Subjective Performance Evaluation and Collusion*

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Abstract

In many employment relationships, employees’ contributions to firm value are not contractible. Firms therefore need to use alternative mechanisms to provide their employees with incentives. This paper investigates and contrasts two alternatives for a firm to provide effort incentives: (i) to subjectively evaluate the employee’s performance; and (ii), to delegate the performance evaluation to a supervisor as a neutral party. Supervision generates contractible information about the employee’s performance, but could result in vertical collusion. This paper demonstrates that supervision can be optimal whenever firms cannot perfectly identify employees’ contributions to firm value. This can be observed despite ensuring collusion-proofness is shown to impose additional cost on firms in form of too low-powered incentives and inefficiently high fixed payments to employees and supervisors. Thus, this paper provides a supplementary rationale for the dominance of multi-level organizational hierarchies in practise.

Keywords: Subjective performance measurement, supervision, collusion, relational contracts, incentives.

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1 Introduction

It is a prevalent phenomenon in many employment relationships that employees’ contributions to firm value are too complex to be quantifiable by third parties [Prendergast and Topel, 1996, p. 958, Prendergast, 1999, p. 57, Kambe, 2006, p. 121]. Since verifiable performance measures form an essential component of court-enforceable incentive contracts, the absence of such measures implies that firms must rely on alternative mechanisms to provide their employees with effort incentives. One alternative is to draw on subjective performance evaluations, which constitute an essential component of incentive schemes in firms [Gibbons, 2005]. For instance, Gibbs [1995] reported that 25 percent of employees in middle management positions receive bonus payments based on subjective evaluations of individual performance.¹ Because such incentive payments are not legally binding, their reliability becomes questionable whenever firms lack sufficient reputation in honoring these (non-enforceable) obligations. Nevertheless, the credibility of these incentive payments can be augmented by involving a supervisor as a neutral party in the evaluation process. Yet, delegating the subjective performance evaluation to a supervisor can impose an additional inefficiency: the involved parties might be tempted to engage in vertical collusion with the aim of swaying the supervisor’s evaluation to their own advantage.

The consideration of supervision as a potential device to strengthen the credibility of subjective performance evaluations raises two important questions: First, how must incentive contracts based upon subjective evaluations be adjusted to prevent vertical side-contracting? Second, what are the conditions permitting supervision to be the superior incentive device despite the possibility of side-contracting? This paper aims to answer these questions by shedding light on the design of incentive contracts in a principal-agent relationship in the absence of verifiable performance measures. In particular, the agent’s contribution to firm value cannot be verified by third parties, but the principal can possibly observe the agent’s performance. This paper investigates and contrasts two alternatives for the principal to provide the agent with credible

¹Similarly, Gibbs, Merchant, Van der Stede, and Vargus [2004] discovered that incentive payments for 23 percent of managers in car dealerships are tied to a subjective appraisal of their performance.
effort incentives: (i) to subjectively evaluate his performance; and (ii), to delegate the performance evaluation to a supervisor. As a supposedly neutral party, the supervisor can attest to the agent’s performance such that it can be incorporated into a court-enforceable incentive contract. As pointed out earlier however, empowering the supervisor to subjectively evaluate the agent’s performance as a basis for incentive payments could create incentives for the involved parties to engage in collusion.\(^2\)

The analysis in this paper offers two important implications with respect to the efficiency of employment contracts. First, a subjective appraisal of the agent’s performance conducted either by the principal or a supervisor is—under certain conditions—accompanied by the provision of too low-powered incentives.\(^3\) This study therefore offers a theoretical explanation of the phenomenon that performance pay is less prevalent in practice than theory predicts [Baker, Jensen, and Murphy, 1988].\(^4\) The rationale for this implication however, is tied to the principal’s preference for providing the agent with credible effort incentives. As well known, low-powered incentives facilitate the reliability of non-enforceable performance payments based upon subjective evaluations conducted by the principal.\(^5\) In other words, the provision of low-powered incentives may be necessary to eliminate the principal’s temptation to renege on promised, but not court-enforceable, incentive payments. This constitutes a safeguard against opportunist behavior on the side of the principal. For supervision however, providing low-powered effort incentives is targeted at deterring the involved parties from side-contracting. In my framework, it is exactly the agent’s incentive payment that potentially provokes collusive behavior since it depends upon the supervisor’s appraisal of his performance. The provision of low-powered


\(^3\)More precisely, incentives are found to be too low-powered as compared to the case where the agent’s contribution to firm value is contractible and could thus be applied in an explicit (i.e. court-enforceable) incentive contract.


\(^5\)See e.g. Baker, Gibbons, and Murphy [1994] for a thorough discussion.
incentives is shown to diminish the benefit of side-contracting for involved parties, and is thus vital under certain conditions to deter collusion.

The second fundamental observation refers to situations where supervision constitutes the principal’s preferred incentive device. Then, collusion-proofness potentially necessitates inefficiently high fixed payments to the agent and supervisor (in addition to low-powered incentives). Fixed payments above their efficient levels provide both parties with sufficiently high economic rents associated with a sustained employment relationship in order to offset their potential gain from collusion. This conclusion can be interpreted as a supplementary explanation of the prevalence of high management compensations in practise. According to this study, high compensations constitute an important safeguard against biased internal performance evaluations, which would jeopardize the effectiveness of any associated incentive payments. Finally, as revealed in my analysis, the aforementioned inefficiencies become more severe in situations where the principal is less informed about the agent’s contribution to firm value. In other words, in the absence of contractible performance measures, less informed principals incur significantly higher costs in providing their employees with effective effort incentives.

There is a growing body of literature investigating the application of subjective performance measures in incentive contracts. One stream, notably Bull [1987], MacLeod and Malcomson [1989], and Levin [2003] considered incentive provision under circumstances where the principal relies exclusively on subjective performance evaluations in the absence of objective (i.e. verifiable) performance measures. Although the corresponding relational incentive contracts cannot be legally enforced, they can be self-enforcing in repeated games. This occurs whenever involved parties have no incentives to violate their implicit (i.e. non-enforceable) agreements as their present value from cooperation preponderates their one-time gain from deviation [Holmstrom, 1981, Bull, 1987, Thomas and Worrall, 1988]. By contrast, Baker et al. [1994], Schmidt and Schnitzer [1995], Pearce and Stacchetti [1998], and Demougin and Fabel [2004] focused on the optimal combination of subjective and objective performance measures in incentive contracts. Despite the availability of objective performance measures, subjective evaluations are
found to be an integral part of incentive schemes in agency relationships characterized by moral hazard.\textsuperscript{6}

The present conceptualization however, differs from preceding studies in one key aspect: Here, the principal cannot perfectly observe the agent’s contribution to firm value. More specifically, the principal is \textit{imperfectly} informed about the agent’s performance, which is modeled by assuming that she can observe the agent’s true contribution only with an exogenous probability. In doing so, this study accentuates the information asymmetry problem in order to capture reality whereby firm owners—as principals—are seldom perfectly informed about each employee’s contribution to firm value.\textsuperscript{7} Furthermore, it should be noted that past research studies restricted their attention to the subjective performance evaluation conducted by the principal. Since it is of high pragmatic relevance, the next logical step is to incorporate a supposedly neutral supervisor as a device to augment the credibility of incentive payments contingent upon subjective evaluations. Then, employment contracts do not only aim at providing sufficient effort incentives, but also at guaranteeing the supervisor’s impartiality by destroying every temptation of involved parties to engage in vertical collusion.

To identify the efficient design of collusion-proof employment contracts, I embed collusive behavior in a three-level hierarchy à la Tirole [1986] into a repeated game environment. Surprisingly, contemporary economic literature dealing with collusion in three-tier agency relationships restricted their investigations to static environments.\textsuperscript{8} It can be conjectured however, that reputational effects in repeated games could render side-contracting unprofitable, even in the absence of ’hard’ (i.e. verifiable) information. This would be the case if one-time gains from side-contracting do not compensate the colluding parties for their forfeited future payoffs. Though this conjecture is proven to be true \textit{generally}, this study demonstrates that collusion-

\textsuperscript{6}See also Hayes and Schaefer [2000] for empirical evidence.

\textsuperscript{7}For instance, the exogenous probability that the principal observes the agent’s contribution to firm value can be interpreted as an indicator of the hierarchical or geographical distance between both parties.

\textsuperscript{8}See e.g. Tirole [1986], Villadsen [1995], Strausz [1997b], Vafaï [2005], and Celik [forthcoming].
proofness cannot be achieved in all situations. In particular, the principal cannot ensure that the effectiveness of incentive contracts is not jeopardized by collusive behavior in every situation.

Previous literature concerned with vertical side-contracting in agency relationships share a common characteristic: the supervisor is assumed to have an information advantage over the principal, i.e. the supervisor is assumed to privately observe either (i) random productivity shocks [Tirole, 1986, Kofman and Lawarree, 1993, Villadsen, 1995], (ii) the true cost of an implemented project [Strausz, 1997a], (iii) the agent’s type [Faure-Grimaud, Laffont, and Martimort, 2003, Celik, forthcoming]; or (iv), the agent’s effort [Kessler, 2000, Vafaï, 2005]. These information asymmetry problems could motivate the agent to collude with the supervisor in order to ensure he withholds or misrepresents his private information to the principal. The approach pursued in this study to model collusive behavior however, differs in one main aspect. Here, the supervisor is charged with confirming the agent’s contribution to firm value, which is potentially observable by all involved parties. Since the agent’s incentive payment is made upon this affirmation, both the agent and the principal may be tempted to collude with the supervisor in order to sway his assessment to their own advantage.

This paper proceeds as follows. Section 2 introduces the model. Section 3 derives the optimal contracts under the principal’s respective alternatives for providing the agent with effort incentives: (i) utilizing a spot contract, (ii) subjectively evaluating the agent’s performance in a repeated game environment; and (iii), delegating the subjective performance evaluation to a supervisor as a neutral party in the evaluation process (supervision). In section 4, the optimal incentive provision from the principal’s perspective is identified and discussed. Section 5 summarizes the key results and concludes.

2 The Model

Consider an infinitely repeated employment relationship between a principal and an agent. Both parties are risk-neutral and their 'patience' is reflected by the mutually shared interest rate \( r \).
The agent is financially constrained and his reservation utility is normalized to zero. In every period, the agent is charged with producing a good. The value of this good $V$ can be either high ($V_H$) or low ($V_L$). For subsequent analysis, let $\Delta V \equiv V_H - V_L$ denote the difference between the high and low values. The realized value of the good $V$ is perfectly observable by the agent but not by the principle. Nevertheless, the principal notices the true realization of $V$ with probability $\theta \in (0, 1]$. However, the good is too complex such that its realized value cannot be verified by third parties.

By implementing effort $e \in \mathbb{R}^+$, the agent determines the likelihood of whether the value of the good will be high or low. Formally, let

$$\text{Prob}\{V = V_H|e\} = \rho(e) \in [0, 1)$$

be the twice-continuously differentiable probability that the high value will be realized, where $\rho(0) = 0$, $\rho'(e) > 0$, and $\rho''(e) < 0$. Effort is non-observable and imposes strictly convex increasing costs $c(e)$ on the agent with $c(0) = c'(0) = 0$. In exchange for his service, the principal offers the agent the payment $w^A$.

Since the realized value of the good $V$ is non-verifiable, the principal cannot use this information in a court-enforceable incentive contract. The principal however, can provide the agent with a relational incentive contract based upon her subjective evaluation of the realized value $V$. Particularly, the principal can promise the agent to pay a bonus $\beta$ in addition to a fixed transfer $\alpha$ in the event she obtains no evidence that the low value $V_L$ is realized.

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9The limited liability constraint ensures that transferring the firm to the agent is not feasible in this framework.

10This occurs, for instance, when the attainment of a specified quality standard is predominant in the valuation of a good. Involved parties are potentially able to assess whether a previously defined quality standard is achieved, but for third parties such as courts it is sometimes either impossible to verify the achieved quality, or the associated costs are prohibitively high.

11At first glance, this appears to be synonymous to the assumption that the bonus $\beta$ is only paid if the principal observes the realization of the high value $V_H$. In this case however, the principal would always claim to be uninformed about $V$ in order to avoid the bonus payment, which in turn would negate any incentive provision based upon subjective evaluations.
Since the payment of $\beta$ cannot be legally enforced, the principal’s promise needs to be reliable from the agents’ perspective. The agent initially trusts the principal but plays a grim trigger strategy: if $V = V_H$ and the principal violates her implicit obligation to pay $\beta$, the agent will never rely on non-enforceable agreements with the principal again. The same applies to other agents in the labor market as the principal earns a bad reputation once she reneged on $\beta$ [Bull, 1987].

As an alternative to subjectively evaluating the agent’s performance, the principal can employ a third party, henceforth referred to as supervisor, who is in charge of confirming the value of the good (supervision).\footnote{The supervisor could also conduct other tasks which contribute to firm value. This paper however, focuses on the principal’s preference for employing the supervisor to obtain contractible measures about the agent’s performance.} The supervisor always observes the true realization of $V$ and thus, enjoys an informational advantage as compared to the principal. The supervisor is risk-neutral and his reservation utility is normalized to zero. In exchange for his service, the principal offers the supervisor the payment $w^S$.

From the perspective of external observers such as courts, the agent and the principal are potentially involved in a dispute over the value of the good, whereas the supervisor is the supposedly neutral entity in such a conflict. Therefore, the statement from the supervisor will have a greater weight in swaying the court’s decision. The supervisor’s confirmation thus guarantees that the realized value of the good can be applied in a court-enforceable incentive contract.

3 Alternative Incentive Provisions

3.1 Spot Contract (SC)

Before I elaborate on the provision of incentives in a repeated game environment, let us first consider the optimal employment contract in a one-shot game. This spot contract constitutes
the principal’s fallback alternative in case the subsequently considered relational contracts are not feasible.

Suppose the principal and the agent interact only for one period. To motivate effort, the principal could promise to pay the agent the bonus $\beta$ if she does not obtain any evidence that the low value of the good $V_L$ is realized. Once this occurs however, the principal would renege on $\beta$ since its payment cannot be legally enforced. By anticipating this opportunistic behavior, the agent refuses to implement effort in the first place. It is therefore optimal from the principal’s perspective to set $\alpha^* = \beta^* = 0$. Consequently, utilizing a spot contract provides the principal with the profit $\Pi^{SC} = V_L$.

3.2 Subjective Performance Evaluation (SE)

If the principal and the agent interact for an infinite number of periods, the principal can utilize a relational incentive contract based upon her subjective evaluation of the agent’ performance. In particular, as shown by Baker et al. [1994], the principal’s promise to pay a bonus $\beta > 0$ can be credible such that the agent could be motivated to implement effort. If the principal reneges on $\beta$ however, the agent will never rely on non-enforceable incentive payments again. Hence, the principal’s best alternative after reneging on $\beta$ can be either to utilize a spot contract ($SC$), or to employ the supervisor ($S$) for evaluating the agent’s performance. Formally, the principal’s best fallback profit is characterized by $\max\{\Pi^{SC}, \Pi^S\}$.

Suppose for a moment that the principal did not obtain any evidence that the low value of the good is realized. Then, she pays $\beta$ if

$$-\beta + \frac{\Pi^{SE}}{r} \geq \frac{\max\{\Pi^{SC}, \Pi^S\}}{r},$$

where $\Pi^{SE}$ denotes the principal’s expected profit under the subjective evaluation of the agent’s performance. The principal adheres to her promise if paying the bonus $\beta$ but sustaining the employment relationship based upon subjective performance evaluations leads to a higher present expected profit than her best fallback.
The principal’s problem is to find a bonus contract \((\alpha^*, \beta^*)\) which maximizes the difference between the expected value of the good and the agent’s expected wage, while simultaneously, ensuring his participation. Note however, that the agent might obtain the bonus \(\beta\) despite the realization of a low value \(V_L\) since the principal cannot perfectly infer \(V\). Let

\[
\bar{\rho}(e, \theta) \equiv \rho(e) + (1 - \rho(e))(1 - \theta)
= 1 - \theta(1 - \rho(e))
\]

denote the adjusted probability of the bonus payment conditional on the agent’s effort \(e\) and the exogenous probability \(\theta\) of the principal being informed about \(V\). Hence, the principal’s problem can be formalized as follows:\(^{13}\)

\[
\max_{\alpha, \beta, e} \Pi_{SE}(\alpha, \beta, e) = V_L + \Delta V \rho(e) - \alpha - \beta \bar{\rho}(e, \theta)
\]  

\[
\text{s.t.}
\]

\[
\alpha + \beta \bar{\rho}(e, \theta) - c(e) \geq 0 
\]  

\[
e \in \arg \max_{\tilde{e}} \alpha + \beta \bar{\rho}(\tilde{e}, \theta) - c(\tilde{e})
\]

\[
\alpha + \beta \geq 0
\]

\[
\alpha \geq 0
\]

\[
\Pi_{SE}(\alpha, \beta, e) - \max\{\Pi_{SC}, \Pi^S\} \geq r\beta.
\]

Condition (4) is the agent’s participation, and (5) his incentive constraint. Furthermore, (6) and (7) are the liability limit constraints guaranteeing that all payments to the agent are non-negative. Finally, (8) is the self-enforcement condition (derived from (2)) ensuring that the principal’s promise to pay \(\beta\) is credible.

Before deriving the optimal bonus contract, let us first consider the agent’s effort choice for a given bonus \(\beta\). Observe that (5) is equivalent to

\[
\beta(e, \theta) = c'(e)/(\theta \rho'(e)), \text{ with } \beta(e, \theta)
\]

\(^{13}\)Note that maximizing the expected profit for a single period is equivalent to maximizing the present value of all future expected profits. This is because reneging does not occur in the reputational equilibrium such that expected profits for every period are identical.
as the required bonus to induce an arbitrary effort level $e(\beta, \theta)$. Thus, the expected bonus $B(e, \theta) \equiv \beta(e, \theta)\bar{\rho}(e, \theta)$ to induce $e$ is

$$B(e, \theta) = \frac{c'(e)\bar{\rho}(e, \theta)}{\theta \rho'(e)},$$

(9)

which is assumed to be convex in $e$.\footnote{It can be shown that assuming $c'''(e) \geq 0$ suffices to ensure convexity of $B(e, \theta)$ for all $e$.} The expected bonus $B(e, \theta)$ is characterized by the likelihood ratio $\bar{\rho}_e(e, \theta)/\bar{\rho}(e, \theta)$, which can be shown to be increasing in $\theta$. Accordingly, if the principal is more likely to observe the true value of the good $V$, she can induce the same effort level with a lower bonus payment $\beta$.

The subsequent proposition characterizes the optimal bonus contract under the subjective evaluation of the agent’s performance by utilizing two threshold interest rates conditional on $\theta$, $r^{SE}(\theta)$ and $\tilde{r}^{SE}(\theta)$. For parsimony, the threshold interest rates for this and subsequent propositions are characterized in the respective proofs in the appendix.

**Proposition 1** For subjective performance evaluation, the optimal fixed payment is $\alpha^* = 0$. The optimal bonus $\beta^*$ is characterized as follows:

(i) If $r \leq r^{SE}(\theta)$, the optimal bonus is $\beta^*(e^*, \theta) = c'(e^*)/(\theta \rho'(e^*))$, where $e^*$ implicitly solves $\Delta V \rho'(e) = B_e(e, \theta)$.

(ii) If $r^{SE}(\theta) < r \leq \tilde{r}^{SE}(\theta)$, the optimal bonus $\beta^*(r, \theta)$ is the highest value of $\beta$ which implicitly solves $\Pi^{SE}(\beta, \theta) - \max\{\Pi^{SC}, \Pi^S\} = r\beta$.

(iii) If $r > \tilde{r}^{SE}(\theta)$, the optimal bonus is $\beta^*(r, \theta) = 0$.

**Proof** All proofs are given in the appendix.

Figure 1 illustrates the optimal bonus contract for different interest rates $r$, where the straight line $r\beta$ represents the right side of the self-enforcement condition (8).\footnote{By using a numerical example, Baker et al. [1994] derived a similar graph to illustrate the optimal adjustment of incentive payments based upon subjective performance measures. I decided to incorporate Figure 1 since it reinforces my subsequent explanations of how the optimal bonus $\beta^*(\cdot)$ is affected by the interest rate $r$.} As one can infer from
Figure 1, the principal can find a credible bonus $\beta > 0$ whenever $r\beta$ either tangents or intersects the adjusted profit curve ($\Pi^{SE}(\beta, \theta) - \max\{\Pi^{SC}, \Pi^{S}\}$). The principal can even credibly commit to pay the efficient bonus $\beta^*(e^*, \theta)$ for a sufficiently low interest rate $r \leq r^{SE}(\theta)$. In this case, the value of a sustained employment relationship based upon a subjective performance evaluation eliminates the principal’s temptation to renege on $\beta^*(e^*, \theta)$. The agent anticipates that the principal would deliver on her promise to pay $\beta^*(e^*, \theta)$ and is therefore motivated to implement the efficient (second-best) effort level $e^*$.\(^{16}\) For $r^{SE}(\theta) < r \leq \hat{r}^{SE}(\theta)$ however, the principal is compelled to adjust $\beta$ in order to ensure it satisfies the self-enforcement condition (8). This follows from the fact that a higher interest rate $r$ imposes a less severe ‘penalty’ on the principal for violating the relational contract with the agent. Specifically, as can be deduced from proposition 1, the principal is forced to offer the agent an inefficiently low bonus $\beta^*(r, \theta)$ with $\beta^*(r, \theta) < \beta^*(e^*, \theta)$, which can be shown to be decreasing in $r$.\(^{17}\) Finally, if $r > \hat{r}^{SE}(\theta)$, the principal is tempted to renege on every strictly positive bonus $\beta$. In other words, she cannot

\(^{16}\)Note that $e^*$ is a function of $\theta$. However, I suppress its argument for parsimony purposes.

\(^{17}\)For a formal proof of this statement see proof of proposition 1 in the appendix.
find a strictly positive bonus which satisfies (8) such that $\beta^*(r, \theta) = 0$. Due to the lack of credible incentives, the agent maximizes his expected utility by implementing $e^* = 0$.

3.3 Supervision (S)

As shown in the preceding section, the agent cannot be motivated to implement the efficient (second-best) effort level $e^*$ if the principal cannot credibly commit to pay the efficient bonus $\beta^*(e^*, \theta)$. As an alternative to subjectively evaluating the agent’s performance, the principal can task a supervisor with affirming the realized value of the good $V$ such that it can be applied in a court-enforceable incentive contract. The supervisor’s confirmation further ensures that the agent obtains his bonus only if the high value of the good is indeed realized, which occurs with probability $\rho(e)$. Thus, as compared to the previously investigated subjective performance evaluation, the principal could induce the same effort level at lower costs, which clearly constitutes an argument for supervision. However, empowering the supervisor to evaluate the agent’s performance can create incentives for the involved parties to engage in vertical side-contracting. The realized value of the good $V$ eventually determines whether the principal or the agent might be better off by colluding with the supervisor. If $V = V_L$, the agent could secure his bonus $\beta$ by bribing the supervisor into affirming a high value $V_H$. In contrast, if $V = V_H$, the principal could bribe the supervisor in order to avoid the payment of $\beta$.

The agent can perfectly infer whether collusion among the principal and the supervisor occurred once the confirmed value of the good deviates from the one he observes. Since the agent plays a grim trigger strategy, the principal’s fallback position after colluding with the supervisor is the application of a spot contract as considered in section 3.1. The principal however, can only detect side-contracting among the agent and the supervisor if she indeed observes the low value $V_L$ (which occurs with probability $\theta$). If the principal discovers collusion, she replaces both colluding parties by employing a new agent and supervisor from the labor market.
To prevent vertical collusion, the principal must to provide the agent and the supervisor with employment contracts ensuring that none of the involved parties can be better off by side-contracting. To derive collusion-proofness conditions which are necessary to identify the optimal employment contracts under supervision, I first elaborate on the side-payments the principal and the agent would offer the supervisor with the aim of swaying his evaluation to their respective advantage. Let $T^A$ denote the bribe the agent offers the supervisor in exchange for affirming a high value of the good $V_H$ despite the realization of the low value $V_L$. If the supervisor accepts $T^A$, he does not deviate from the stipulated behavior and confirms the requested value.\(^{18}\)

However, if

$$T^A \leq \frac{\theta}{r} w^S, \quad (10)$$

the supervisor refuses to engage in side-contracting with the agent since $T^A$ does not compensate him for his expected loss of prospective income. Likewise, let $T^P$ denote the bribe the principal potentially offers the supervisor in order to ensure he affirms the low value $V_L$. The supervisor rejects the principal’s bribe $T^P$ however, if

$$T^P \leq \frac{1}{r} w^S. \quad (11)$$

Collusion-proofness thus requires that (10) and (11) are satisfied for the maximum bribes the principal and the agent are willing to pay. Suppose for a moment that the low value $V_L$ is realized such that the agent might be tempted to bribe the supervisor with the aim of obtaining his bonus $\beta$. The maximum bribe $\bar{T}^A(r, \theta)$ the agent is willing to pay equals his one-time gain $\beta$ minus his discounted loss of expected utility once collusion has been discovered by the principal:

$$\bar{T}^A(r, \theta) = \beta - \frac{\theta}{r} [\alpha + \beta \rho(e) - c(e)]. \quad (12)$$

It is straightforward to infer from (10) and (13) that the agent and the supervisor refrain from side-contracting if $\bar{T}^A(r, \theta) \leq \theta w^S/r$.

\(^{18}\)There exists experimental evidence that promises are honored among agents, see Dawes and Thaler [1988] for a survey.
By contrast, if the high value $V_H$ is realized, the principal could avoid the payment of $\beta$ by bribing the supervisor into confirming the low value of the good $V_L$. The maximum bribe $\bar{T}^P(r)$ the principal is willing to pay equals her one-time gain $\beta$ minus her discounted loss of expected profit after she engaged in side-contracting and forfeited her reputation in the labor market:

$$\bar{T}^P(r) = \beta - \frac{\Pi^S - \Pi^{SC}}{r}.$$  \hspace{1cm} (13)

By combining (11) and (13), it becomes apparent that the principal and the supervisor resist the temptation to collude if $T^P(r) \leq w^S/r$. A violation of this collusion-proofness condition would convince the agent that he will never obtain the bonus $\beta$ despite a potential realization of the high value $V_H$. Thus, the agent would refuse to implement effort such that $V = V_L$.

The principal’s objective is to find collusion-proof employment contracts $w^{A^*}(\alpha^*, \beta^*)$ and $w^{S^*}$, which maximize her expected profit while ensuring the participation of both the agent and the supervisor. Hence, the optimal collusion-proof contracts solve

$$\max_{\alpha, \beta, e, w^S} \Pi^S = V_L + \Delta V \rho(e) - \alpha - \beta \rho(e) - w^S$$  \hspace{1cm} (14)

s.t.

$$\alpha + \beta \rho(e) - c(e) \geq 0$$  \hspace{1cm} (15)

$$e \in \arg \max_{\bar{e}} \alpha + \beta \rho(\bar{e}) - c(\bar{e})$$  \hspace{1cm} (16)

$$w^S \geq 0$$  \hspace{1cm} (17)

$$T^A(r, \theta) \leq \theta w^S / r$$  \hspace{1cm} (18)

$$T^P(r) \leq w^S / r.$$  \hspace{1cm} (19)

This maximization problem differs from the one in section 3.2 (subjective performance evaluation) in three aspects. First, the agent’s probability to obtain the bonus $\beta$ is now $\rho(e)$, where $\rho(e) \leq \bar{\rho}(e, \theta)$. This is because the supervisor’s confirmation ensures that the bonus $\beta$ is only paid if the good is indeed of high value. Second, the self-enforcement condition (8) for subjective performance evaluation is not relevant here as a result of the supervisor’s affirmation of...
the realized value $V$. Finally, the present maximization problem additionally takes the supervisor’s participation constraint (17) as well as the two previously derived collusion-proofness conditions (18) and (19) into account.

As a benchmark for subsequent analysis, let us first assume that the interest rate $r$ is sufficiently low such that the collusion-proofness conditions (18) and (19) are satisfied for the optimal contracts $w^A^*(\alpha^*, \beta^*)$ and $w^S^*$. Then, cost minimization demands that the principal sets $w^S^* = 0$ and $\alpha^* = 0$. Moreover, we can infer from (16) that the required bonus $\beta(e)$ to induce an arbitrary effort level $e$ is characterized by $\beta(e) = c'(e)/\rho'(e)$. The expected bonus $B(e) = \beta(e)\rho(e)$ to induce $e$ thus becomes

$$B(e) = \frac{c'(e)\rho(e)}{\rho'(e)}. \quad (20)$$

Observe that we obtain the same expected bonus as for the subjective performance evaluation with a perfectly informed principal ($\theta = 1$). Thus, we can infer from proposition 1 that the optimal effort $e^*$ implicitly solves $\Delta V \rho'(e) = B'(e)$.

If the mutually shared interest rate $r$ is not sufficiently low, at least one of the collusion-proofness conditions becomes binding. The subsequent proposition emphasizes the appropriate contract adjustments for all interest rates $r$ required inter alia to ensure collusion-proofness. For this proposition, keep in mind that the threshold interest rate $r^P$ refers to the principal’s temptation to collude with the supervisor, and $r^A(\theta)$ to the agent’s respectively.

**Proposition 2** The optimal collusion-proof contracts $w^A^*(\alpha^*, \beta^*)$ and $w^S^*$ are characterized as follows:

(i) If $r \leq r^S \equiv \min\{r^A(\theta), r^P\}$, the principal sets $w^S^* = 0$, $\alpha^* = 0$, and $\beta^*(e^*) = c'(e^*)/\rho'(e^*)$.

(ii) If $r^S < r \leq \hat{r}^S(\theta)$, the optimal fixed transfers $w^S^* \geq 0$ and $\alpha^* \geq 0$ are characterized by

$$\begin{cases} 
\alpha^* = 0, \quad w^S^* = 0, & \text{if } r \leq r^A(\theta) \\
\alpha^* + w^S^* = \beta^*(r, \theta) \left[\frac{r}{\theta} - \rho(\cdot)\right] + c(\cdot), & \text{if } r^A(\theta) < r,
\end{cases}$$

15
and $\beta^*$ as the highest value of $\beta$ which implicitly solves

$$
\begin{align*}
\beta^*(r, \theta) & : \quad r = \theta \left[ \Delta V \rho'(e(\beta)) - c'(e(\beta)) \right] \frac{\partial e}{\partial \beta}, \quad \text{if} \quad r^A(\theta) < r \leq r^P \\
\beta^*(r) & : \quad \Delta V \rho(e(\beta)) = \beta \left[ r + \rho(e(\beta)) \right], \quad \text{if} \quad r^P < r \leq r^A(\theta) \\
\beta^* & = \min \{ \beta^*(r, \theta), \beta^*(r) \}, \quad \text{if} \quad r^P, r^A(\theta) < r.
\end{align*}
$$

(iii) If $r > \hat{r}^S(\theta)$, the principal sets $w^{S*}, \alpha^*, \beta^*(r, \theta) = 0$.

If $r \leq r^S$, all involved parties sufficiently value a sustained employment relationship under supervision such that no one is tempted to collude. This in turn enables the principal to provide the agent with the efficient bonus $\beta^*(e^*) = c'(e^*)/\rho'(e^*)$ without provoking collusion. In contrast, if $r^S < r \leq \hat{r}^S(\theta)$, the principal is forced to adjust the agent’s—and possibly the supervisor’s—contract to prevent vertical side-contracting. For a brief discussion of the different cases exposed by proposition 2, recall that $r^A(\theta)$ refers to the agent’s temptation to collude with the supervisor, and $r^P$ to the principal’s respectively. Suppose for moment that $r^A(\theta) < r$, i.e. the agent is tempted to collude with the supervisor. For this case, we can infer from proposition 2 that the principal is compelled to increase the supervisor’s payment $w^{S*}$ and the agent’s fixed compensation $\alpha^*$ above their efficient levels (i.e., $w^{S*}, \alpha^* > 0$). This in turn leads to the extraction of higher economic rents, which in the context of this study is essential for deterring the agent and the supervisor from side-contracting. Moreover, it can be deduced from (18) that a less informed principal (lower $\theta$)—who is less likely to detect collusion among the agent and supervisor—is required to offer even higher fixed payments in order to ensure collusion-proofness. Over and above the provision of inefficiently high fixed payments, proposition 2 further implies that the principal is required to offer the agent a bonus $\beta^*(r, \theta)$ below the efficient level (i.e., $\beta^*(r, \theta) < \beta^*(e^*)$). By implicit differentiating $\beta^*(r, \theta)$, one can show that a less informed principal (lower $\theta$) is compelled to offer the agent an even lower incentive payment. The provision of too low-powered incentives aims at curbing the agent’s temptation to bribe the supervisor into spuriously affirming the high value of the good. As emphasized by
proposition 2, the latter contract adjustment is also necessary in the event that $r^P < r$ so as to eliminate the principal’s temptation to collude with the supervisor.

Finally, for $r > \tilde{r}^S(\theta)$, the agent’s optimal contract $w^A*$ under supervision does not comprise any incentive payments. The reason is as follows.\textsuperscript{19} For small values of $\theta$, the provision of a strictly positive and collusion-proof bonus $\beta$ involves inefficiently high fixed payments to the agent and supervisor. If these fixed payments eventually exceed the agent’s expected contribution to firm value (which occurs for sufficiently high values of $r$), the principal is better off by refraining from providing the agent with effort incentives. Moreover, for sufficiently high values of $r$, the principal cannot find a strictly positive bonus which eliminates her temptation to collude with the supervisor.\textsuperscript{20} In both cases, the principal is compelled to set $\beta^*(r, \theta) = 0$. Without the provision of effort incentives, the agent implements $e^* = 0$.

4 Informativeness and Incentive Provision

This section identifies the optimal incentive provision from the principal’s perspective, and illustrates how it is affected by her ability to obtain information about the agent’s contribution to firm value. This in turn facilitates a thorough discussion of how incentive contracts are adjusted in order to ensure their credibility (subjective evaluation) or collusion-proofness (supervision).

To unravel how the principal’s informativeness—parameterized by $\theta$—affects the optimal incentive provision, let us first consider the extreme cases: (i) the principal is fully informed ($\theta = 1$); and (ii), she does not receive any information about the agent’s contribution to firm value ($\theta = 0$). The next proposition identifies the optimal incentive provision for these two extreme cases.

\textsuperscript{19}For a formal analysis refer to proof of proposition 2 in the appendix.
\textsuperscript{20}The principal can nevertheless find a strictly positive bonus for all values of $r$ which deters the agent from side-contracting, see proof of proposition 2 in the appendix.
**Proposition 3** If the principal is fully informed about the agent’s contribution to firm value ($\theta = 1$), she

(i) is indifferent between a subjective performance evaluation and supervision, if $r \leq r^A(1)$

(ii) applies a subjective performance evaluation, if $r^A(1) < r \leq \hat{r}^{SE}(1)$

(iii) utilizes a spot contract, if $\hat{r}^{SE}(1) < r$.

In contrast, if the principal does not receive any information about the agent’s contribution to firm value ($\theta = 0$), she applies a spot contract for all values of $r$.

Suppose for a moment that the principal can perfectly observe the agent’s contribution to firm value ($\theta = 1$). Then, as pointed out by proposition 3, a subjective performance evaluation and supervision are equally profitable for the principal as long as $r \leq r^A(1)$. This can be deduced from the fact that the agent’s incentive contract $w^A*$ is identical under both alternatives, and $w^{S*} = 0$.21 For $r^A(1) < r \leq \hat{r}^{SE}(1)$ however, supervision requires inefficiently high fixed payments to the agent and the supervisor in order to deter them from side-contracting. Moreover, as can be inferred from proposition 2, collusion-proofness potentially calls for a lower bonus payment under supervision than under subjective performance evaluation. Consequently, the principal strictly prefers a subjective evaluation of the agent’s performance as a means of providing him with credible effort incentives. For sufficiently high interest rates ($r > \hat{r}^{SE}(1)$), any strictly positive bonus $\beta$ is neither credible under subjective evaluation, nor collusion-proof under supervision. In this case, the principal is forced to utilize a spot contract, which in turn eliminates incentives for the agent to implement effort.

If the principal cannot observe the agent’s contribution to firm value ($\theta = 0$), utilizing a spot contract is her superior strategy. To understand the rationale of this result, let us briefly

21This conclusion rests on the assumption that the supervisor’s reservation utility is zero. With a strictly positive reservation utility (which would imply that $w^{S*} > 0$), the principal would exhibit strong preferences towards subjective performance evaluation. However, incorporating a strictly positive reservation utility for the supervisor in this framework would complicate the comparison of alternative incentive provisions without offering additional insights.
discuss the effects of an uninformed principal on subjective performance evaluation and supervision. First, a completely uninformed principal cannot detect when the agent shirks. Due to the absence of feasible punishment mechanisms, the agent cannot be motivated under subjective performance evaluation to implement effort. Second, if the principal cannot observe the agent’s true contribution to firm value, she will never be able to discover collusion between the agent and the supervisor. Thus, the principal cannot apply employment contracts which effectively deter both parties from side-contracting.

Next, let us delve into the optimal incentive provision in case the principal is only partially informed about the agent’s true contribution to firm value ($0 < \theta < 1$). It is important for subsequent discussion to keep in mind that for some parameter values, only one alternative—either subjective performance evaluation or supervision—allows the principal to profitably offer the agent a strictly positive and credible bonus payment $\beta$. For other parameter values, both incentive devices are applicable, and the principal chooses naturally the one which leads to a higher expected profit. For the latter case, let $\hat{r}(\theta)$ denote the threshold interest rate which makes the principal indifferent between a subjective evaluation of the agent’s performance and supervision. Formally, $\hat{r}(\theta)$ satisfies $\Pi^{SE}(\hat{r}(\theta), \theta) = \Pi^{S}(\hat{r}(\theta), \theta)$. By utilizing this additional threshold, the next proposition identifies the optimal incentive provision for a partially informed principal.

**Proposition 4** If the principal is partially informed about the agent’s contribution ($0 < \theta < 1$), the optimal incentive provision is characterized as follows:

(i) If $r \leq \hat{r}(\theta)$, the principal utilizes supervision.

(ii) If $\hat{r}(\theta) < r \leq \min\{\hat{r}^{SE}(\theta), \hat{r}^{S}(\theta)\}$, she prefers a subjective performance evaluation.

(iii) If $\min\{\hat{r}^{SE}(\theta), \hat{r}^{S}(\theta)\} < r \leq \max\{\hat{r}^{SE}(\theta), \hat{r}^{S}(\theta)\}$, the principal adopts

(a) supervision, if $\hat{r}^{S}(\theta) > \hat{r}^{SE}(\theta)$

(b) a subjective performance evaluation, if $\hat{r}^{S}(\theta) \leq \hat{r}^{SE}(\theta)$.

(iv) If $r > \max\{\hat{r}^{SE}(\theta), \hat{r}^{S}(\theta)\}$, the principal applies a spot contract.
At first glance, proposition 4 implies that the principal’s contract choice is determined by the mutually shared interest rate \( r \), which—as previously demonstrated—constitutes a crucial factor of the reputational equilibria under subjective performance evaluation as well as under supervision. Less obvious but at least as important for the optimal contract choice however, is the extent to which the principal is informed about the agent’s true contribution to firm value—parameterized by \( \theta \). The optimal incentive provision thus hinges on these two exogenous parameters and their specific effects on (i) the corresponding costs to induce an arbitrary effort level under a subjective performance evaluation; and (ii), the associated costs to ensure collusion-proofness under supervision. To facilitate subsequent discussion, Figure 2 illustrates the optimal incentive provision for different interest rates \( r \) (vertical axis) and informativeness parameters \( \theta \) (horizontal axis).²²

²²To understand the shape of \( \hat{\varphi}^S(\theta) \), notice that it is a function of two additional thresholds \( \hat{\varphi}_1^S \) and \( \hat{\varphi}_2^S(\theta) \), with \( \hat{\varphi}^S(\theta) = \min\{\hat{\varphi}_1^S, \hat{\varphi}_2^S(\theta)\} \). From a closer inspection of these two thresholds characterized in proof of proposition 2 in the appendix, it becomes clear that \( \hat{\varphi}_2^S(\theta) \) is increasing, and \( \hat{\varphi}_1^S \) is constant in \( \theta \). Moreover, it can be shown that \( \hat{\varphi}_1^S = \hat{\varphi}^{SE}(1) \). Finally note that Figure 2 applies to the case where \( \hat{\varphi}_2^S(\theta) > \hat{\varphi}_1^{SE}(\theta) \) for all \( \theta \in (0, 1) \).
As can be deduced from Figure 2, supervision is strictly preferred by the principal as long as the mutually shared interest rate $r$ is sufficiently low ($r \leq \hat{r}(\theta)$). This is true despite collusion-proofness calls for inefficiently high fixed payments $w^{S*} > 0$ and $\alpha^* > 0$ for $r^A(\theta) < r \leq \hat{r}(\theta)$. Thus, even though deterring the involved parties from side-contracting imposes additional costs on the principal, supervision is the superior incentive device since it constitutes a safeguard against mistaken bonus payments. For intermediate interest rates ($\hat{r}(\theta) < r \leq \min\{\hat{r}^{SE}(\theta), \hat{r}^{S}(\theta)\}$), the principal is better off by providing the agent with a relational incentive contract based upon a subjective evaluation of his performance. In this case, taking the risk of mistaken bonus payments is less costly for the principal than preventing side-contracting under supervision.

In contrast to the previously discussed cases, only one of the two considered incentive mechanisms is feasible if $\min\{\hat{r}^{SE}(\theta), \hat{r}^{S}(\theta)\} < r \leq \max\{\hat{r}^{SE}(\theta), \hat{r}^{S}(\theta)\}$. In particular, if $\hat{r}^{S}(\theta) > \hat{r}^{SE}(\theta)$, the principal’s promise to pay a strictly positive bonus $\beta$ contingent on her subjective evaluation is not credible from the agent’s perspective, but the principal can profitably utilize supervision to provide the agent with credible effort incentives. As a logical consequence, delegating the appraisal of the agent’s performance to a supervisor constitutes the principal’s preferred alternative. By contrast, if $\hat{r}^{S}(\theta) \leq \hat{r}^{SE}(\theta)$, only subjective performance evaluation facilitates the effective provision of effort incentives. The principal thus strictly prefers tying the agent’s incentive payment to the subjective evaluation of his performance.

Finally, if $r > \max\{\hat{r}^{SE}(\theta), \hat{r}^{S}(\theta)\}$, the principal is forced to utilize a spot contract for two reasons. First, the principal cannot credibly commit to pay the agent a strictly positive bonus $\beta$ under subjective performance evaluation. A relational contract based upon a subjective evaluation thus becomes equivalent to a spot contract due to the lack of credible effort incentives. Second, under supervision, either the principal cannot find a strictly positive bonus $\beta$ which eliminates her temptation to collude with the supervisor, or ensuring collusion-proofness necessitates inefficiently high fixed payments to the agent and supervisor which eventually exceed the agent’s expected contribution to firm value, and thus render supervision unprofitable.
In addition to identifying the optimal incentive provision, it is insightful to shed light on the efficiency of the agent’s and the supervisor’s employment contracts for different values of the interest rate $r$ and the principal’s informativeness measured by $\theta$. Figure 3 visualizes the previously discussed contract adjustments under subjective performance evaluation and supervision for different parameter values. The efficient contract elements are indicated by $w^S$, $\alpha^*$, and $\beta^*$. Moreover, inefficiently high fixed payments to the supervisor and agent are denoted by $\pi^S$ and $\pi$, and the provision of too low-powered incentives by $\beta$.

As can be inferred from Figure 3, supervision enables the principal to provide the agent with the efficient incentive contract $(\alpha^*, \beta^*(e^*))$ if the mutually shared interest rate $r$ is sufficiently low ($r \leq \min\{r^A(\theta), r^P\}$). Otherwise, the principal is forced to provide the agent with too low-powered incentives, which in turn induces only a suboptimal effort level. This particular phenomenon however, hinges on the chosen incentive device. If supervision is the principal’s superior alternative, low-powered incentives are indispensable for deterring involved parties from side-contracting, which would have a devastating effect on the agent’s incentives to im-

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23Specifically, the efficient contract elements are characterized by $w^S$, $\alpha^* = 0$, and $\beta^*(e^*) = c'(e^*)/\rho'(e^*)$, where $e^*$ implicitly solves $\Delta V\rho'(e) = B'(e)$. 
plement effort. For subjective performance evaluation however, the provision of low-powered incentives is aimed at eliminating the principal’s reneging temptation, and thus assures the effectiveness of the relational incentive contract.

Figure 3 points to another fundamental observation whenever supervision is the principal’s superior incentive device. For \( r > r^A(\theta) \), ensuring collusion-proofness entails inefficiently high fixed payments to the agent and the supervisor (see also proposition 2). These escalated fixed payments not only ensure the participation of the agent and the supervisor, but also deter both parties from side-contracting. One can infer from Figure 3 that this is especially the case if the principal is relatively uninformed about the agent’s contribution to firm value (low \( \theta \)), and is thus less likely to detect collusion between the supervisor and the agent.

5 Conclusion

This paper investigates the optimal provision of incentives in a principal-agent relationship in the absence of contractible measures about the agent’s performance. It analyzes and compares two alternative mechanisms for the principal to provide effort incentives: (i) to subjectively evaluate the agent’s performance; and (ii), to delegate the performance evaluation to a supervisor as a neutral party. Supervision generates contractible performance measures, but could create incentives for involved parties to engage in vertical side-contracting.

The analysis in this paper points to two fundamental observations with respect to the contract efficiency. First, both considered incentive mechanisms—subjective performance evaluation and supervision—potentially call for the provision of too low-powered incentives. The reasons however, are notably different. For subjective performance evaluation, providing low-powered incentives is aimed at eliminating the principal’s temptation to renge on promised but not court-enforceable incentive payments, and is thus vital for the credibility of relational incentive contracts. For supervision however, utilizing low-powered incentives is a viable means to deter involved parties from side-contracting. The second observation refers to the case where
supervision is the principal’s preferred incentive device. Then, ensuring collusion-proofness possibly necessitates inefficiently high fixed payments to both the agent and the supervisor. Fixed payments above their efficient levels are necessary to maintain the supervisor’s neutrality in assessing the agent’s performance as a basis for incentive payments. As shown, both explicated inefficiencies become more severe if the principal becomes less informed about the agent’s contribution to firm value.

The comparison of both considered incentive mechanisms further reveals that supervision can only be optimal from the principal’s perspective if she cannot perfectly observe the agent’s true contribution to firm value. This implies—according to the proposed model—that a fully informed principal does not rely on supervision as a means to provide the agent with credible effort incentives. In other words, a subjective evaluation of the agent’s performance conducted directly by the principal attains at least the same contract efficiency despite potentially entailing too low-powered incentives. Things are considerably different however, if the principal cannot perfectly observe the agent’s contribution to firm value. If so, supervision can dominate a subjective evaluation even though ensuring collusion-proofness imposes additional costs on the principal. In the present context, supervision might be strictly preferred since it constitutes a safeguard against erroneous incentive payments, which could occur under subjective performance evaluation with an incomplete informed principal.

In conclusion, this study provides a theoretical underpinning of why firms commonly consist of multiple hierarchy levels: supervisors (or middle managers) obtain better information about their subordinates’ performance, which in turn facilitates the design of more effective incentive contracts. This paper suggests that incorporating supervisors in corporate hierarchies can be profitable for firms, even if their sole responsibility is the performance evaluation of their subordinates. Nevertheless, empowering supervisors to appraise their subordinates’ performances as basis for incentive payments comes at a cost: to ensure supervisors’ impartiality, employment contracts are characterized by inefficiently high fixed payments and too low-powered incentives.
Appendix

Proof of Proposition 1.

Note first that \( e > 0 \) requires \( \beta > 0 \). Thus, (6) is satisfied if (7) holds, and can therefore be omitted. Assume for a moment that (8) is satisfied for the optimal bonus contract. Let \( \lambda \) and \( \xi \) denote Lagrange multipliers. Then, the Lagrangian is

\[
\mathcal{L}(\alpha, e) = V_L + \Delta V\rho(e) - \alpha - B(e, \theta) + \lambda [\alpha + B(e, \theta) - c(e)] + \xi \alpha.
\]  

(21)

The first-order conditions with respect to \( \alpha \) and \( e \) are

\[
-1 + \lambda + \xi = 0, \quad (22)
\]

\[
\Delta V\rho'(e) - B_e(e, \theta) + \lambda [B_e(e, \theta) - c'(e)] = 0. \quad (23)
\]

Suppose \( \lambda > 0 \). Then, \( \alpha + B(e, \theta) - c(e) = 0 \) due to complementary slackness. Since \( \alpha \geq 0 \), this would imply that \( B(e, \theta) \leq c(e) \), and hence \( e = 0 \). Thus, \( \lambda > 0 \) cannot be a solution such that \( \lambda = 0 \). We can then infer from (22) that \( \xi = 1 \). Consequently, \( \alpha^* = 0 \) due to complementary slackness. Since \( \lambda = 0 \), it follows from (23) that \( e^* \) solves \( \Delta V\rho'(e) = B_e(e, \theta) \).

Concavity of \( \rho(e) \) and convexity of \( B(e, \theta) \) in \( e \) ensure that the first-order approach is sufficient. Substituting \( \alpha^* = 0 \) and \( B(e^*, \theta) \) in the principal’s objective function leads to \( \Pi^{SE}(e^*, \theta) = V_L + \Delta V\rho(e^*) - B(e^*, \theta) \). Moreover, substituting \( \Pi^{SE}(e^*, \theta), B(e^*, \theta) = \beta^*(e^*, \theta)\tilde{\rho}(e^*, \theta) \), and \( \beta^*(e^*, \theta) = c'(e^*)/(\theta\rho'(e^*)) \) in (8) yields

\[
r \leq \frac{\theta\rho'(e^*)}{c'(e^*)} [V_L + \Delta V\rho(e^*) - \max\{\Pi^{SC}, \Pi^{S}\}] - \tilde{\rho}(e^*, \theta) \equiv r^{SE}(\theta). \quad (24)
\]

If \( r > r^{SE}(\theta) \), the efficient bonus \( \beta^*(e^*, \theta) \) would violate (8). In this case, the principal chooses the highest feasible \( \beta \) such that (8) binds:

\[
\frac{V_L + \Delta V\rho(\beta, \theta) - \beta\tilde{\rho}(\beta, \theta)}{\Pi^{SE}(\beta, \theta)} - \max\{\Pi^{SC}, \Pi^{S}\} = r\beta. \quad (25)
\]

Note that the left side of (25) is concave increasing in \( \beta \) for \( \beta < \beta^*(e^*, \theta) \), whereas the right side is linear increasing with slope \( r \). Thus, depending on \( r \), there are potentially two values of
\[ \beta \] which solve (25). Let \( \beta^*(r, \theta) \) denote the maximum value of \( \beta \) which implicitly solves (25), or equivalently,

\[ \beta = \frac{1}{r + \bar{\rho}(e(\beta, \theta), \theta)} \left[ V_L + \Delta V \rho(e(\beta, \theta)) - \max\{\Pi^{SC}, \Pi^S\} \right]. \tag{26} \]

Next, implicit differentiating (25) gives

\[ \frac{\partial \beta^*(r, \theta)}{\partial r} = \frac{\beta}{\frac{\partial \Pi^{SE}(\beta, \theta)}{\partial \beta}|_{\beta = \beta^*(r, \theta)} - r}. \tag{27} \]

Recall that \( \Pi^{SE}(\beta, \theta) \) is concave increasing in \( \beta \) as long as \( \beta < \beta^*(e^*, \theta) \), whereas the right side of (25) is linear increasing in \( \beta \) with slope \( r \). As a consequence, \( \frac{\partial \Pi^{SE}(\beta, \theta)}{\partial \beta}|_{\beta = \beta^*(r, \theta)} < r \) for \( r^{SE}(\theta) < r \leq \hat{r}^{SE}(\theta) \), where \( \hat{r}^{SE}(\theta) \) is characterized below. Hence, \( \frac{\partial \beta^*(\cdot)}{\partial r} < 0 \) for \( r^{SE}(\theta) < r \leq \hat{r}^{SE}(\theta) \).

Finally, there exists a threshold \( \hat{r}^{SE}(\theta) \) such that every \( \beta > 0 \) would violate (25) for \( r > \hat{r}^{SE}(\theta) \). Thus, \( \beta^*(r, \theta) = 0 \) for \( r > \hat{r}^{SE}(\theta) \). Because the left side of (25) is concave increasing in \( \beta \) as long as \( \beta < \beta^*(e^*, \theta) \), and the right side is linear increasing, the threshold \( \hat{r}^{SE}(\theta) \) implies that \( \hat{r}^{SE}(\theta) \) tangents (\( \Pi^{SE}(\beta, \theta) - \max\{\Pi^{SC}, \Pi^S\} \)). Hence, \( \hat{r}^{SE}(\theta) \) is defined by the tangent condition \( r = \frac{\partial \Pi^{SE}(\beta, \theta)}{\partial \beta}|_{\beta = \beta^*(r, \theta)} - \bar{\rho}(e(\beta^*(r, \theta), \theta), \theta) \).

\[ \operatorname{Tangency} \]

\[ r = \Delta V - \theta \beta^*(r, \theta)| \rho'(e(\beta^*(r, \theta), \theta)) \frac{\partial e}{\partial \beta}|_{\beta = \beta^*(r, \theta)} - \bar{\rho}(e(\beta^*(r, \theta), \theta), \theta). \tag{28} \]

Prove of Proposition 2.

Suppose for moment that \( w^{A*}(\alpha^*, \beta^*) \) and \( w^{S*} \) are collusion-proof such that (18) and (19) can be temporarily ignored. Recall that in this case the optimal contracts are characterized by \( w^{S*} = 0 \), \( \alpha^* = 0 \), and \( \beta^*(e^*) = e'(e^*)/\rho'(e^*) \), where \( e^* \) solves \( \Delta V \rho'(e) = B(e) \). Consequently, \( \Pi^S(e^*) = V_L + \Delta V \rho(e^*) - B(e^*) \). Since \( w^{S*} = 0 \), we can infer from (10) and (11) that the supervisor colludes if \( T^A, T^P > 0 \). Hence, collusion-proofness requires \( \bar{T}^A(r, \theta), \bar{T}^P(r) \leq 0 \).
Substituting $\alpha^* = 0$ and $\beta^*(e^*) = c'(e^*)/\rho'(e^*)$ in (12) yields the condition ensuring the agent refrains from side-contracting:

$$r \leq \theta \left[ \rho(e^*) - \frac{\rho'(e^*) c(e^*)}{c'(e^*)} \right] \equiv r^A(\theta). \tag{29}$$

Likewise, substituting $\Pi^S(e^*), \beta^*(e^*)$, and $\Pi^{SC} = V_L$ in (13) leads to the condition guaranteeing that the principal has no incentives to collude:

$$r \leq \frac{\rho'(e^*)}{c'(e^*)} \Delta V \rho(e^*) - \rho(e^*) \equiv r^P. \tag{30}$$

Thus, $(\alpha^*, \beta^*)$ and $w^{S*} = 0$ are collusion-proof if $r \leq r^S \equiv \min\{r^A(\theta), r^P\}$.

For $r > r^S$, there are three cases to discuss: (i) $r^A(\theta) < r \leq r^P$, (ii) $r^P < r \leq r^A(\theta)$; and (iii), $r^P, r^A(\theta) < r$. Consider first case (i). Then, (18) is equivalent to

$$\alpha + w^S \geq \beta \left[ \frac{T}{\theta} - \rho(\cdot) \right] + c(\cdot). \tag{31}$$

To minimize costs, the principal sets $\alpha$ and $w^S$ such that (31) binds, given that $\alpha^* \geq 0$ and $w^{S*} \geq 0$. Substituting $\alpha + w^S$ from (31) in the principal’s objective function yields the simplified problem for $r^A(\theta) < r \leq r^P$:

$$\max_{\beta} \Pi^S = V_L + \Delta V \rho(e(\beta)) - \frac{r}{\theta} \beta - c(e(\beta)). \tag{32}$$

The first-order condition implies that $\beta^*(r, \theta)$ solves

$$r = \theta \left[ \Delta V \rho'(e(\beta)) - c'(e(\beta)) \right] \frac{\partial e}{\partial \beta}. \tag{33}$$

Implicit differentiating (33) gives

$$\frac{\partial \beta^*(r, \theta)}{\partial r} = \frac{1}{\frac{\partial}{\partial \beta} \left[ \theta \left[ \Delta V \rho'(e(\beta)) - c'(e(\beta)) \right] \frac{\partial e}{\partial \beta} \right]}, \tag{34}$$

where the denominator is strictly negative due to the second-order condition. Hence, $\partial \beta^*(\cdot)/\partial r < 0$. Next, consider case (ii) where $r^P < r \leq r^A(\theta)$. We can infer from (13) that increasing $\alpha$ would reduce $\Pi^S$ and thus, raise $T_P^P(r)$. Consequently, $\alpha^* = 0$. Moreover, the marginal effect
of raising \( w^S \) on the collusion-proofness conditions (11) and (13) is \( 1/r \), and thus identical for both conditions. Consequently, adjusting \( w^S \) does not support collusion-proofness. Thus, to minimize costs, the principal sets \( w^{S*} = 0 \). Furthermore, the principal chooses the highest feasible \( \beta \) such that \( \bar{T}^P(r) = 0 \), which is equivalent to

\[
\frac{V_L + \Delta V(\rho(e(\beta))) - \beta \rho(e(\beta))}{\Pi^S(\beta)} - \Pi^{SC} = r\beta. \tag{35}
\]

Note that the left side of (35) is concave increasing in \( \beta \) as long as \( \beta < \beta^*(e^*) \), whereas the right side is linear increasing in \( \beta \). Consequently, depending on \( r \), there are potentially two values of \( \beta \) solving (35). Let \( \beta^*(r) \) denote the maximum value of \( \beta \) which implicitly solves (35). Since \( \Pi^{SC} = V_L \), it follows that \( \beta^*(r) \) solves

\[
\Delta V(\rho(e(\beta))) = \beta [r + \rho(e(\beta))]. \tag{36}
\]

Implicit differentiating (35) gives

\[
\frac{\partial \beta^*(r)}{\partial r} = \frac{\beta}{\left. \frac{\partial \Pi^S(\beta)}{\partial \beta} \right|_{\beta = \beta^*(r)} - r}. \tag{37}
\]

Recall that \( \Pi^S(\beta) \) is concave increasing in \( \beta \) for \( \beta < \beta^*(e^*) \), whereas the right side of (35) is linear increasing with slope \( r \). Hence, \( \partial \Pi^S(\beta)/\partial \beta \big|_{\beta = \beta^*(r)} < r \) for \( r^P < r \leq \hat{r}^S(\theta) \), where \( \hat{r}^S(\theta) \) is characterized below. As a result, \( \partial \beta^*(r)/\partial r < 0 \). Finally, consider case (iii) where \( r^P, r^A(\theta) < r \). Here, the principal needs to set \( \alpha \) and \( w^S \) as for case (i) with \( r^A(\theta) < r \leq r^P \) in order to deter the agent from collusion. Moreover, to ensure that neither the principal nor the agent colludes with the supervisor, it is necessary to choose the lowest of the two bonuses implicitly characterized by (33) and (36), i.e. \( \beta^* = \min\{\beta^*(r, \theta), \beta^*(r)\} \).

Finally note that the principal can always find a strictly positive bonus \( \beta \) which satisfies (33) for \( r > r^A(\theta) \). In contrast, if \( r > r^P \), there exists a threshold \( \hat{r}_1^S \) such that for \( r > \hat{r}_1^S \), every \( \beta > 0 \) would violate (35). Thus, \( \beta^*(r, \theta) = 0 \) for all \( r > \hat{r}_1^S \). The threshold \( \hat{r}_1^S \) thereby implies that
\(^\hat{r}_1^S\beta\) tangents \((\Pi^S(\beta) - \Pi^{SC})\), see (35). Hence, \(^\hat{r}_1^S\) is implicitly characterized by the tangent condition \(r = \frac{\partial \Pi^S(\beta)}{\partial \beta}\big|_{\beta = \beta^*(r)}\), which is equivalent to

\[
r = [\Delta V - \beta^*(r)] \rho'(e(\beta^*(r))) \frac{\partial e}{\partial \beta}\big|_{\beta = \beta^*(r)} - \rho(e(\beta^*(r))). \tag{38}
\]

Moreover, recall that \(\alpha^* + w^{S*} > 0\) for \(r > r^A(\theta)\), which is decreasing in \(\theta\), see (31). Hence, there exists a threshold \(^\hat{r}_2^S(\theta)\) satisfying \(\Pi^S(\^\hat{r}_2^S(\theta), \theta) = \Pi^{SC}\). Thus, \(^\hat{r}_2^S(\theta)\) is implicitly characterized by

\[
V_L + \Delta V \rho(e(\cdot)) - \beta^*(\^\hat{r}_2^S(\theta), \theta) \rho(e(\cdot)) - \alpha^*(\^\hat{r}_2^S(\theta), \theta) - w^{S*}(\^\hat{r}_2^S(\theta), \theta) = V_L, \tag{39}
\]

which is equivalent to

\[
\alpha^*(\^\hat{r}_2^S(\theta), \theta) + w^{S*}(\^\hat{r}_2^S(\theta), \theta) = \Delta V \rho(e(\cdot)) - \beta^*(\^\hat{r}_2^S(\theta), \theta) \rho(e(\cdot)). \tag{40}
\]

Combining the previous observations, the principal sets \(w^{S*}, \alpha^*, \beta^*(r, \theta) = 0\) if \(r > \^\hat{r}_2^S(\theta) \equiv \min\{\^\hat{r}_1^S, \^\hat{r}_2^S(\theta)\}\). \(\Box\)

**Proof of Proposition 3.**

Consider first the case \(\theta = 1\). Then, we can infer from proposition 1 that \(\alpha^* = 0\) and \(\beta^*(e^*, 1) = c'(e^*)/\rho'(e^*)\) under a subjective performance evaluation as long as \(r \leq r^{SE}(1)\). Moreover, proposition 2 implies that supervision leads to the same incentive contract for the agent and \(w^{S*} = 0\) as long as \(r \leq r^S\). Thus, \(\Pi^{SE}(e^*, 1) = \Pi^S(e^*)\) for \(r \leq \min\{r^{SE}(1), r^S\}\). For \(r > \min\{r^{SE}(1), r^S\}\), we need to consider two cases: (i) \(r^P \leq r^A(1)\); and (ii) \(r^A(1) < r^P\).

It is essential for both cases to demonstrate that \(r^{SE}(1) = r^P\). Since the agent plays a grim trigger strategy, utilizing a spot contract is the principal’s best fallback after she either reneged on \(\beta\) (subjective performance evaluation) or colluded with the supervisor (supervision). Hence, \(r^{SE}(1) = r^P\) is equivalent to

\[
\frac{\Pi^{SE} - \Pi^{SC}}{\beta^*(e^*, 1)} = \frac{\Pi^S - \Pi^{SC}}{\beta^*(e^*)}, \tag{41}
\]

which is satisfied because \(\beta^*(e^*, 1) = \beta^*(e^*)\) and \(\Pi^{SE} = \Pi^S\) for \(r \leq r^{SE}(1), r^P\).
Now suppose for a moment that (i) applies. For \( r > r^{SE}(1) = r^{P} \), the principal needs to adjust \( \beta^*(\cdot) \) for a subjective evaluation as well as for supervision. As (27) in connection with (25), and (37) in connection with (35) indicate, \( \beta^*(\cdot) \) is decreasing in \( r \) with the same rate under a subjective evaluation as under supervision. Thus, \( \Pi^{SE}(\cdot) \) is decreasing in \( r \) for \( r^{SE}(1) < r \leq \hat{r}^{SE}(1) \) with the same rate as \( \Pi^{S}(\cdot) \) for \( r^{P} < r \leq r^{A}(1) \). Hence, \( \Pi^{SE}(\cdot) = \Pi^{S}(\cdot) \) for \( r \leq r^{A}(1) \). Now suppose that (ii) applies. Then, for \( r^{A}(1) < r \leq r^{P} = r^{SE}(1) \), the principal needs to adjust \( \beta \) under supervision, but not under a subjective performance evaluation. Moreover, collusion-proofness requires to set \( \alpha \) and/or \( w^{S} \) above their efficient levels. Hence, \( \Pi^{SE}(\cdot) > \Pi^{S}(\cdot) \) for \( r \leq r^{A}(1) \), and \( \Pi^{SE}(\cdot) > \Pi^{S}(\cdot) \) for \( r^{A}(1) < r \leq \hat{r}^{SE}(1) \). For \( r > \hat{r}^{SE}(1) \) however, any strictly positive bonus \( \beta \) would neither be credible under a subjective performance evaluation, nor (profitably) collusion-proof under supervision. Hence, for \( r > \hat{r}^{SE}(1) \), the principal utilizes a spot contract.

Finally, consider the case \( \theta = 0 \). A completely uninformed principal (\( \theta = 0 \)) has the two subsequent implications. First, \( \bar{\rho}(e, 0) = 1 \), i.e. the agent would always obtain \( \beta \) under a subjective performance evaluation. Second, the agent’s collusion-proofness condition (18) becomes \( \beta \leq 0 \). Consequently, any strictly positive bonus \( \beta \) would lead to collusion between the agent and supervisor. Hence, the principal utilizes a spot contract for all values of \( r \) if \( \theta = 0 \).

**Proof of Proposition 4.**

First, recall from proposition 3 that \( \Pi^{SE}(\cdot) = \Pi^{S}(\cdot) \) for \( \theta = 1 \) and \( r \leq r^{A}(\theta) \). To demonstrate that \( \Pi^{SE}(\cdot) < \Pi^{S}(\cdot) \) for \( 0 < \theta < 1 \) and \( r \leq r^{A}(\theta) \), it is important to first verify that \( \partial \Pi^{SE}(\cdot)/\partial \theta > 0 \). To do so, we need to consider two cases: (i) \( r \leq r^{SE}(\theta) \); and (ii), \( r^{SE}(\theta) < r \leq \hat{r}^{SE}(\theta) \). For case (i), applying the Envelope Theorem yields

\[
\frac{\partial \Pi^{SE}(\cdot)}{\partial \theta} = -B_{\theta}(e, \theta) = \frac{-c'(e)\bar{\rho}(e, \theta)\theta \rho'(e) + c'(e)\bar{\rho}(e, \theta)\rho'(e)}{[\theta \rho'(e)]^2},
\]

which is strictly positive since \( \bar{\rho}(e, \theta) = -(1 - \rho(e)) < 0 \). For case (ii), note first that the principal’s expected cost of inducing an arbitrary effort level \( e \), \( \beta \bar{\rho}(\cdot) \), is decreasing in \( \theta \).
Thus, for any given bonus $\beta$, $\Pi^{SE}(\cdot)$ is increasing in $\theta$. Because the left side of the self-enforcement condition (8) is increasing in $\theta$, a higher value of $\theta$ implies that the principal can credibly commit to pay a higher (i.e., more efficient) bonus $\beta$. Hence, $\Pi^{SE}(\cdot)$ is increasing in $\theta$ for $r^{SE}(\theta) < r \leq \hat{r}^{SE}(\theta)$. In contrast, $\Pi^{S}(\cdot)$ is constant in $\theta$ for $r \leq r^{A}$. As a result, $\Pi^{SE}(\cdot) < \Pi^{S}(\cdot)$ for $0 < \theta < 1$ and $r \leq r^{A}(\theta)$.

For $r^{A}(\theta) < r \leq \min\{\hat{r}^{SE}(\theta), \hat{r}^{S}(\theta)\}$, supervision and a subjective performance evaluation lead to strictly higher expected profits than the application of a spot contract. To identify the superior incentive provision for this range, let $\hat{r}(\theta)$ denote the threshold interest rate which satisfies $\Pi^{SE}(\hat{r}(\theta), \theta) = \Pi^{S}(\hat{r}(\theta), \theta)$. Recall from proposition 3 that $\Pi^{SE}(\cdot) = \Pi^{S}(\cdot)$ if $r \leq r^{A}(1)$. Thus, $\Pi^{SE}(r, 1) > \Pi^{S}(r, 1)$ if $r > \hat{r}(1) = r^{A}(1)$, and vice versa. Consequently, we can infer that $\Pi^{SE}(\cdot) > \Pi^{S}(\cdot)$ if $r > \hat{r}(\theta)$, and $\Pi^{SE}(\cdot) \leq \Pi^{S}(\cdot)$ otherwise. To summarize the previous observations, supervision is preferred for $r \leq \hat{r}(\theta)$, and a subjective performance evaluation for $\hat{r}(\theta) < r \leq \min\{\hat{r}^{SE}(\theta), \hat{r}^{S}(\theta)\}$ respectively.

For $\min\{\hat{r}^{SE}(\theta), \hat{r}^{S}(\theta)\} < r \leq \max\{\hat{r}^{SE}(\theta), \hat{r}^{S}(\theta)\}$, either a subjective performance evaluation (if $\hat{r}^{SE}(\theta) \geq \hat{r}^{S}(\theta)$), or supervision (if $\hat{r}^{S}(\theta) > \hat{r}^{SE}(\theta)$) yields a strictly higher expected profit than the application of a spot contract. In contrast, if $r > \max\{\hat{r}^{SE}(\theta), \hat{r}^{S}(\theta)\}$, we can infer from propositions 1 and 2 that the principal utilizes a spot contract. 

\[\square\]
References


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Derivative of likelihood ratio.

Define \( R(e, \theta) \equiv \bar{\rho}(e, \theta)/\bar{\rho}(e, \theta) \). Since \( \bar{\rho}(e, \theta) = 1 - \theta(1 - \rho(e)) \), we have \( \bar{\rho}(\cdot) = \theta \rho'(e) \). Hence,
\[
R(e, \theta) = \frac{\theta \rho'(e)}{1 - \theta(1 - \rho(e))}.
\]
Taking the first derivative with respect to \( \theta \) yields
\[
\frac{\partial R(e, \theta)}{\partial \theta} = \frac{\rho'(e) \bar{\rho}(e, \theta) + \theta \rho'(e) [1 - \rho(e)]}{[\bar{\rho}(e, \theta)]^2},
\]
which is strictly positive. Thus, the likelihood ratio is increasing in \( \theta \).

Second-order condition for optimal effort level.

I subsequently demonstrate that the second-order condition for the optimal effort level \( e^* \) under a subjective performance evaluation is satisfied. By setting \( \theta = 1 \), one can make the same observation for supervision.

Since \( \rho(e) \) (from the principal’s payoff function) is concave increasing in \( e \), the first-order approach to identify \( e^* \) is also sufficient if the expected bonus \( B(e, \theta) \) is convex increasing in \( e \). Because \( \bar{\rho}_e(e, \theta) = \theta \rho'(e) \), the first derivative of \( B(e, \theta) \) with respect to \( e \) is
\[
\frac{\partial B(\cdot)}{\partial e} = c'(e) + \frac{\bar{\rho}(e, \theta)c''(e)}{\theta \rho'(e)} + \frac{\bar{\rho}(e, \theta)c'(e)\theta(-\rho''(e))}{[\theta \rho'(e)]^2},
\]
which is strictly positive for all \( e > 0 \). The second derivative leads to
\[
\frac{\partial^2 B(\cdot)}{\partial e^2} = c''(e) + \frac{[\bar{\rho}_e(e, \theta)c''(e) + \bar{\rho}(e, \theta)c''(e)]\theta \rho'(e) - \bar{\rho}(e, \theta)c''(e)\theta \rho''(e)}{[\theta \rho'(e)]^2}
\]
\[
\quad + \frac{[\bar{\rho}_e(e, \theta)c'(e) + \bar{\rho}(e, \theta)c''(e)]\theta(-\rho''(e))}{[\theta \rho'(e)]^4}
\]
\[
\quad - 2\bar{\rho}(e, \theta)c'(e)\theta(-\rho''(e))\theta \rho'(e)\rho''(e)
\]
\[
\quad \left[\theta \rho'(e)\right]^4
\]
\[
\quad = 2c''(e) + \frac{\bar{\rho}(e, \theta)c''(e)}{\theta \rho'(e)} + \frac{\bar{\rho}(e, \theta)c''(e)\theta(-\rho''(e))}{[\theta \rho'(e)]^2}
\]
\[
\quad + \frac{[\bar{\rho}_e(e, \theta)c'(e) + \bar{\rho}(e, \theta)c''(e)]\theta [-\rho''(e)]}{[\theta \rho'(e)]^2} + \frac{2\bar{\rho}(e, \theta)c'(e)\theta(-\rho''(e))^2}{[\theta \rho'(e)]^3}.
\]

Apparently, \( \partial^2 B(\cdot)/\partial e^2 > 0 \) for all \( e \) is guaranteed whenever \( c''(e) \geq 0 \). Hence, assuming \( c''(e) \geq 0 \) suffices to ensure that the first-order approach for identifying \( e^* \) is sufficient.
Implicit differentiation of $\beta^*(e^*, \theta)$ with respect to $\theta$ (supervision).

First, by utilizing (33), we can define

$$F \equiv \theta [\Delta V \rho'(e(\beta)) - c'(e(\beta))] \frac{\partial e}{\partial \beta} - r. \quad (45)$$

By applying the Implicit Function Theorem one get

$$\frac{\partial \beta^*(\cdot)}{\partial \theta} = -\frac{\partial F}{\partial \theta} = -\frac{\partial F}{\partial \beta} \left[ \frac{\partial e}{\partial \beta} \right]$$

$$= G$$

$$= \theta G$$

$$\frac{\partial \beta^*(\cdot)}{\partial \beta}$$

$$= \frac{\partial e}{\partial \beta}$$

$$= \frac{\partial}{\partial \beta} \left[ \theta [\Delta V \rho'(e(\beta)) - c'(e(\beta))] \frac{\partial e}{\partial \beta} - r \right], \quad (46)$$

where the denominator is strictly negative due to the second-order condition. Moreover, we can infer from (33) that $\theta G = r$. Since $\theta$ and $r$ are strictly positive, it must hold that $G > 0$. Hence, $\partial \beta^*(\cdot)/\partial \theta > 0.$ \hfill $\Box$