on the specific functional forms.

It is also interesting to have a closer look at the promotion probability of a type \( B \) worker in an \( AB \)-match, which is given by \( 1 - G(e^*_A - e^*_B) \), and hence, positive. In contrast, if the principal could observe the agents’ types, she would never select the type \( B \) agent for the management position. Therefore, type \( B \)’s promotion probability is inefficiently high under the contract solving the optimization problem (15). Now suppose that the principal is still not able to observe the agents’ types, but knows whether a heterogenous or homogenous match occurred. In the former case, she would induce the effort levels \( \hat{e}_A \) and \( \hat{e}_B \) that maximize \( \pi_{AB} \) as given by (16). More specifically, we now compare \( e^*_A \) and \( e^*_B \) (heterogenous match) with a benchmark that accounts for the fact that type \( A \) implementing a higher effort level than type \( B \) improves selection, and is therefore beneficial to the principal. The benchmark effort levels \( \hat{e}_A \) and \( \hat{e}_B \) are implicitly given by

\[
\begin{align*}
\hat{c}'(\hat{e}_A) &= 1 + g(\Delta\hat{e})(\Pi_A - \Pi_B + \delta), \\
\hat{c}'(\hat{e}_B) &= 1 - g(\Delta\hat{e})(\Pi_A - \Pi_B + \delta).
\end{align*}
\]

\(^{21}\)Note that effort would be first-best in case of a homogeneous tournament match.
Clearly, to maximize $\pi_{AB}$, the principal prefers the type $A$ agent to work harder than type $B$, i.e., $\hat{e}_A > \hat{e}_B$. Moreover, we have $\hat{e}_A > e^{FB} > \hat{e}_B$. The next corollary points out that type $B$’s promotion probability is not only too high relative to the first-best solution, but also compared to the previously defined benchmark.

**Corollary 1** Compared to the benchmark effort levels $\hat{e}_A$ and $\hat{e}_B$, type $A$ works too little while type $B$ works too hard in an $AB$-match, i.e.,

$$e^*_A < \hat{e}_A \text{ and } \hat{e}_B < e^*_B.$$  

Thus, type $B$’s promotion probability is inefficiently high relative to the benchmark promotion probability, i.e., $1 - G(\hat{e}_A - \hat{e}_B) < 1 - G(e^*_A - e^*_B)$.

There are two driving forces behind this result. First, the principal is simply not able to induce the benchmark effort levels $\hat{e}_A$ and $\hat{e}_B$ because she cannot contract upon effort, and is therefore not able to provide agents with the appropriate effort incentives.\(^{22}\)

Nevertheless, it would still be possible for the principal to induce the benchmark promotion probability, which is characterized by the effort difference $\Delta \hat{e} = \hat{e}_A - \hat{e}_B$. Here, the second driving force comes into play: the conflict between incentive provision and selection. To implement the benchmark effort difference $\Delta \hat{e}$, both agents’ effort levels would have to be either higher or lower than $\hat{e}_A$ and $\hat{e}_B$, respectively. Clearly, this is not optimal from an incentive perspective. The principal therefore compromises on selection, and induces a lower effort difference by making the type $A$ agent working less, and the type $B$ agent working harder than in the benchmark case. This in turn implies that type $B$ will be promoted too often compared to the benchmark. The principal deliberately accepts this inefficiency as a necessary consequence of balancing incentive and selection effects appropriately.

The next proposition points out how the principal adjusts the contract elements $r$ and $w_2$ when selection becomes more important.

**Proposition 2** Suppose the principal’s problem has an interior solution. If $c''(e^*_A) > c''(e^*_B)$, the principal offers a lower piece rate $r^*$ and a higher management wage $w_2^*$ when $\Pi_A - \Pi_B$ increases, i.e., when assigning a type $A$ agent to the management position becomes more desirable. Overall, the induced effort levels $e^*_A$ and $e^*_B$ decrease, while the induced effort difference

\(^{22}\)Formally, the incentive compatibility constraints for the heterogeneous match, (10) and (11), imply that $c'(e^*_A) - c'(e^*_B) = g(\Delta e^*)\delta$. It then follows from (20) and (21) that $c'(e^*_A) - c'(e^*_B) \neq c'(\hat{e}_A) - c'(\hat{e}_B)$.  

18
\(\Delta e^* = e^*_A - e^*_B\) increases. As a consequence, in a heterogeneous tournament match, type A’s promotion probability increases. The effort levels in homogeneous tournaments, \(e^*_{AA}\) and \(e^*_{BB}\), remain unchanged.

Recall from Lemma 1 that both lowering the piece rate \(r\) and the management wage \(w_2\) improves selection if the harder working type A agent responds less strongly to intensified effort incentives than the type B agent (i.e., \(c''(e^*_A) > c''(e^*_B)\)). Why, then, does the principal decrease \(r^*\) and increase \(w^*_2\) when promoting a type A agent becomes more important? The answer can be found in the optimal incentive structure for the homogeneous tournaments. Since selection is irrelevant in these matches, the implemented effort levels should be independent of \(\Pi_A - \Pi_B\). Indeed, the adjustments of \(r^*\) and \(w^*_2\) are such that \(e^*_{AA}\) and \(e^*_{BB}\) remain constant. This can only be achieved if any change in effort incentives caused by a higher piece rate \(r\) is offset by adjusting the management wage \(w_2\) appropriately. According to the incentive compatibility constraints for the homogeneous tournament matches, (4) and (5), an agent’s marginal benefit from raising effort increases by 1 under a marginally higher piece rate \(r\). By contrast, the marginal benefit from implementing more effort under a marginally higher management wage \(w_2\) increases by \(g(0)\). Thus, the effects on effort from adjusting the piece rate and the management wage just cancel out if

\[
\frac{\partial r^*}{\partial(\Pi_A - \Pi_B)} + g(0) \frac{\partial w^*_2}{\partial(\Pi_A - \Pi_B)} = 0. \tag{22}
\]

An increase in \(r^*\) must therefore be accompanied by a lower \(w^*_2\), and vice versa.

In the heterogeneous tournament match, an increase of the effort difference \(\Delta e^*\) – which aims at improving selection – is achieved when overall effort incentives decrease. We can infer from the respective incentive compatibility constraints, (10) and (11), that this is the case if

\[
\frac{\partial r^*}{\partial(\Pi_A - \Pi_B)} + g(\Delta e^*) \frac{\partial w^*_2}{\partial(\Pi_A - \Pi_B)} < 0. \tag{23}
\]

Since \(g(\cdot)\) is single-peaked at zero, we have \(g(\Delta e^*) < g(0)\). This implies that agents in the heterogeneous tournament match respond less strongly to adjustments of the management wage \(w_2\) than agents in a homogeneous match. Intuitively, since heterogeneous agents exert different effort levels, their promotion probabilities are less sensitive to changes in effort than those of homogeneous agents. Consequently, a higher management wage \(w_2\) has a weaker incentive effect when agents differ with respect to their management skills. Thus, given that \(r^*\) and \(w^*_2\) are adjusted such that (22) holds, the effect on the effort levels in a heterogeneous match is
determined by the sign of the change in \( r \) rather than \( w_2 \). As a result, a reduction in \( e^*_A \) and \( e^*_B \) – while holding \( e^*_{AA} \) and \( e^*_{BB} \) constant – can only be achieved by lowering the piece rate \( r^* \) and raising the management wage \( w^*_2 \).

The driving force behind this result is the tension between incentive and selection considerations, which can be summarized as follows: the principal knows that selection is not always important. Sometimes, the principal hires production workers who would be equally good managers (\( AA \)- or \( BB \)-match), but sometimes production workers differ substantially in their management skills (\( AB \)-match). In the latter case, the compensation scheme should support the selection of good managers. At the same time, however, effort incentives for homogeneous workers should not too heavily be distorted. If \( c''(e^*_A) > c''(e^*_B) \), this objective can be achieved by offering both workers a lower piece rate and a higher management wage whenever selection becomes more important (i.e., \( \Pi_A - \Pi_B \) increases).

Now we impose the additional (and certainly realistic) restriction that piece rates should be non-negative. We then obtain the following result.

**Corollary 2** Suppose that the principal prefers to restrict the piece rate scheme \( r \) to non-negative values. If \( c''(e_A) > c''(e_B) \) for all effort levels \( e_A \) and \( e_B \) with \( e_A > e_B \), and \( \Pi_A - \Pi_B \) is sufficiently large, then the principal does not provide individual performance pay, i.e., \( r^* = 0 \).

Corollary 2 implies that the principal refrains from providing the agents with individual performance pay if selecting high-ability managers is sufficiently important, and the effort choice of these already harder working high-ability types is less sensitive to intensified effort incentives. In this case, the introduction of individual performance pay would encourage low-ability types to catch up, and thus, to increase their chances of promotion.

Proposition 2 and Corollary 2 suggest that the difference between workers with respect to their potential management skills and the resulting effect on firm profits – quantified by \( \Pi_A - \Pi_B \) – is the main driving force for the firm-specific combination of promotion tournaments and individual performance pay. It is therefore essential to identify firm characteristics which are likely to affect the importance of managerial ability for firm success, and thus determine the

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\[23\] We focus in our explanations on the case \( c''(e^*_A) > c''(e^*_B) \) because we believe it is more relevant for a situation captured by our framework (see also our explanations in Section 3). Note, however, that our results would be reversed in case \( c''(e^*_A) < c''(e^*_B) \): the principal would then offer a higher piece rate \( r^* \) and a lower management wage \( w^*_2 \) if selection becomes more important. In the special case where \( c''(e^*_A) = c''(e^*_B) \), the principal is not able to affect \( \Delta e^* \) by adjusting the employment contract. Then, \( \Delta e^* \) is constant for all \( r \) and \( w_2 \), and therefore, each employment contract has the same selection properties.
properties of firm-internal incentives schemes. This, of course, first requires to identify the
types of firms which are captured by our framework. Recall that we focus on the two lowest
hierarchy levels of a firm, where contractible individual performance measures are available on
the first level (production stage). Moreover, the production task differs substantially from the
management job on the second tier. Consequently, performance of production workers is not
per se an informative signal about their individual management skills. Finally, we considered
firms where firm-specific human capital is crucial for managerial success, and these firm thus
prefer internal promotions to external recruitment. These characteristics suggest that our results
primarily (but not exclusively) apply to manufacturing firms such as input suppliers or produc-
ers of consumer goods. This can be deduced because in manufacturing firms, (i) it is frequently
possible to measure workers’ individual output, (ii) production and management typically re-
quire fundamentally different skills; and (iii), managers usually need extensive knowledge on
the firm’s specific production process to perform their tasks effectively. Firms of the service
industry that also fit our framework are, e.g., parcel services, car repair shops, fast food chains,
or call centers.24

Our analysis specifically implies that a high importance of managerial ability (i.e., a large
difference $\Pi_A - \Pi_B$) is associated with high wage differentials across hierarchy levels,25 and
low-powered individual performance pay on lower levels. The opposite applies if $\Pi_A - \Pi_B$
is low. The difference $\Pi_A - \Pi_B$ is likely to be large in firms whose profits are highly sensi-
tive to managerial performance. This is the case whenever a firm’s success crucially hinges on
managers’ individual expertise and abilities. This particularly applies to firms that produce com-
plex, custom-tailored goods (e.g., input suppliers in the automobile or aircraft industry). In such
firms, middle managers do not simply supervise a standardized production process, but perform
substantially more involved tasks. They need to be capable of coordinating the implementation
of customer-specific solutions, which may require frequent and timely adjustments of the
production process. Moreover, managers in these firms must serve as intermediaries between
manufacturing and product development. Therefore, they should be able to early recognize un-

24 By contrast, our results are not applicable to firms where team work is prevalent, and therefore, individual
performance measures are hardly available. Also, our model is not suitable to explain compensation schemes in
firms in the service industry such as consultancies and law firms. The reason is that, in these firms, tasks are
very similar across different hierarchy levels (e.g., associate level vs. partner). Therefore, the performance of
employees on lower levels serves as a good signal about their suitability for promotion. Thus, ability with respect
to management jobs does not remain private information of employees.

25 By using the binding participation constraint (13), it can be shown that the optimal fixed payment $w^*_1$ for the
production stage is decreasing in $\Pi_A - \Pi_B$. Thus, the wage differential $\Delta w \equiv w^*_2 - w^*_1$ is increasing in $\Pi_A - \Pi_B$. 
foreseen problems on the production floor, and to communicate those problems effectively to higher tiers of the firm. Similarly, \( \Pi_A - \Pi_B \) is large for firms where poor performance of lower management can have severe detrimental effects on the firm’s reputation, and consequently, on firm profits. For example, the reputation of an entire fast food chain suffers if only a few restaurants offer poor food or service. The promotion of able managers is also crucial to parcel services, whose success is based on a reputation for timely delivery.

Clearly, high individual skills of managers are critical for these firms as poor managerial decision making can have substantial detrimental effects on their performance, corresponding to a large loss of \( \Pi_A - \Pi_B \) in our framework. Since promoting highly skilled employees to management positions is of vital importance for these firms, our framework offers the following two predictions regarding the properties of their internal incentive schemes: First, the compensation structure within these firms is characterized by comparatively high wage differentials across hierarchy levels. As shown, this does not only provide effort incentives to employees on lower levels, but also facilitates the selection of suitable employees for promotion. Second, these firms can be expected to provide only low-powered individual incentives to employees on lower levels, or even refrain from offering them individual performance pay. Our analysis shows that this adjustment improves the selection effect of promotion tournaments, and thus, increases the chance of filling management positions with highly skilled employees.

The opposite predictions can be made for firms whose profits are less sensitive to managerial performance, which relates to a small difference \( \Pi_A - \Pi_B \) in our model. This can mainly be expected for firms specialized in producing highly standardized products, where a manager is primarily required to supervise the production process (e.g., the installation of windshields as in the firm analyzed by Lazear [2000]). Hence, managerial tasks are less demanding compared to firms producing custom-tailored inputs, and consequently, require substantially less skills on the side of managers. As a result, differences in managers’ individual skills and expertise are relatively less critical for the performance of these firms.

To summarize, our previous argumentation highlights that the complexity of managerial tasks is likely to affect the properties of firm-internal compensation schemes. This further suggests that firms, whose managers are in charge of supervising many subordinates responsible for conducting very diverse tasks (which clearly implies a high task-complexity for managers), should put more emphasis on promotion tournaments relative to individual performance pay in order to improve the selection of highly skilled managers. Indeed, there is empirical support
for this prediction: Rajan and Wulf [2003] find that ‘flat’ firms (i.e., firms with only a few hierarchy levels) are characterized by higher wage differentials across levels, and lower-powered individual performance pay at lower levels, in comparison to ‘deep’ hierarchical firms.\(^{26}\)

Finally, the importance of selecting suitable employees for management positions is not only determined by the previously emphasized firm-specific characteristics, but also by the heterogeneity of employees with respect to their individual management skills (i.e., employee-specific characteristics). A low heterogeneity in terms of these skills, resulting in a small difference \(\Pi_A - \Pi_B\), may be expected for highly educated employees, presuming that education harmonizes managerial skills (e.g., organizing and communication ability) among individuals. Because selection is then of comparatively little importance, our model predicts that those employees face relatively small wage differentials when moving up the corporate hierarchy, and receive high-powered individual performance pay. Indeed, several empirical studies suggest that individual performance pay is more prevalent in firms with highly-educated workforces (see, e.g., Tremblay and Chenevert [2005], Long and Shields [2005], Barth et al. [2008]).

5 Extensions and Discussion

In this section, we discuss the effect of some of our assumptions on the results stated above.

So far, we have analyzed a situation where both types of agents have identical skills in production. Now suppose instead that type \(A\) has effort costs \(\alpha c(e_i), \alpha > 0\), at the production stage, whereas type \(B\)’s effort costs are still \(c(e_j)\). If \(\alpha = 1\), we are clearly in the case of equally skilled agents as analyzed above. If \(\alpha > 1\), type \(A\) is a better manager but a worse production worker than type \(B\). This reflects a situation where agents have different abilities and/or preferences for different tasks. For example, type \(B\) might be satisfied with a job in production because he prefers simple tasks, but would dislike a more responsible and demanding occupation. By contrast, type \(A\) may be more ambitious and perceives simple tasks as stultifying. He has therefore higher marginal cost of effort for the production task than type \(B\).\(^{27}\) If, on the other hand, \(\alpha < 1\), type \(A\) is not only the better manager, but also the more efficient production worker. This corresponds to a situation where good types are characterized by higher produc-

\(^{26}\)Moreover, Rajan and Wulf [2003] find that divisional managers in ‘flat’ firms have relatively greater authority, which reinforces our conjecture that managerial tasks are more complex in these firms.

\(^{27}\)For our framework to remain meaningful, we assume that \(B\)’s cost advantage in production is not too strong such that, in equilibrium, type \(A\) still chooses a higher effort level. Otherwise, the principal would always want to promote the agent with the lower output, who is then more likely of type \(A\).
tivity in each job. For our framework, we perceive the case $\alpha \geq 1$ to be more relevant since then the incentive and selection aspect of the promotion tournament is crucial. The reason is that – without employing a promotion tournament – type $A$ would have only low effort incentives at the production stage. We proceed by showing that all our results continue to hold for $\alpha \geq 1$. In contrast, for $\alpha < 1$, our results remain if $\alpha$ is not too small.

For a heterogenous tournament match, it is straightforward to verify that type $A$’s effort is decreasing, and type $B$’s effort is increasing in $\alpha$. Thus, ceteris paribus, a higher value of $\alpha$ deteriorates selection. However, the principal can counteract this effect by adjusting the incentive instruments $w_2$ and $r$ appropriately. Analogous to the condition emphasized by Lemma 1, type $A$’s promotion probability is decreasing in $w_2$ and $r$ if and only if

$$\alpha c''(e_A) > c''(e_B).$$  \hspace{1cm} (24)$$

Consequently, if this condition holds for the optimal effort levels $e_A = e^*_A$ and $e_B = e^*_B$, the principal still decreases $r$ and increases $w_2$ when selecting a type $A$ agent for the management position becomes more important. Thus, the results of Lemma 1 and Proposition 2 immediately extend if we replaced the condition $c''(e_A) > c''(e_B)$ with condition (24).\footnote{The derivation of this condition is equivalent to the derivation of equation (29) in the Appendix.} Clearly, condition (24) is more likely to hold if $\alpha$ is large. The reason is that, the higher $A$’s marginal effort costs compared to $B$’s, the less sensitive is $A$’s effort to intensified incentives relative to $B$’s effort. In particular, provided that (24) holds for identical skills in production as we assumed above (i.e., if $c''(e_A) > c''(e_B)$), condition (24) will continue to hold for $\alpha > 1$. Therefore, if agents are either good managers or good production workers, our results are reinforced. Intuitively, since type $A$ is a worse production worker compared to type $B$, intensifying incentives leads to an even stronger deterioration of the selection effect than for identical abilities in production.

Now consider the case $\alpha < 1$ and assume that (24) holds for $\alpha = 1$. A lower $\alpha$ has then two opposite effects. On the one hand, the effort difference $e_A - e_B$ increases. Then, type $A$ works much harder than type $B$, which could even further decrease the former’s relative responsiveness to intensified effort incentives. On the other hand, type $A$ has lower marginal costs, which counteracts the first effect. Overall, our results will continue to hold if $\alpha$ is sufficiently close to one. For small enough values of $\alpha$, however, (24) will be violated. In this case, our results

\footnote{The results summarized by Proposition 1 continue to hold for all values of $\alpha$, and are independent of whether condition (24) is satisfied or not. The only exception is, if agents differ in their skills for the manufacturing task, that $\hat{e}_{AA} \neq \hat{e}_{BB}$.}
from Lemma 1 and Proposition 2 are reversed: intensifying incentives then increases the effort difference $e_A - e_B$, and thus, improves selection. As a consequence, $r^*$ is then increasing and $w^*_2$ decreasing in $\Pi_A - \Pi_B$. Intuitively, if $\alpha$ is small, type $A$ already enjoys a significantly higher ability at the production stage. The incentive and selection effect of the promotion tournament, which stems from type $A$’s higher ability with respect to the the management job, then becomes less important. The principal therefore puts more emphasis on individual incentives at the production stage. The same is true when we return to the case of identical agents in production ($\alpha = 1$), but now assume that the harder working agent’s effort is more responsive to enhanced incentives (i.e., $c''(e_A) < c''(e_B)$).

Finally, in our model, being of a superior type with respect to the management task means to have a higher productivity and lower effort costs. Typically, only one of these assumptions is made to differentiate between types of agents. However, to ensure that the principal prefers to promote a type $A$ agent and that a type $A$ agent has a higher valuation for promotion than a type $B$ agent, we need to impose both assumptions. This is because we only allow for a fixed management wage. However, in a framework where incentive contracts for managers are feasible, it would be sufficient that type $A$ has either a higher marginal productivity or lower marginal effort costs. Then, at the end of the first period, the principal offers the promoted agent a menu of contracts as in a standard adverse selection model. Regardless of being more productive or more cost efficient, a high-skilled type earns a higher rent than a low-skilled type under their respective preferred contracts. Therefore, type $A$ still benefits relatively more from being employed as a manager. Furthermore, despite extracting a higher rent, type $A$ contributes relatively more to firm value. The principal therefore benefits from using a promotion tournament which increases the likelihood of selecting a type $A$ agent for the management position. If selection is sufficiently important, it should still be the case that the principal refrains from offering a piece rate scheme for the manufacturing task.

6 Conclusion

We investigate the optimal combination of linear individual performance pay and promotion tournaments which is aimed at motivating high effort (motivation challenge) and, concurrently, facilitating efficient job assignments (selection challenge). We find that individual performance pay and promotion tournaments are substitutes in the provision of effort incentives. The specific
intensity of individual performance pay, however, is determined by the relative importance of the selection effect of promotion tournaments for firm performance. In our analysis, we focus on a situation where harder working employees are relatively less responsive to intensified effort incentives. Then, the more important it is to promote the most suitable worker to the management position, the higher is the management wage, and the lower-powered are individual effort incentives. Moreover, although contractible performance measures are available at the production stage, we find that it can be even optimal for a firm to refrain from providing individual performance pay if the efficient job assignment is sufficiently crucial for firm performance. This is because individual rewards dilute the selection effect of promotion tournaments.

In our analysis, we focus on firms which utilize internal promotion tournaments to fill vacancies in management, rather than hiring potential candidates from the external labor market. As emphasized in the Introduction, firms predominantly use their internal labor markets when the acquisition of firm-specific human capital is a crucial factor for managerial performance. For these firms, our model predicts a negative relationship between individual performance pay and wage differentials across hierarchy levels. Moreover, the specific combination of both incentive schemes is determined by the importance of individual skills of employees for their respective performance as managers. If individual skills have a significant effect on managerial performance, higher wage differentials – which constitute prizes for internal promotion tournaments – are accompanied by low-powered individual performance pay at lower-level jobs. This observation is consistent with several empirical studies (see, e.g., Baker et al. [1988] and further references therein) which find that promotion-based incentive schemes are predominant in most firms. For these firms, our analysis suggests that management skills – and therefore efficient job assignments – may be particularly important for firm performance.

\footnote{In our framework, if firm-specific human capital played no role and hence, internal promotion was not important, the firm would induce first-best effort on the production stage by setting $r = 1$ and $w_2 = -\delta$. Such a management wage implies that only a type $A$ worker is willing to work as a manager. In case of a $BB$-match, no worker is willing to fill the management position. The firm then recruits a manager from the external labor market by offering the wage $-p\delta$ to an arbitrarily chosen candidate.}
7 Appendix

Proof of Lemma 1. Using matrix notation, implicit differentiation of (10) and (11) yields

\[
H \begin{pmatrix} \frac{\partial e_A}{\partial r} \\ \frac{\partial e_B}{\partial r} \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix},
\]

(25)

where

\[
H := \begin{pmatrix} g'(\Delta e)(w_2 + \delta) - c''(e_A) & -g'(\Delta e)(w_2 + \delta) \\ g'(\Delta e)w_2 & -g'(\Delta e)w_2 - c''(e_B) \end{pmatrix}.
\]

(26)

Since \( \Delta e > 0 \), we have \( g'(\Delta e) < 0 \). Together with (6), (7), and \(-g'(\Delta e) = g'(-\Delta e)\) it follows that \( \det(H) > 0 \). Applying Cramer’s Rule to (25) yields

\[
\frac{\partial e_A}{\partial r} = \frac{-g'(\Delta e)\delta + c''(e_B)}{\det(H)} > 0,
\]

(27)

\[
\frac{\partial e_B}{\partial r} = \frac{-g'(\Delta e)\delta + c''(e_A)}{\det(H)} > 0.
\]

(28)

Consequently,

\[
\frac{\partial \Delta e}{\partial r} = \frac{c''(e_B) - c''(e_A)}{\det(H)}.
\]

(29)

If \( c''(e_B) < c''(e_A) \), \( \Delta e \) is strictly decreasing in \( r \). Otherwise, \( \Delta e \) is (weakly) increasing in \( r \). Applying the same procedure with respect to \( w_2 \), we obtain

\[
\frac{\partial e_A}{\partial w_2} = g(\Delta e) \frac{\partial e_A}{\partial r},
\]

(30)

\[
\frac{\partial e_B}{\partial w_2} = g(\Delta e) \frac{\partial e_B}{\partial r},
\]

(31)

and thus, Lemma 1 follows. \( \square \)

Proof of Proposition 1. Let \( \mathcal{L} \) denote the Lagrangian of problem (15), and \( \lambda_1, \ldots, \lambda_4 \) the Lagrange multipliers for the constraints (4), (5), (10), and (11), respectively. The corresponding
first-order conditions are

\[
\frac{\partial L}{\partial r} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 0, \tag{32}
\]

\[
\frac{\partial L}{\partial w_2} = [\lambda_1 + \lambda_2] \cdot g(0) + [\lambda_3 + \lambda_4] \cdot g(\Delta e^*) = 0, \tag{33}
\]

\[
\frac{\partial L}{\partial e_A} \bigg|_{e_A = e_A^*} = 2p(1 - p) \left[ 1 + g(\Delta e^*)(\Pi_A - \Pi_B + \delta) - c'(e_A^*) \right] + \lambda_3 \left[ g'(\Delta e^*)(w_2 + \delta) - c''(e_A^*) \right] + \lambda_4 g(\Delta e^*)w_2 = 0, \tag{34}
\]

\[
\frac{\partial L}{\partial e_B} \bigg|_{e_B = e_B^*} = 2p(1 - p) \left[ 1 - g(\Delta e^*)(\Pi_A - \Pi_B + \delta) - c'(e_B^*) \right] - \lambda_3 g'(\Delta e^*)(w_2 + \delta) - \lambda_4 \left[ g'(\Delta e^*)w_2 + c''(e_B^*) \right] = 0, \tag{35}
\]

\[
\frac{\partial L}{\partial e_{AA}} \bigg|_{e_{AA} = e_{AA}^*} = 2p^2 \left[ 1 - c'(e_{AA}^*) \right] - \lambda_1 c''(e_{AA}^*) = 0, \tag{36}
\]

\[
\frac{\partial L}{\partial e_{BB}} \bigg|_{e_{BB} = e_{BB}^*} = 2(1 - p)^2 \left[ 1 - c'(e_{BB}^*) \right] - \lambda_2 c''(e_{BB}^*) = 0. \tag{37}
\]

Since \(g(\Delta e^*) < g(0)\), it follows from (32) and (33) that \(\lambda_1 + \lambda_2 = \lambda_3 + \lambda_4 = 0\). Suppose for a moment that \(\lambda_1 = \lambda_2 = 0\). In this case, (36) and (37) imply that \(c'(e_{AA}^*) = c'(e_{BB}^*)\), which is a contradiction to (4) and (5). Thus, \(\lambda_1 = -\lambda_2 \neq 0\). Consequently, by (36) and (37), \(1 - c'(e_{AA}^*)\) and \(1 - c'(e_{BB}^*)\) must have opposite signs. Moreover, (4) and (5) entail \(c'(e_{AA}^*) > c'(e_{BB}^*)\). Therefore, \(1 - c'(e_{AA}^*) < 0\) and \(0 < 1 - c'(e_{BB}^*)\). Since \(c'(e_{FB}^*) = 1\), this implies that \(e_{FB}^* < e_{AA}^*\) and \(e_{BB}^* < e_{FB}^*\). By comparing the incentive compatibility constraints for the \(BB\)-match and the \(AB\)-match, (5) and (11), we can infer that, \(e_B^* < e_{BB}^*\). Hence, \(e_B^* < e_{FB}^*\). Similarly, (4) and (10) imply \(e_A^* < e_{AA}^*\). However, depending on the specific functional forms, \(e_A^*\) can be smaller or greater than \(e_{FB}^*\). \(\square\)

**Proof of Corollary 1.** Consider the first-order conditions for the principal’s problem (15) w.r.t. \(e_A\) and \(e_B\), i.e., equations (34) and (35). Suppose for a moment that \(\lambda_3 = \lambda_4 = 0\). Then, (34) and (35) in conjunction with (4) and (5) imply

\[
1 + g(\Delta e^*)(\Pi_A - \Pi_B + \delta) = g(\Delta e^*)(w_2 + \delta) + r, \tag{38}
\]

\[
1 - g(\Delta e^*)(\Pi_A - \Pi_B + \delta) = g(\Delta e^*)w_2 + r. \tag{39}
\]

Subtracting the second equation from the first equation yields \(2(\Pi_A - \Pi_B) + \delta = 0\), which is a contradiction to \(\Pi_A > \Pi_B\) and \(\delta > 0\). Thus, \(\lambda_3 = -\lambda_4 \neq 0\). Using this observation, (34) and
(35) can be transformed to

\[
2p(1 - p) \left[ 1 + g(\Delta e^*)(\Pi_A - \Pi_B + \delta) - c'(e_A^*) \right] + \lambda_3 \left[ g'(\Delta e^*)\delta - c''(e_A^*) \right] = 0, \quad \text{or} \quad F_1 = 0. \tag{40}
\]

\[
2p(1 - p) \left[ 1 - g(\Delta e^*)(\Pi_A - \Pi_B + \delta) - c'(e_B^*) \right] - \lambda_3 \left[ g'(\Delta e^*)\delta - c''(e_B^*) \right] = 0. \quad \text{or} \quad G_2 = 0. \tag{41}
\]

Applying (10) and (11) yields

\[
F_1 = 1 + g(\Delta e^*)(\Pi_A - \Pi_B - w_2) - r; \quad G_1 = 1 - g(\Delta e^*)(\Pi_A - \Pi_B + w_2 + \delta) - r. \tag{42}
\]

Hence, \( F_1 > G_1 \). Furthermore, we can infer from (6) and (7) that \( F_2 \) and \( G_2 \) have opposite signs, so that the same must be true for \( F_1 \) and \( G_1 \). As a result, \( F_1 > 0 > G_1 \), implying \( e_A^* < \hat{e}_A \) and \( \hat{e}_B < e_B^* \).

**Proof of Proposition 2.** The principal’s problem (15) can be further simplified to

\[
\max_{r, w_2} \Pi(r, w_2) \equiv \pi_{AB}(r, w_2) + \pi_{AA}(r, w_2) + \pi_{BB}(r, w_2), \tag{44}
\]

where \( \pi_{AB}(r, w_2) \), \( \pi_{AA}(r, w_2) \), and \( \pi_{BB}(r, w_2) \) are defined as in (16)-(18). The only difference is that \( e_{AA}, e_{BB}, e_A, \) and \( e_B \) are now expressed as functions of \( r \) and \( w_2 \), which are implicitly characterized by (4), (5), (10), and (11), respectively. We assume that the functional forms are such that \( \Pi(r, w_2) \) is concave for all \( p \in (0, 1) \).

Provided that the optimal solution is an interior one, the optimal piece rate \( r^* \) and management wage \( w_2^* \) are characterized by the first-order conditions

\[
\frac{\partial \Pi}{\partial r} = \frac{\partial \pi_{AB}}{\partial r} + \frac{\partial \pi_{AA}}{\partial r} + \frac{\partial \pi_{BB}}{\partial r} = 0, \tag{45}
\]

\[
\frac{\partial \Pi}{\partial w_2} = \frac{\partial \pi_{AB}}{\partial w_2} + \frac{\partial \pi_{AA}}{\partial w_2} + \frac{\partial \pi_{BB}}{\partial w_2} = 0. \tag{46}
\]

\[31\text{Due to the complexity of this problem, it is not possible to provide a meaningful general condition for } \Pi(r, w_2) \text{ to be concave. However, it follows from the incentive compatibility constraints for } e_{AA}^* \text{ and } e_{BB}^* \text{ that it is sufficient for } \pi_{AA}(r, w_2) \text{ and } \pi_{BB}(r, w_2) \text{ to be concave if } c'' \text{ is positive but not too large.} \]
For \( y \in \{r, w_2\} \) we obtain
\[
\frac{\partial \pi_{AB}}{\partial y} = 2p(1-p) \left[ (1 - c'(e_A^*)) \frac{\partial e_A^*}{\partial y} + (1 - c'(e_B^*)) \frac{\partial e_B^*}{\partial y} \right. \\
\left. + g(\Delta e^*) \frac{\partial (\Delta e^*)}{\partial y} (\Pi_A - \Pi_B + \delta) \right],
\]
(47)
\[
\frac{\partial \pi_{AA}}{\partial y} = 2p^2[1 - c'(e_{AA}^*)] \frac{\partial e_{AA}^*}{\partial y},
\]
(48)
\[
\frac{\partial \pi_{BB}}{\partial y} = 2(1-p)^2[1 - c'(e_{BB}^*)] \frac{\partial e_{BB}^*}{\partial y}.
\]
(49)

Then, by using matrix notation, (45) and (46) imply
\[
K \begin{pmatrix}
\frac{\partial r^*}{\partial (\Pi_A - \Pi_B)} \\
\frac{\partial w_2^*}{\partial (\Pi_A - \Pi_B)}
\end{pmatrix} = \begin{pmatrix}
-2p(1-p)g(\Delta e^*) \frac{\partial (\Delta e^*)}{\partial r} \\
-2p(1-p)g(\Delta e^*) \frac{\partial (\Delta e^*)}{\partial w_2}
\end{pmatrix},
\]
(50)
where
\[
K := \begin{pmatrix}
\frac{\partial^2 \Pi}{\partial r^2} & \frac{\partial^2 \Pi}{\partial r \partial w_2} \\
\frac{\partial^2 \Pi}{\partial w_2^2} & \frac{\partial^2 \Pi}{\partial w_2^2}
\end{pmatrix}.
\]
(51)

From (30) and (31) it follows that \( \frac{\partial (\Delta e^*)}{\partial w_2} = g(\Delta e^*) \frac{\partial (\Delta e^*)}{\partial r} \). Using this relationship and applying Cramer’s Rule to (50) yields
\[
\frac{\partial r^*}{\partial (\Pi_A - \Pi_B)} \det(K) = 2p(1-p)g(\Delta e^*) \frac{\partial (\Delta e^*)}{\partial r} \left[ g(\Delta e^*) \frac{\partial^2 \Pi}{\partial r \partial w_2} - \frac{\partial^2 \Pi}{\partial w_2^2} \right],
\]
(52)
\[
\frac{\partial w_2^*}{\partial (\Pi_A - \Pi_B)} \det(K) = 2p(1-p)g(\Delta e^*) \frac{\partial (\Delta e^*)}{\partial r} \left[ \frac{\partial^2 \Pi}{\partial r \partial w_2} - g(\Delta e^*) \frac{\partial^2 \Pi}{\partial r^2} \right].
\]
(53)

These expressions can be transformed to \(^{32}\)
\[
\frac{\partial r^*}{\partial (\Pi_A - \Pi_B)} \det(K) = -2p(1-p)g(\Delta e^*) \frac{\partial (\Delta e^*)}{\partial r} \left[ g(0)[g(0) - g(\Delta e^*)] \left[ \frac{\partial^2 \pi_{AA}}{\partial r^2} + \frac{\partial^2 \pi_{BB}}{\partial r^2} \right] \right]
\]
(54)

\(^{32}\)The derivation is available from the authors upon request.
\[
\frac{\partial w_2^*}{\partial (\Pi_A - \Pi_B)} \det(K) = \\
2p(1 - p)g(\Delta e^*) \frac{\partial(\Delta e^*)}{\partial r} [g(0) - g(\Delta e^*)] \left[ \frac{\partial^2 \pi_{AA}}{\partial r^2} + \frac{\partial^2 \pi_{BB}}{\partial r^2} \right].
\] (55)

Since \( \Pi \) is concave, \( K \) must be negative definite. Thus, \( \det(K) > 0 \). According to Lemma 1, \( \frac{\partial \Delta e^*}{\partial r} < 0 \) if \( c''(e_A^*) > c''(e_B^*) \). Furthermore, since \( \Pi = \pi_{AA} \) for \( p = 1 \) and \( \Pi = \pi_{BB} \) for \( p = 0 \), concavity of \( \Pi \) for all \( p \in (0,1) \) implies concavity of \( \pi_{AA} \) and \( \pi_{BB} \). Thus, \( \frac{\partial^2 \pi_i}{\partial r^2} < 0 \) for \( i = A, B \). Overall, we therefore obtain

\[
\frac{\partial r^*}{\partial (\Pi_A - \Pi_B)} < 0, \quad \frac{\partial w_2^*}{\partial (\Pi_A - \Pi_B)} > 0.
\] (56)

From equations (54) and (55) it follows immediately that

\[
\frac{\partial r^*}{\partial (\Pi_A - \Pi_B)} = -\frac{g(0)}{\partial \Pi_A} - \frac{\partial w_2^*}{\partial (\Pi_A - \Pi_B)}.
\] (57)

Using (57) in a comparative statics analysis applied to (10) and (11), it can be easily verified that \( \frac{\partial e_A^*}{\partial (\Pi_A - \Pi_B)} \cdot \frac{\partial e_B^*}{\partial (\Pi_A - \Pi_B)} < 0 \), and \( \frac{\partial (\Delta e^*)}{\partial (\Pi_A - \Pi_B)} > 0 \). Moreover, using (57) in conjunction with (4) and (5), it is straightforward to verify that \( \frac{\partial e_A^*}{\partial (\Pi_A - \Pi_B)} = \frac{\partial e_B^*}{\partial (\Pi_A - \Pi_B)} = 0. \) \( \Box \)

**Proof of Corollary 2.** First, recall that \( \frac{\partial \Delta e^*}{\partial r} < 0 \). Then, we can infer from (45) and (47)-(49) that there exists a pair \( \bar{\Pi}_A, \bar{\Pi}_B \) such that \( \max_{w_2} \frac{\partial \Pi}{\partial r} \big|_{r=0} < 0 \) for all \( \Pi_A - \Pi_B > \bar{\Pi}_A - \bar{\Pi}_B \). Since \( \Pi \) is concave, \( \frac{\partial \Pi}{\partial r} \) is decreasing in \( r \) for all \( w_2 \). Thus, \( \frac{\partial \Pi}{\partial r} < 0 \) for all \( r > 0 \) and \( \Pi_A - \Pi_B > \bar{\Pi}_A - \bar{\Pi}_B \). Hence, \( r^* = 0 \) for all \( \Pi_A - \Pi_B > \bar{\Pi}_A - \bar{\Pi}_B \) if \( r \) is required to be non-negative. \( \Box \)

**References**


