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# Dynamic Pricing in the Presence of Strategic Consumers and Oligopolistic Competition

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We present a dynamic pricing model for oligopolistic firms selling differentiated perishable goods to multiple finite segments of *strategic consumers* who are aware that pricing is dynamic and may time their purchases accordingly. This model encompasses strategic behavior by both firms and consumers in a unified stochastic dynamic game in which each firm's objective is to maximize its total expected revenues, and each consumer responds according to a shopping-intensity-allocation consumer choice model. We prove the existence of a unique subgame-perfect equilibrium, provide equilibrium optimality conditions, and prove monotonicity results for special cases. The model provides insights about equilibrium price dynamics under different levels of competition, asymmetry between firms, and multiple market segments with varying properties. We demonstrate that strategic behavior by consumers can have serious impacts on revenues if firms ignore that behavior in their dynamic pricing policies. Moreover, ideal equilibrium responses to consumer strategic behavior can recover only a portion of the lost revenues. A key conclusion is that firms may benefit more from limiting the information available to consumers than from allowing full information and responding to the resulting strategic behavior in an optimal fashion.

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## 1. Introduction

The rapid growth in Internet sales channels and point-of-sale technologies has given many firms a new capability for revenue management (RM)—they can now monitor demand for their products in real time and adjust prices dynamically in response to changes in demand patterns. In many settings, such dynamic pricing (DyP) can augment or replace traditional *capacity-control* RM in which multiple product "classes" are offered at different posted prices, and revenues are controlled by allocating capacity to the different price classes over time. Unfortunately, the increased flexibility and simplicity of DyP brings with it a new danger—consumers or third party brokers are now able to track price changes and, in some cases, can also track available capacity (for example, many online airline booking systems allow consumers to choose preferred seats from the remaining seats on a given flight). Experienced consumers may now behave *strategically* by timing their purchases to anticipated periods of lower price.

The difficult problem of price competition faced by firms operating in an oligopoly is made an order of magnitude more complex by the potential for strategic behavior of consumers. In a multifirm marketplace with strategic consumers, three types of strategic interactions must be considered: competition among firms for revenues, competition between consumers for capacity at favorable prices, and strategic interactions between firms and their consumers. Furthermore, in most settings, firms' products are *differentiated* to some degree, for example, by product or service quality or convenience attributes. Thus, another important type of strategic interaction that needs to be captured is *consumer choice*, that is, how consumers choose among different products.

Most classical DyP models assume that consumer behavior is *myopic*—a consumer makes a purchase as soon as the price is below his/her valuation for the product. A model that incorporates strategic consumers must move beyond this assumption to allow for consumers who assess future possible valuations and prices and aim to maximize some measure of utility for the purchase. In addition, most models avoid explicit consideration of competitors and assume a monopolistic market. The assumption of monopoly may be (partly) justified if a reasonable approximation of the effects of competitor response can be captured by a price-sensitive demand model. However, pure monopoly is rarely observed, and it is desirable to explicitly capture the effects of competition between firms on their pricing policies. This requires some form of dynamic differentiated products duopoly or oligopoly model that also captures

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strategic interactions of the firms with consumers. Underlying all of these considerations is an inherent stochasticity in the market that reflects the demand uncertainty so typical of practical situations in which RM techniques are employed.

A distinctive feature of this paper is that it explores, in a unified model, all strategic interactions in a marketplace characterized by multiple firms selling fixed stocks of perishable products to a finite population of consumers. The products are differentiated, and the firms employ DyP. Competition as well as intertemporal and discrete choice behavior of consumers are considered, and we allow for multiple consumer segments that are internally homogeneous in a stochastic sense but may differ with respect to degree of strategic behavior, product valuation, and price sensitivity. The model described here is a perfect information stochastic dynamic game in which each firm's objective is to maximize its total expected revenues. To model consumer response to the observed prices, we add to the classical random utility-based choice model an explicit option to delay purchase. Consumers evaluate this option using the expected present value of their utility.

The consumer choice model is based on smoothing, in a probabilistic sense, either a *specific choice* or a *multiple choice* rule. Under the specific choice rule, consumers allocate all of their willingness to purchase (which we call *eagerness*) to a specific product, whereas under the multiple choice rule they can be equally eager to purchase several of the available products. We show that the multiple choice model is more tractable both analytically and computationally than the specific choice model. For example, under multiple choice, we are able to establish the existence and uniqueness of a Markov-perfect equilibrium under very general conditions. We demonstrate in numerical experiments that, in the presence of strategic consumers, the difference in equilibria resulting from these two choice models can be small. (Whereas the model and most existence and structural results apply to a general valuation distribution, the numerical study focuses on the exponential case.) Because all other model components remain unchanged, using the multiple instead of specific choice model is attractive if our primary interest is the study of strategic behavior by consumers. We examine the structure of the equilibrium in a "high product supply" case, where each firm has enough capacity to supply the entire market, and show that equilibrium pricing policies are independent of population size. We also measure the effect of strategic behavior by comparing the expected revenues at two equilibria: with and without strategic consumer behavior. This difference in revenues tells how much a firm may lose if consumers become strategic. Numerical illustrations show that the drop may be significant: 30%–40% in the symmetric and more than 50% in the asymmetric equilibrium cases. Moreover, a firm that deviates from equilibrium by ignoring strategic behavior runs a risk of significant additional losses in revenues. This effect is stronger in an asymmetric equilibrium for a firm providing a product with lower consumer valuations than another firm, and is also stronger in a duopoly than in a monopoly. We show that the model is robust with respect to errors in estimation of competitor capacity. The model can also be used to study qualitative features of price dynamics under various settings. In particular, we see that strategic behavior reduces pricing flexibility of firms during most of the selling season while leading to significant price volatility at the end. The experiments suggest that only a portion of the revenue loss resulting from strategic behavior can be recovered by a proper response to it. Thus, firms should focus on limiting opportunities for strategic consumer behavior because there are only limited means for countering its effects.

In the next section, we provide a literature review. Section 3 describes the basic model setting and notation, and §4 presents our fundamental modelling assumptions. Section 5 focuses on the details of the game that models strategic interactions in the market and its analysis. Section 6 discusses a generalized view of the consumer choice model, and §7 outlines managerial insight that can be obtained from the model using numerical experiments with specific selections of the model parameters. Section EC.2 (provided in the e-companion)[1] examines the structural and asymptotic properties of equilibrium for the high-supply case. We conclude and indicate directions for further research in §8.

## 2. Literature Review

There is an extensive literature on DyP. For surveys, see Bitran and Caldentey (2003) and Elmaghraby and Keskinocak (2003). Research on coordinated pricing and inventory decisions is surveyed by Chan et al. (2004) and Yano and Gilbert (2003). Broad discussions of RM and pricing can be found in Talluri and van Ryzin (2004).

Monopolistic dynamic pricing models for an infinite population of myopic consumers is a well-researched area in RM. A number of papers, starting with Gallego and van Ryzin (1994), considered stochastic models with a Poisson arrival stream of price-sensitive myopic consumers. Monopoly models with strategic consumers are more complex, however,

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and are often considered in the deterministic form. For example, Besanko and Winston (1990) present a general deterministic DyP model. They show that the subgame-perfect equilibrium policy for the firm is to lower prices over time in a manner similar to *price skimming*. Su (2007) considers a deterministic demand model with consumers partitioned into four segments according to their valuation level and whether they are strategic or myopic. Consumers arrive continuously with fixed rates, and the seller looks for the optimal price and capacity rationing schedule. The article shows when the seller should use markdowns or markups. Aviv and Pazgal (2008) study the optimal pricing of fashion-like seasonal goods in the presence of forward-looking consumers who arrive according to a Poisson process with constant rate and have declining valuations for the product over the course of the season. This work considers the Nash equilibrium between a seller and strategic consumers for the cases of inventory-contingent pricing strategies and announced fixed discount strategies. Levin et al. (2007) study optimal DyP of perishable items by a monopolist facing strategic consumers.

A number of authors take a mechanism design approach to the problem of pricing in the presence of strategic consumers. For example, Harris and Raviv (1981) consider pricing for a monopolist facing rational consumers with unknown valuations and single-unit demands. Utilizing the revelation principle, the paper derives the form of the optimal mechanism. Elmaghraby et al. (2008) study markdown mechanisms in the presence of rational consumers with multiunit demands. Optimal mechanisms are analyzed both under known and unknown valuations. In both of the above two papers, all consumers are present in the market from the beginning. Gallien (2006) considers a dynamic pricing mechanism design problem for a monopolist selling identical nonperishable items to time-sensitive buyers with random independent valuations arriving over an infinite time horizon.

A related problem was considered by Liu and van Ryzin (2008), who study a two-period model with strategic consumers and quantity decisions (rather than pricing), which are used to induce early purchases. Ovchinnikov and Milner (2007) study a pricing model with consumers who, over multiple selling seasons, learn to delay their purchases after observing recurrent last-minute discounts.

Consumer choice models are studied extensively in the marketing literature. A review of a large portion of this work can be found in the book by Anderson et al. (1992). A survey of consumer choice and strategic behavior models in RM can be found in Shen and Su (2007).

The economics literature on static as well as dynamic models of duopolistic and oligopolistic competition is also extensive. We only mention representative papers considering dynamic models of competition by firms selling differentiated products and refer readers to the book by Vives (1999) for a comprehensive survey. Fershtman and Pakes (2000) consider a stochastic dynamic oligopoly model that can describe both collusive behavior and price wars. This work uses a traditional logit form for the consumer choice model. The paper uses numerical experiments to compare collusive to noncollusive environments and discovers sets of industry states where collusion does occur. Chintagunta and Rao (1996) consider a differential game model for a duopoly in which brand choice probabilities are in logit form, and derive equilibrium price paths as well as steady state prices. Both papers assume that supply is unlimited. In the RM field, Xu and Hopp (2006) study a model of one-time inventory and dynamic price competition for the cases of piecewise deterministic and geometric Brownian motion arrival processes and nonstrategic consumers. The products sold by different retailers in this model are not differentiated. Perakis and Sood (2006) consider a model of dynamic price and protection level competition in a market with perishable differentiated products under uncertain demand. The authors use robust optimization ideas to address the competitive aspect together with demand uncertainty. Gallego and Hu (2006) consider a choice-based, multiplayer, game-theoretic dynamic pricing model for perishable products in the case of stationary demand. The pricing policies of the firms considered in this work are in the open-loop form. Granot et al. (2006) study a multiperiod model of competition between independent retailers selling identical perishable goods to myopic consumers who purchase only if the observed price is below their valuations, and may otherwise return to the same retailer in the following period.

The literature on demand learning in the context of dynamic pricing of a stock of perishable items includes Aviv and Pazgal (2005) and other articles for the case of monopoly with myopic consumers, Levina et al. (2008) for the case of monopoly with strategic consumers, and Bertsimas and Perakis (2006), who treat both a monopoly and an oligopoly with linear demand functions.

The articles most relevant to our work are Talluri (2003) and Lin and Sibdari (2008), which consider dynamic competition under multinomial logit discrete consumer choice models. The first paper examines duopolistic competition in terms of the sets of available fare products (a capacity control approach). Both observable and unobservable (in a simplified form) competitor capacity cases are analyzed. The second

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paper considers oligopolistic price competition with observable capacities. The main contribution of the current paper is that we consider all aspects of strategic behavior by both firms and the consumers in a unified stochastic model.

## 3. Basic Model Elements

Fundamental elements of the model described in this section include a planning horizon (sales season), firms and their characteristics, and the consumer population and its structure.

### 3.1. Planning Horizon

Consider a planning horizon of $T$ decision periods indexed by $t \in \{0, \dots, T-1\}$. The number of decision periods is sufficiently large that any continuous-time counting process in the model can be well approximated by its discrete-time analogue. Under such a choice of $T$, the maximum intensity of any process has to be relatively small, and we can assume that at most one event in any of the processes can occur per decision period. The time unit is equal to the length of a single decision period. Thus, the intensity of a process is represented by the probability of one event of this process occurring per decision period.

### 3.2. Firms

There are $m$ firms selling perishable, differentiated products, and competing in the same market. Each firm $j$ offers a single product at unit cost $c_j$ and price $p_j$ (adjusted dynamically), and has initial integer capacity $Y_j > 0$. As sales progress, the initial capacities of the firms are depleted. The remaining capacity is $y_j \geq 0$. At time $T$, all remaining capacity is lost.

### 3.3. Consumer Population

Real-world consumer populations are typically heterogeneous, and it is important to understand how this heterogeneity can affect pricing. Because we focus on product and intertemporal (strategic) choice behavior, our model needs to capture differences of consumers both in their product preferences and in their tendency toward strategic behavior.

We model differences in product preferences through consumers' valuations, interpreted as willingness to pay. This is a common approach to modelling heterogeneity that is present, in some form, in the single-product (monopoly) models of Besanko and Winston (1990), Su (2007), Aviv and Pazgal (2008), and others. Valuations are not as important as the resulting heterogeneous consumer choices, but the notion of valuation is convenient and widely used. Thus, like Su (2007), who considered two fixed valuation levels, we consider a finite number of consumer groups (called *segments*) that differ in their valuations. We are also consistent with Su (2007) in allowing consumer segments to differ in their strategic behavior. We model differences in strategic behavior through differences in a discount factor on future utilities.

In our model, segments are *stochastically homogeneous* in the sense that all consumers within a segment have the same distributional and parametric characteristics. We disregard consumer heterogeneity in time of arrival (considered, for example, by Su 2007 and Aviv and Pazgal 2008) and assume, like Besanko and Winston (1990), that there is a finite number of consumers who are present in the market from the beginning of the sales season. (The model can be extended to incorporate consumer arrivals and departures without service.)

Let $N$ be the total number of consumers in the market, and $s$ the number of consumer segments. The initial segment $r$ capacity is $N_r$ (so that $N = \sum_{r=1}^{s} N_r$), and the remaining capacity at future times is $n_r \geq 0$. Each consumer requires one unit of the product. As consumers acquire the items, capacities of firms and market segments are depleted.

### 3.4. Consumer Product Choice Characteristics

At time $t \in \{0, 1, \dots, T-1\}$, a segment $r$ consumer evaluates a purchase of product $j$ at price $p_j$ according to a *linear random utility* of the form $a_{trj} + \epsilon_{trj} - p_j$, where $a_{trj}$ is a known constant describing consumer perception of quality or value of the product, and $\epsilon_{trj}$ is a random variable with mean zero (see §3.2 of Anderson et al. 1992). The distribution of the random vector $\boldsymbol{\epsilon}_{tr} = (\epsilon_{tr1}, \dots, \epsilon_{trm})$ is continuous with a given density function $f_{tr}(\boldsymbol{\epsilon})$. The quantity $B_{trj} = a_{trj} + \epsilon_{trj}$ formally corresponds to the valuation of product $j$ at time $t$ by a segment $r$ consumer. It is equivalent to specify the density $f_{tr}(\mathbf{b})$ of $\mathbf{B}_{tr} = (B_{tr1}, \dots, B_{trm})$ instead of $f_{tr}(\boldsymbol{\epsilon})$. In a stochastic sales model like this, reducing consumer heterogeneity to a few segments is a significant simplification compared to the approach used, for example, by Besanko and Winston (1990), in which each consumer in the population has a fixed valuation. Even if the initial valuations were perfectly known, under a stochastic sales process, it would be impossible to tell which consumer has just purchased a product, leaving the firms with an effectively unknown valuation distribution and, thus, an imperfect information problem under competition (for a brief discussion of mechanism design as an alternative approach, see §EC.1.1 of the e-companion). However, differences in the $a_{trj}$'s still provide a way to model heterogeneity in consumer preferences.

### 3.5. Consumer Strategic Characteristics

For a segment $r$ consumer, the utility of buying an item in the future is discounted by a factor $\beta_r \in [0, 1]$ per unit of time, which can be interpreted as the *degree of strategic behavior* of the consumer. Indeed, the value $\beta_r = 0$ means that the consumer completely disregards the possibility of a future purchase; that is, the

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consumer is *myopic*. The value $\beta_r = 1$ means that the consumer values the current purchase the same as a purchase at any point in the future and thus exhibits fully strategic behavior. Intermediate values of $\beta_r \in (0, 1)$ determine how long consumers can postpone their purchase without excessive loss of utility. Under appropriate assumptions on the degree of consumer sophistication, this leads to a recursive relation for the utility of delaying a purchase.

The notion of consumer segments is common in marketing practice. Consumers are considered relatively homogeneous within segments but may differ widely between segments. For example, in airline booking, one simple segmentation is between business and leisure travelers. Business travelers typically book on short notice, place higher valuation on a timely booking, and tend not to be strategic. In contrast, price-sensitive leisure travelers will often have schedule flexibility, can begin watching price and availability well in advance of their intended departure, and are more likely to be strategic. We show how this situation can be represented by the consumer choice specifications of our model in §EC.1.2 of the e-companion.

# 4. Modelling Assumptions

To define precisely how the sales process unfolds, how consumers choose between products and strategize over time to make their purchases, and how the firms set their prices, we first need to discuss a number of modelling assumptions. These assumptions can be broadly classified into the following categories: information availability, rationality, demand structure, and consumer choice.

## 4.1. Information Availability

Because our primary focus is on strategic consumer behavior, we model DyP in markets in which consumers and firms have access to enough information to exhibit rational behavior in the economic sense. In current markets, such information may be only partially available, but full information is an important limiting case with significant potential for management insight regarding pricing and other policies (for further discussion of information availability in practical contexts, see §EC.1.3 of the e-companion). Thus, we assume

Perfect Information. *Firms and consumers have perfect knowledge of all market information, including the remaining capacities of the firms, the market segments, and all their distributional and parametric characteristics.*

The perfect information assumption is common because of its analytical advantages. Models of DyP under competition and stochastic demand considered by Gallego and Hu (2006) and Lin and Sibdari (2008) contain this assumption. Moreover, all deterministic models have to employ this assumption either implicitly or explicitly. Alternative modelling approaches include those of demand learning (see Levina et al. 2008) and the robust framework (see Perakis and Sood 2006). However, none of these settings have been fully explored under a general pattern of uncertainty in market characteristics and strategic consumer behavior. We also assume

Zero Information Cost. *Information relevant for decisions of firms and consumers, including current prices in the case of consumers, is available at no cost.*

For firms, this assumption is a logical development of the perfect information one. For consumers, some models assume that there are *search costs* related to collecting price and other information. However, in current e-markets, the cost of information for consumers becomes negligible.

## 4.2. Rationality

We model firms' behavior as equilibrium strategic decisions in a dynamic stochastic game, followed by consumer responses to these decisions (in a leader-follower sense). In terms of decision-making capabilities of firms, we assume full rationality in the game-theoretic sense, a standard modelling assumption. For the consumers, we assume an ability for or access to a calculation of equilibrium strategies for all market participants as well as their own expected utility in any information state (this is somewhat weaker than full rationality):

Perfect Foresight. *All market participants are sophisticated or can employ third-party services so that they correctly anticipate the strategic behavior of their opponents and compute resulting event probabilities and expected payoffs.*

## 4.3. Demand Structure

We next describe two key assumptions on demand structure.

Demand as a Counting Process. *Aggregate product demand from each segment of the population is a counting process with intensity dependent on time, price, and other market conditions. Each segment process is a sum of independent demand processes originating from individual consumers.*

The model of demand as a continuous-time or discrete-time counting process is widely used by the research community. It is also natural to assume that the demand process from each market segment is a sum of individual consumer demand processes because such an assumption conforms both to our intuition and the statistical properties of continuous-time counting (Poisson) processes. The

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assumption implies that the demand *intensity* for each segment of the population and each product is the sum of the individual demand intensities from all remaining consumers in the segment. Henceforth, we will use the term *shopping intensity* for individual consumer demand intensity.

Shopping Intensity Control. *Consumers respond to current prices and other market conditions such, as the remaining capacities of firms and market segments, by controlling their shopping intensities.*

That is, consumers cannot control the precise timing of their purchases, but more favorable prices make them more eager to purchase; hence, purchases *tend* to occur sooner when prices are relatively low. This assumption is motivated by two practical considerations. First, in a real marketplace, all interested consumers cannot make their purchases at exactly the same time—both the capacity of the purchasing channel and the timing of access by consumers to that channel are constrained. (Many consumers may know that the price is favorable but be unable to take the time to complete the purchasing process until later.) Second, real-world consumers have a tendency to procrastinate (see Su 2006). One of the possible reasons for procrastination is the *status quo bias* (see Samuelson and Zeckhauser 1988). This decision bias is a tendency of a decision maker to stick with the default or status quo option, which, in our case, is the option of delaying the purchase.

Because, in practice, the total segment demand intensity is bounded across all products, the individual consumer shopping intensity for each segment and product is also bounded. The value of this upper bound captures transaction uncertainties in the market as well as consumer tendency toward procrastination, and can be interpreted as the *inverse of the average time to acquire* an item of a particular product by an eager consumer. Because of our representation of intensities by probabilities of a single event occurrence, we formally make an assumption of

Maximum Shopping Intensity. *There exists a maximum probability that a consumer who is eager to acquire a particular product is able to do so in a given decision period. This probability bound is the same for all consumers.*

Note that the above assumptions ensure that the competition between firms can be described by a Markovian model. The subsequent assumptions specify the precise form of consumer shopping intensity control.

### 4.4. Choice Model

A description of consumer behavior in terms of shopping intensities also allows us to capture uncertainty in the choices of individual consumers. The underlying assumption is

Intensity Allocation as a Choice Model. *Consumers allocate their shopping intensity among products according to a discrete-choice model with the outside alternative of delay in purchase.*

Preferences and shopping behavior of real-world consumers may vary both across consumers and across time. The linear random utility choice model whose basic elements were described in §3 captures two distinct sources of this variability: variation between market segments and random variation due to consumer indecisiveness.

Because consumers do not migrate between different market segments during the course of a typical sales season, the variability of preferences by segment is deterministic in nature. However, empirical research has also indicated inconsistent consumer behavior as a potential source of demand uncertainty. For example, Tversky (1972, p. 281) indicates, "When faced with a choice among several alternatives, people often experience uncertainty and inconsistency. That is, people are often not sure which alternative they should select, nor do they always take the same choice under seemingly identical conditions." Tversky (1972, p. 281) concludes that, "In order to account for the observed inconsistency and the reported uncertainty, choice behavior has to be viewed as a probabilistic process." Thus, in a situation of choice between several dynamically priced products, the final product purchased by each particular consumer may only be predicted up to some probability. This residual uncertainty constitutes the second level of preference variability mentioned above. Preferences are determined by random utility, and randomness in utility is equivalent to randomness in valuations. Therefore, we assume that

Valuations Are Known Up to Their Distribution. *Given the consumer's segment, the state of his/her valuation is known only as a distribution rather than a specific value. This limitation to the knowledge of the precise valuation applies both to the firms and to all consumers.*

In a static setting, given a standard interpretation of valuations, this assumption is a logical equivalent of the statement that consumer behavior, even on the individual level, is described by a probabilistic choice model. As pointed out in §3, valuations are formally the components of random utility. In a dynamic setting, this assumption means that there is a residual level of uncertainty in valuations corresponding to the random component of utility, and this uncertainty cannot be resolved by the consumers until the purchase is complete. Nevertheless, upon making a purchase, a consumer must feel a sense of commitment to his/her decision: "if I buy this product, I must value it more than any other option." In other words, consumers experience an increase in perceived valuation for a product after the purchase, see Cohen

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and Goldberg (1970) and Winter (1974). Formally, we assume that

Consumer Perceptions Are Conditional on Purchase. *After the purchase, the distribution of valuations perceived by the consumers is the conditional valuation distribution given that the purchased product is preferred to any other option. Consumers and firms account for this shift in distributions in their decision making.*

Together with "perfect foresight" on the part of the consumers, this assumption ensures consistency of the expected utility expression derived in this paper for the dynamic case with the expression for the static case provided in Anderson et al. (1992). The final element of consumer preference required is an exact expression for the value of delay in purchase. We give this formula after the derivation of a recursive relation for the expected utility in §5, where we also describe the competitive game and its equilibrium. In §6, we present a generalized view of the consumer choice that covers both the model in §4 and an alternative one with strategic consumer behavior but a simpler equilibrium structure.

## 5. The Competitive Game

### 5.1. Firms' Game and Consumer Response

We state the firms' game by describing its subgame for each decision period and each possible information state. (For a general treatment of game theory and equilibrium concepts, see Fudenberg and Tirole 1991.) At time $t$, let the remaining capacities of the firms be given by a vector $\mathbf{y} = (y_1, \ldots, y_m)$ and the vector of remaining numbers of consumers across segments be given by $\mathbf{n} = (n_1, \ldots, n_s)$ (henceforth called segment sizes). All market participants are aware of this information. Then, the following sequence of events occurs: the firms simultaneously announce their product prices, and the consumers in each segment respond with their shopping intensities. The information state at the time of the firms' decisions is given by $(\mathbf{y}, \mathbf{n})$, whereas the information state at the time of consumer decisions is $(\mathbf{y}, \mathbf{n}, \mathbf{p})$. The move of the firms is the $m$-vector of prices $\mathbf{p}(t, \mathbf{y}, \mathbf{n}) = (p_1(t, \mathbf{y}, \mathbf{n}), \ldots, p_m(t, \mathbf{y}, \mathbf{n}))$, and the response of segment $r$ consumers is the $m$-vector of shopping intensities $\boldsymbol{\lambda}_r(t, \mathbf{y}, \mathbf{n}, \mathbf{p}) = (\lambda_{r1}(t, \mathbf{y}, \mathbf{n}, \mathbf{p}), \ldots, \lambda_{rm}(t, \mathbf{y}, \mathbf{n}, \mathbf{p}))$, $r = 1, \ldots, s$. Following a consumer decision, a sale of one unit to one consumer may occur with the probability (shopping intensity) chosen by this consumer. The probability that a unit of product $j$ is sold to an individual consumer of segment $r$ is $\lambda_{rj}(t, \mathbf{y}, \mathbf{n}, \mathbf{p})$ for each consumer in that segment. The probability that no sale occurs to any of the segments is $1 - \sum_{r,j} n_r \lambda_{rj}(t, \mathbf{y}, \mathbf{n}, \mathbf{p})$.

If there exists a unique Markov-perfect equilibrium, then the equilibrium expected payoffs in each information state are uniquely defined. We defer for the moment the question of the existence of equilibrium beyond the current period $t$ and suppose that the expected payoffs of the firms and the expected utilities of consumers are uniquely defined for all information states in period $t+1$. Then, for each $(\mathbf{y}, \mathbf{n})$, we can determine the expected payoffs at time $t$ in a stochastic game $\mathcal{G}(t, \mathbf{y}, \mathbf{n})$ of simultaneous pricing decisions by firms followed by consumer shopping intensity responses.

Let $\bar{\lambda}$ be the maximum shopping intensity specified in the "maximum shopping intensity" assumption. Then, $\lambda_{rj}(t, \mathbf{y}, \mathbf{n}, \mathbf{p}) \leq \bar{\lambda}$ for all $t$, $r$, $j$ and $(\mathbf{y}, \mathbf{n}, \mathbf{p})$. To ensure that the discrete-time model provides an adequate representation of a continuous-time demand process, we choose the granularity of the discretization so that $\bar{\lambda} mN$ is sufficiently small compared to 1. This implies that the probability of more than one purchase occurring in a given decision period is negligible. The function of $\bar{\lambda}$ in our model is to relate product choice probabilities, discussed next, to consumer shopping intensity responses.

For the case of myopic consumers, consumer choice theory implies that, given prices $\mathbf{p} = (p_1, \ldots, p_m)$, the probability of a segment $r$ consumer choosing product $j$ (*choice probability*) is

$$P\Big(B_{trj} - p_j = \max_{j'=1,\ldots,m} (B_{trj'} - p_{j'})\Big).$$

(See equation (3.2) on p. 66 of Anderson et al. 1992.) In this expression, an option of no purchase is disregarded, and the consumer has to choose at least one product. The expected surplus of a representative myopic consumer in segment $r$ is

$$\mathrm{E}_{\mathbf{B}_{tr}}\Big[\max_{j=1,\ldots,m} (B_{trj} - p_j)\Big] = \sum_{j=1}^{m} \int_{b_j - p_j \geq b_{j'} - p_{j'},\, j' \neq j} (b_j - p_j) f_{tr}(\mathbf{b})\, d\mathbf{b}. \qquad (1)$$

(Expression (1) matches the surplus component of an aggregate welfare function defined by equation (3.10) on p. 71 of Anderson et al. 1992.) This expected utility corresponds to a static model of the market and is conditional on successful purchase of one of the items. Our task is to generalize both the static choice probabilities and the static expected utility to their dynamic equivalents, and we use a constructive approach to present these generalizations. We start by generalizing the choice probabilities.

At time $t$, let $Q_r(t, \mathbf{y}, \mathbf{n}, \mathbf{p})$ be the *certainty equivalent of a future purchase* in information state $(\mathbf{y}, \mathbf{n}, \mathbf{p})$ as evaluated by a segment $r$ consumer (exact form to be determined based on an assumption to follow). The certainty equivalent captures the value of a potential future purchase. It is affected by the time remaining to complete the purchase, product availability, and, most

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importantly, the level of competition from other consumers. Therefore, a consumer can take into account future behavior of firms and responses from the market, given an appropriate model for $Q_r(t, \mathbf{y}, \mathbf{n}, \mathbf{p})$. In the context of a discrete choice model, $Q_r(t, \mathbf{y}, \mathbf{n}, \mathbf{p})$ is precisely the value of the option of delaying purchase for the consumer. Adding this option to the set of alternatives, we immediately obtain that a segment $r$ strategic consumer chooses product $j$ with probability $P(\mathbf{B}_{tr} \in A_{rj}(t, \mathbf{y}, \mathbf{n}, \mathbf{p}))$, where

$$A_{rj}(t, \mathbf{y}, \mathbf{n}, \mathbf{p}) = \Big\{ \mathbf{b} \in \mathbb{R}^m \colon b_j - p_j = \max_{j'=1,\ldots,m} (b_{j'} - p_{j'}),\; b_j - p_j \geq Q_r(t, \mathbf{y}, \mathbf{n}, \mathbf{p}) \Big\}. \quad (2)$$

The condition $b_j - p_j = \max_{j'=1,\ldots,m}(b_{j'} - p_{j'})$ ensures that product $j$ utility is maximal among all products, whereas the condition $b_j - p_j \geq Q_r(t, \mathbf{y}, \mathbf{n}, \mathbf{p})$ ensures that it also exceeds the value of a possible future purchase.

Because of the "maximum shopping intensity" assumption, the resulting purchase probability for product $j$ is proportional to the corresponding choice probability with coefficient $\bar{\lambda}$:

$$\lambda_{rj}(t, \mathbf{y}, \mathbf{n}, \mathbf{p}) = \bar{\lambda} P(\mathbf{B}_{tr} \in A_{rj}(t, \mathbf{y}, \mathbf{n}, \mathbf{p})). \quad (3)$$

The use of a scaling coefficient such as $\bar{\lambda}$ is necessary because we move from a static model of consumer choice to a dynamic one. An important implication of expressions (2) and (3) is that, all prices and the current state being the same, a lower value of $Q_r(t, \mathbf{y}, \mathbf{n}, \mathbf{p})$ will result in a higher shopping intensity by the consumer. This discussion can be summarized as follows:

Proposition 1. *The responses of any of the $n_r$ segment $r = 1, \ldots, s$ consumers in information state* $(\mathbf{y}, \mathbf{n}, \mathbf{p})$ *are given by* (3).

We next generalize the expected utility derived by consumers (1) to the dynamic strategic case.

### 5.2. Expected Utility of a Future Purchase

Let the expected utility of a segment $r$ consumer in information states $(\mathbf{y}, \mathbf{n})$ and $(\mathbf{y}, \mathbf{n}, \mathbf{p})$ at time $t$ be $U_r(t, \mathbf{y}, \mathbf{n})$ and $U_r(t, \mathbf{y}, \mathbf{n}, \mathbf{p})$, respectively. At the end of the planning horizon, consumers have the expected utility of 0: $U_r(T, \mathbf{y}, \mathbf{n}) = 0$ for all $r$ and $(\mathbf{y}, \mathbf{n})$. Also, if all firms run out of capacity, the expected utilities are 0: $U_r(t, \mathbf{0}, \mathbf{n}) = 0$ for all $t$, $r$, and $\mathbf{n}$. Thus, utilities are uniquely defined in all "terminal" states. If there exists a (possibly mixed) equilibrium strategy profile $\mathbf{p}^*(t, \mathbf{y}, \mathbf{n})$, then the expected utility as a function of information state $(\mathbf{y}, \mathbf{n})$ is related to that of $(\mathbf{y}, \mathbf{n}, \mathbf{p})$ through $U_r(t, \mathbf{y}, \mathbf{n}) = \mathrm{E}[U_r(t, \mathbf{y}, \mathbf{n}, \mathbf{p}^*(t, \mathbf{y}, \mathbf{n}))]$, with expectation taken over $\mathbf{p}^*(t, \mathbf{y}, \mathbf{n})$. To complete the recursion, we now need to find $U_r(t, \mathbf{y}, \mathbf{n}, \mathbf{p})$ as a function of expected utilities at time $t+1$.

In the myopic case, a representative consumer averages the maximum utility over the valuation distribution. The average in the dynamic strategic case is taken over all possible consumer choices, over the valuation realizations given those choices, and over all possible sale events. The result of this averaging is an expression for $U_r(t, \mathbf{y}, \mathbf{n}, \mathbf{p})$ which, as we shall see, also suggests a reasonable assumption on the form of $Q_r(t, \mathbf{y}, \mathbf{n}, \mathbf{p})$. We state the recursion in the following proposition, where we employ the following standard notation: given a $k$ dimensional vector $\mathbf{z}$, the $k$ dimensional vector $(\hat{z}, \mathbf{z}_{-l})$ is obtained by replacing the $l$th component of $\mathbf{z}$ with $\hat{z}$. A proof of this proposition and other formal statements, unless they are immediate, can be found in §EC.3 of the e-companion.

Proposition 2. *Consider decision period $t$ and suppose that there exists a unique equilibrium in all subsequent periods $t+1, t+2, \ldots, T-1$. The expected utility of a segment $r$ consumer in information state* $(\mathbf{y}, \mathbf{n}, \mathbf{p})$ *is given uniquely by*

$$\begin{aligned} U_r(t, \mathbf{y}, \mathbf{n}, \mathbf{p}) &= \bar{\lambda} \sum_{j=1}^{m} \int_{\mathbf{b} \in A_{rj}(t, \mathbf{y}, \mathbf{n}, \mathbf{p})} (b_j - p_j - \beta_r U_r(t+1, \mathbf{y}, \mathbf{n})) f_{tr}(\mathbf{b})\, d\mathbf{b} \\ &\quad - \beta_r \sum_{r' \neq r} (n_{r'} - I(r' = r)) \boldsymbol{\lambda}_{r'}(t, \mathbf{y}, \mathbf{n}, \mathbf{p})^\top \boldsymbol{\Delta}_{r'} U_r(t+1, \mathbf{y}, \mathbf{n}) \\ &\quad + \beta_r U_r(t+1, \mathbf{y}, \mathbf{n}), \end{aligned} \quad (4)$$

*where* $\boldsymbol{\Delta}_{r'} U_r(t+1, \mathbf{y}, \mathbf{n})$, $r' = 1, \ldots, s$, *is a vector of terms* $\Delta_{r'j} U_r(t+1, \mathbf{y}, \mathbf{n}) = U_r(t+1, \mathbf{y}, \mathbf{n}) - U_r(t+1, (y_j - 1, \mathbf{y}_{-j}), (n_{r'} - 1, \mathbf{n}_{-r'}))$, $j = 1, \ldots, m$.

Expression (4) is consistent with the expected utility expression (1) in the static myopic case. Indeed, if $\beta_r = 0$, then all terms in (4) except the first one are zero. The first term is identical to (1) up to a scaling coefficient $\bar{\lambda}$ if $Q_r(t, \mathbf{y}, \mathbf{n}, \mathbf{p}) = 0$, which is assumed by definition in the myopic case. Expression (4) also suggests a reasonable form for $Q_r(t, \mathbf{y}, \mathbf{n}, \mathbf{p})$. The first term of (4) is the only part of the expected utility that is under direct control of the consumer in the current state. It is natural to assume that the consumers expect to obtain a positive utility from the purchase. For example, in the mechanism design literature, nonnegativity of consumer utility is a constraint on the mechanism that ensures individual rationality of the consumers (see Gallien 2006). The expression under the integral is guaranteed to be positive for every $\mathbf{b} \in A_{rj}(t, \mathbf{y}, \mathbf{n}, \mathbf{p})$ if we assume the following:

Value of an Explicit Delay. *For a segment $r$ consumer at time $t$ in state* $(\mathbf{y}, \mathbf{n}, \mathbf{p})$, *the value of the option of explicitly delaying the purchase is given by*

$$Q_r(t, \mathbf{y}, \mathbf{n}, \mathbf{p}) = \beta_r U_r(t+1, \mathbf{y}, \mathbf{n}). \quad (5)$$

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Although it is possible to assume other forms for $Q_r(t, \mathbf{y}, \mathbf{n}, \mathbf{p})$, the resulting insights would not change significantly as long as it is close to $U_r(t, \mathbf{y}, \mathbf{n}, \mathbf{p})$. From an analytical point of view, excluding a possible dependence on $\mathbf{p}$ is a significant simplification, because the expression for consumer response is then more straightforward, and one does not need to worry about the continuity of response with respect to prices (which is essential for the existence proof). Moreover, for small values of $\bar{\lambda}$, the contribution of the first two terms in (4) is small compared to the last one, which is precisely $\beta_r U_r(t+1, \mathbf{y}, \mathbf{n})$. From this point on, we omit $\mathbf{p}$ in the notation for certainty equivalent.

Given that firms use their equilibrium pricing strategies in the future, and the market responds according to our model, expression (4), together with a "value of an explicit delay" assumption, allows a strategic consumer to account for the behavior of other consumers. Indeed, more intense shopping from other consumers would lead to a smaller product supply, and would thus reduce the expected utility. According to our model, the given consumer would then increase shopping intensity as well.

### 5.3. Segment Demand, Firms' Payoffs, and Equilibrium

An immediate by-product of Propositions 1 and 2 is an expression for the demand intensity of a segment (demand function):

Corollary 1. *Under the conditions of Proposition* 2, *the demand intensity for product j from segment r in information state* $(\mathbf{y}, \mathbf{n}, \mathbf{p})$ *can be expressed as*

$$D_{rj}(t, \mathbf{y}, \mathbf{n}, \mathbf{p}) = n_r \bar{\lambda} P(\mathbf{B}_{tr} \in A_{rj}(t, \mathbf{y}, \mathbf{n}, \mathbf{p})). \qquad (6)$$

Next, we examine the expected profits of firm $j$ in information state $(\mathbf{y}, \mathbf{n})$ at time $t$ assuming that all future prices follow equilibrium policies. Let firm $j$ equilibrium expected payoff be $R_j(t, \mathbf{y}, \mathbf{n})$. At the end of the planning horizon (time $T$), the firms have future expected payoffs of 0: $R_j(T, \mathbf{y}, \mathbf{n}) = 0$ for all $(\mathbf{y}, \mathbf{n})$. If the firms run out of capacity or consumers, their expected payoffs are 0: for all $j$ and $t$, $R_j(t, \mathbf{y}, \mathbf{n}) = 0$ for all $(\mathbf{y}, \mathbf{n})$ such that $y_j = 0$ or $\mathbf{n} = \mathbf{0}$.

Suppose that, for $t' > t$, the equilibrium expected profits, $R_j(t', \mathbf{y}, \mathbf{n})$ are uniquely defined, and the firms use prices $\mathbf{p}$ at time $t$. Then the expected future profit of firm $j$ is obtained by taking the expected value over all possible purchase realizations:

$$\begin{aligned} R_j(t, \mathbf{y}, \mathbf{n}, \mathbf{p}) &= \sum_{r=1}^{s} D_{rj}(t, \mathbf{y}, \mathbf{n}, \mathbf{p}) \\ &\quad \cdot (p_j + R_j(t+1, (y_j - 1, \mathbf{y}_{-j}), (n_r - 1, n_{-r})) - c_j) \\ &\quad + \sum_{r=1}^{s} \sum_{j' \neq j} D_{rj'}(t, \mathbf{y}, \mathbf{n}, \mathbf{p}) \\ &\quad \cdot R_j(t+1, (y_{j'} - 1, \mathbf{y}_{-j'}), (n_r - 1, \mathbf{n}_{-r})) \\ &\quad + \left(1 - \sum_{r=1}^{s} \sum_{j'=1}^{m} D_{rj'}(t, \mathbf{y}, \mathbf{n}, \mathbf{p})\right) R(t+1, \mathbf{y}, \mathbf{n}). \end{aligned}$$

After collecting terms and introducing $\Delta_{rj'} R_j(t+1, \mathbf{y}, \mathbf{n}) = R_j(t+1, \mathbf{y}, \mathbf{n}) - R_j(t+1, (y_{j'} - 1, \mathbf{y}_{-j'}), (n_r - 1, \mathbf{n}_{-r}))$ (the firm $j$ marginal value of a segment $r$ consumer paired with a sale to firm $j'$), we can rewrite this expression as

$$\begin{aligned} R_j(t, \mathbf{y}, \mathbf{n}, \mathbf{p}) &= \sum_{r=1}^{s} D_{rj}(t, \mathbf{y}, \mathbf{n}, \mathbf{p})(p_j - \Delta_{rj} R_j(t+1, \mathbf{y}, \mathbf{n}) - c_j) \\ &\quad - \sum_{r=1}^{s} \sum_{j' \neq j} D_{rj'}(t, \mathbf{y}, \mathbf{n}, \mathbf{p}) \Delta_{rj'} R_j(t+1, \mathbf{y}, \mathbf{n}) \\ &\quad + R_j(t+1, \mathbf{y}, \mathbf{n}). \end{aligned} \qquad (7)$$

Expression (7) determines the payoff of firm $j$ corresponding to pure strategy profile $\mathbf{p}$ in game $\mathcal{G}(t, \mathbf{y}, \mathbf{n})$. General proofs of existence of a Markov-perfect equilibrium in pure strategies for dynamic oligopoly models are difficult to obtain. The difficulty is increased by the absence of easily verifiable conditions for uniqueness of the equilibrium in a subgame starting from a particular state—in our case, $\mathcal{G}(t, \mathbf{y}, \mathbf{n})$ describes first moves in this subgame. Often, additional restrictions on the model are needed such as a specific functional form for a consumer choice or demand model. Our first existence result is for a Markov-perfect equilibrium in mixed strategies (probability measures on prices). For this we require the following relatively weak assumptions:

(A) Bounded Prices. *The prices used are bounded by a sufficiently large constant* $\bar{p}$.

(B) Tie-Breaker Mechanism. *There is a mechanism that selects an equilibrium to be implemented by the firms if* $\mathcal{G}(t, \mathbf{y}, \mathbf{n})$ *has multiple equilibria. This mechanism ensures that equilibrium payoffs in* $\mathcal{G}(t, \mathbf{y}, \mathbf{n})$ *are uniquely defined.*

An assumption similar to (B) was also used by Lin and Sibdari (2008) in addition to a specific (multinomial logit) consumer choice model. As far as the equilibrium existence is concerned, this assumption is made only to enable notational convenience in defining the expected payoffs in every $\mathcal{G}(t, \mathbf{y}, \mathbf{n})$. The general modelling issue of determining which equilibrium is played when there are several equilibria is well known in game theory. Schelling (1960) proposed a theory of "focal points," which assumes that in real-life situations players may be able to coordinate using

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information that is not captured by the formal statement of the game. In our case, the firms may, for example, choose an equilibrium with the lowest average or lowest maximum price in the market. It is now straightforward to establish the general existence statement that follows.

Theorem 1. *Under assumptions* (A) *and* (B), *there exists a Markov-perfect equilibrium in mixed strategies.*

Given equilibrium strategy profile $\mathbf{p}^*(t,\mathbf{y},\mathbf{n})$, $R_j(t,\mathbf{y},\mathbf{n})$ is computed from $R_j(t,\mathbf{y},\mathbf{n},\mathbf{p})$ as $R_j(t,\mathbf{y},\mathbf{n}) = \mathrm{E}[R_j(t,\mathbf{y},\mathbf{n},\mathbf{p}^*(t,\mathbf{y},\mathbf{n}))]$. Next, we generalize the consumer choice assumptions of §4 to obtain a simpler equilibrium structure.

## 6. Generalization of the Choice Model

Choice behavior of consumers can also be interpreted as explicit allocation of their efforts to acquire one or more products. Let $\mathbf{x}$ be the nonnegative $m$-vector of shopping intensities reflecting these efforts. A modest generalization of the "maximum shopping intensity" assumption is as follows:

Generalized Maximum Shopping Intensity. $\|\mathbf{x}\|_q \le \bar{\lambda}$, *where* $\|\mathbf{x}\|_q$ *is a q-norm of* $\mathbf{x}$.

We explicitly consider two cases: $q=1$ so that $\|\mathbf{x}\|_1 = \sum_{j=1}^m |x_j|$, and $q=\infty$ so that $\|\mathbf{x}\|_\infty = \max_{j=1,\dots,m}|x_j|$. We show that the case $q=1$ corresponds to the classical choice model based on linear random utility and can be termed as *specific choice* behavior, whereas the case $q=\infty$ corresponds to *multiple choice* behavior and provides an alternative model that is more tractable analytically and computationally than the case $q=1$. The case $q=\infty$ preserves most of the model features except for its different view of consumer choice behavior. Thus, it can be used as a proxy in the study of competitive behavior of firms and strategic behavior of consumers. The numerical experiments show that the effects of strategic behavior are very similar in these two cases. We also replace all consumer choice assumptions of §4 and the "value of an explicit delay" with the following:

Averaging Behavior. *We assume that a strategic consumer behaves as follows: first, for each possible realization of the valuation vector a consumer determines his/her optimal shopping intensity, and second, the consumer averages those intensities over the valuation distribution. In the valuation of the purchase delay option the consumer also averages the expected utility corresponding to each valuation vector over the valuation distribution.*

This assumption generalizes a standard construction of discrete consumer choice theory because the choice probabilities are obtained precisely by averaging consumer choices over the valuation vector (thus, replacing "intensity allocation as a choice model"). It implicitly contains "valuations are known up to their distribution," because the uncertainty in valuations is not resolved until a purchase is complete. Finally, it also provides a (somewhat stronger) alternative to the "consumer perceptions are conditional on purchase" because the expected utility is computed as if consumers zero in on specific valuation vectors after the purchase. A general version of Proposition 2 is

Proposition 3. *Consider decision period t and suppose that there exists a unique equilibrium in all subsequent periods* $t+1, t+2, \dots, T-1$. *The equilibrium responses of any of the* $n_r$ *consumers in segment r in the information state* $(\mathbf{y},\mathbf{n},\mathbf{p})$ *are given by* $\lambda_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p}) = \mathrm{E}[x^*_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p},\mathbf{B}_{tr})]$, *where the optimal shopping intensity* $\mathbf{x}^*_r(t,\mathbf{y},\mathbf{n},\mathbf{p},\mathbf{B}_{tr})$ *in this information state for given valuation vector* $\mathbf{B}_{tr}$ *is, for* $q=\infty$,

$$x^*_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p},\mathbf{B}_{tr}) = \bar{\lambda} I(B_{trj} - p_j \ge \beta_r U_r(t+1,\mathbf{y},\mathbf{n})), \quad j=1,\dots,m, \tag{8}$$

*and, for* $q=1$,

$$x^*_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p},\mathbf{B}_{tr}) = \bar{\lambda} I(\mathbf{B}_{tr} \in A_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p})), \quad j=1,\dots,m, \tag{9}$$

*where* $A_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p})$ *is defined by* (2). *Moreover, the equilibrium expected utility of a segment r consumer in information state* $(\mathbf{y},\mathbf{n},\mathbf{p})$ *is given uniquely by*

$$\begin{aligned}U_r(t,\mathbf{y},\mathbf{n},\mathbf{p}) &= \mathrm{E}_{\mathbf{B}_{tr}}\big[\mathbf{x}^*_r(t,\mathbf{y},\mathbf{n},\mathbf{p},\mathbf{B}_{tr})^\top\big(\mathbf{B}_{tr} - \mathbf{p} - \mathbf{1}\beta_r U_r(t+1,\mathbf{y},\mathbf{n})\big)\big] \\ &\quad -\beta_r \sum_{r'\ne r}(n_{r'} - I(r'=r))\boldsymbol{\lambda}_{r'}(t,\mathbf{y},\mathbf{n},\mathbf{p})^\top \boldsymbol{\Delta}_{r'} U_r(t+1,\mathbf{y},\mathbf{n}) \\ &\quad +\beta_r U_r(t+1,\mathbf{y},\mathbf{n}),\end{aligned} \tag{10}$$

*where* $\mathbf{1}$ *is an m-vector of ones, and* $\boldsymbol{\Delta}_{r'} U_r(t+1,\mathbf{y},\mathbf{n})$ *is defined as before.*

We can interpret the consumer response identified in the above proposition as follows (from the viewpoint of a consumer who knows his/her postpurchase valuation vector). In the case $q=\infty$, she would shop with the maximum possible intensity $\bar{\lambda}$ for any of the products for which the utility of the valuation-price difference exceeds the discounted future expected utility; that is, the consumer would exhibit *multiple* choice behavior. In the case $q=1$ (our base model), the consumer would pursue with maximum possible intensity a product with the highest valuation-price difference—*specific* choice behavior. Corollary 1 is modified for the case of multiple choice as follows:

Corollary 2. *Let* $q=\infty$ *and consider decision period t, and suppose that there exists a unique equilibrium*

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*in all subsequent periods $t+1, t+2, \ldots, T-1$. Then, the demand intensity for product $j$ from segment $r$ in information state $(\mathbf{y}, \mathbf{n}, \mathbf{p})$ can be expressed as*

$$D_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p}) = n_r \bar{\lambda} P(B_{trj} - p_j \geq \beta_r U_r(t+1,\mathbf{y},\mathbf{n})). \quad (11)$$

Because demand functions for product $j$ in this case depend only on $p_j$, we use a specialized notation $D^M_{rj}(t,\mathbf{y},\mathbf{n},p_j)$ instead of $D_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p})$. An important immediate implication of Corollary 2 is that the objective of each firm becomes separable in prices under the multiple choice model:

Corollary 3. *Under conditions of Corollary 2, the firm $j$ profit expression (7) is separable in the components of $\mathbf{p}$ for all $j = 1, \ldots, m$.*

This result is important, because it shows that each firm can set its price independently from others in each information state. Because of this, Theorem 1 can be strengthened as follows:

Theorem 2. *For the case $q = \infty$ and under assumptions (A) and (B), there exists a Markov-perfect equilibrium in pure strategies.*

Existence of equilibrium in pure strategies is important because it is much easier to interpret. We can also show the existence of a *unique* Markov-perfect equilibrium in pure strategies for the multiple choice case and one segment of consumers under milder assumptions than (A) and (B):

Theorem 3. *If $q = \infty$, $s = 1$, and, for all $t$, the distribution of $\mathbf{B}_{tr}$ satisfies the assumptions of*

(C) Logconcavity. *Marginal density $f_{trj}(b)$ of each component of $\mathbf{B}_{tr}$ is logconcave, and*

(D) Regularity. *The marginal distribution of each component of $\mathbf{B}_{tr}$ satisfies the regularity condition $\lim_{b\to\infty} bP(B_{trj} \geq b) = 0$,*
*then there exists a unique Markov-perfect equilibrium in pure strategies.*

We remark that the family of logconcave densities includes the exponential, normal, and uniform, among others. Also, the regularity condition is standard in the RM literature (see, for example, Gallego and van Ryzin 1994), and can be interpreted as vanishing of expected revenue from period $t$ sales of product $j$ to a consumer when the price tends to infinity.

# 7. Managerial Insight from Numerical Illustrations

In our illustrations, we focus on the case when the unit costs are negligible (the standard case of RM), and, for each $t$ and $r$, the $B_{trj}$, $j = 1, \ldots, m$ are independent and identically distributed according to the exponential distribution with mean $a_{rj}$:

$$f_{tr}(b_j) = \frac{1}{a_{rj}} e^{-b/a_{rj}}. \quad (12)$$

The value of the mean $a_{rj}$ is independent of time, and may be associated with a constant perception of quality of product $j$ by segment $r$ consumers. When $B_{trj}$s are interpreted as valuations, $a_{rj}$ loosely corresponds to the average price a myopic consumer in segment $r$ would pay for product $j$. Although this choice of the form of distribution is very specific, it does not affect the qualitative nature of the insights as long as the variability of $B_{trj}$s is comparable in magnitude to the presented case. However, some aspects of real markets that we have not captured in this paper can affect the results. For example, if there are information or waiting costs for the consumers, they are likely to reduce strategic behavior. Consumer arrivals and departures without a purchase are also likely to reduce strategic behavior and change the dynamics of prices (but it is easy to incorporate these two aspects into our model). If the firms could ration the product sales, this would also reduce the negative effects of strategic behavior or perhaps even invert them. Limited information availability for the consumers or the firms (or both) can lead to quite different market dynamics. Nevertheless, the option to include myopic consumer segments can approximate incomplete information for some consumers, and insights derived from our model are interesting as a limiting case.

In this section, we review a number of numerical experiments that study the effects of strategic behavior on equilibrium and the effects of deviation from equilibrium. Because the critical performance indicator for the model is expected revenues, we define the effect of strategic behavior as the percentage difference in revenues between the strategic and purely myopic cases for each firm relative to the myopic case. We refer to this measure as the *percentage of revenue difference*.

Our first finding, presented in §7.1, is that the difference between equilibria under specific and multiple choice consumer behavior is quite modest. This is seen in a variety of settings for symmetric and asymmetric duopolistic equilibria. The difference is particularly small *when consumers behave strategically*. Moreover, the difference in effects of strategic behavior on equilibria is small. Thus, in many cases we can safely use a multiple choice-based model for a qualitative study of the consequences of consumer strategic behavior on equilibrium (and our further experiments are restricted to this case).

In §7.2, we show that the effects of strategic behavior become stronger when the level of competition

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increases from monopoly to duopoly and then three-firm oligopoly. The strongest strategic behavior effects are observed when firms approximately divide the market, and this characteristic is observed even in heterogeneous markets where only a fraction of consumers are strategic.

In §7.3, we examine the losses suffered by a firm that deviates from equilibrium by wrongly assuming that the consumers are myopic when they are in fact strategic. The loss in our experiments reaches 7% and is typically the least severe for the firm providing a better quality product. Also, the loss is higher in a duopoly than in monopoly and occurs over a wider range of market sizes.

E-companion §EC.5 describes additional experiments. §EC.5.1 indicates that asymmetry in consumer willingness to pay has a much stronger effect on equilibrium than asymmetry in the firms' capacities, and that the firm that provides the lower quality product stands to lose most if its consumers become strategic (over 50% of revenue). The effects of model settings on prices are easiest to study with typical realizations of the equilibrium pricing policies. Computer simulations of the policies presented in §EC.5.2 show that consumer strategic behavior *reduces flexibility of the policy*, and this effect is stronger for a weaker company. Moreover, consumer strategic behavior, coupled with stochastic demand, causes the price to be very volatile at the end of the selling season, with a possibility of very significant increases. Additional experiments illustrate the robustness of our model and examine the effects of deviation from equilibrium when a firm makes an incorrect assumption about competitor capacity. We also illustrate interactions of competition with strategic consumer behavior in a heterogeneous market (with two different market segments). We find that strategic behavior of the high-valuation segment has the greatest effect on equilibrium revenues and pricing policies. When only the low-valuation segment is strategic, the impact is smaller, but the firms have to "price out" low-valuation consumers at the beginning of the season. We also observe that the dynamics of prices vary remarkably under these scenarios.

In the subsequent numerical experiments for the case of a one-segment market, we use the following base settings: a symmetric equilibrium with all initial inventories $Y_j = 20$ and price sensitivities $a_j = 1$. The time horizon is $T = 200{,}000$ with $\bar{\lambda} = 0.00005$. Thus, the expected "number of purchase possibilities" of each consumer is $\bar{\lambda}T = 10$. In two-segment market experiments, as well as those involving simulation of pricing policies, we used a coarser discretization with $T = 20{,}000$ and $\bar{\lambda} = 0.0005$ (still resulting in $\bar{\lambda}T = 10$). In §EC.4 of the e-companion, we provide mathematical expressions for demand functions and best-response mappings of the firms for the case of exponential valuation distributions (12) and discuss numerical aspects of our calculations.

### 7.1. Effects of Strategic Behavior in Symmetric and Asymmetric Equilibria Under the Multiple and Specific Consumer Choice Models

In this section, we examine equilibrium revenues in asymmetric and symmetric duopoly and study how asymmetry and choice behavior interact with the effects of strategic behavior. Expected revenues are computed for the following cases: a symmetric case with $a_1 = a_2 = 1$ and $Y_1 = Y_2 = 20$, as well as two asymmetric cases, $a_1 = 2$, $a_2 = 1$, $Y_1 = Y_2 = 20$ (the first firm provides a higher quality product) and $a_1 = a_2 = 1$, $Y_1 = 20$, $Y_2 = 40$ (the second firm has a larger capacity). All three equilibria are examined for either choice model, both $\beta = 0$ and $\beta = 1$, and the population size $N$ in the set $\{20, 30, \ldots, 80\}$. The relative difference in multiple choice equilibrium revenues, as a percentage of revenues in the specific choice case, is shown in Table 1 (the values of the expected revenues used to produce Table 1 are provided in Table EC.1 of the e-companion). Importantly, the difference in the specific and multiple choice equilibrium revenues is small in all scenarios except for the case of myopic consumers with $N = 20$ or 30. Moreover, this difference is much smaller in the case of fully strategic consumers. Table 2 shows the effects of strategic behavior as the percentage of revenue difference. For all population sizes except $N = 30$, and independently of the initial capacity of the firms or the relative quality of their products, this effect is very close for the two choice models. The strategic effect is strongest when $N = 40$ or 50—a situation in which the firms roughly

**Table 1** Percentage Increase in Equilibrium Revenues for the Multiple Choice Case over the Specific Choice Case in a Duopoly in the Cases of Symmetric and Asymmetric Equilibria

| | | Equilibrium (%) | | |
|---|---|---|---|---|
| $N$ | $\beta$ | Symmetric | $a_1 = 2a_2$ | $Y_1 = Y_2/2$ |
| 20 | 0 | 8.1 | (4.5, 12.0) | (8.1, 8.1) |
| 20 | 1 | 3.7 | (1.2, 7.8) | (3.7, 3.7) |
| 30 | 0 | 14.8 | (4.6, 21.0) | (10.3, 8.9) |
| 30 | 1 | 3.7 | (1.1, 7.6) | (3.7, 3.7) |
| 40 | 0 | 3.8 | (2.1, 5.3) | (1.6, 3.8) |
| 40 | 1 | 3.6 | (0.5, 4.4) | (3.2, 3.8) |
| 50 | 0 | 1.5 | (1.0, 2.1) | (2.1, 2.7) |
| 50 | 1 | 2.7 | (0.8, 3.3) | (2.1, 2.7) |
| 60 | 0 | 1.0 | (0.6, 1.3) | (2.1, 2.2) |
| 60 | 1 | 1.6 | (0.9, 2.3) | (1.9, 2.1) |
| 70 | 0 | 0.7 | (0.5, 0.9) | (2.0, 1.5) |
| 70 | 1 | 1.0 | (0.7, 1.4) | (1.8, 1.7) |
| 80 | 0 | 0.5 | (0.4, 0.7) | (1.5, 1.0) |
| 80 | 1 | 0.7 | (0.5, 1.0) | (2.2, 1.9) |

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**Table 2** Effect of Strategic Behavior as the Percentage Difference in Equilibrium Revenues in the Strategic and Myopic Consumer Cases Relative to the Myopic Case

| | Equilibrium with multiple choice (%) | | | Equilibrium with specific choice (%) | | |
|---|---|---|---|---|---|---|
| $N$ | Symmetric | $a_1 = 2a_2$ | $Y_1 = Y_2/2$ | Symmetric | $a_1 = 2a_2$ | $Y_1 = Y_2/2$ |
| 20 | 32.5 | (8.6, 58.1) | (32.5, 32.5) | 29.7 | (5.7, 56.5) | (29.7, 29.7) |
| 30 | 31.2 | (6.1, 56.9) | (32.1, 31.8) | 23.8 | (2.8, 51.6) | (27.7, 28.4) |
| 40 | 39.3 | (16.7, 54.7) | (38.4, 38.7) | 39.2 | (15.4, 54.3) | (39.4, 38.7) |
| 50 | 32.4 | (19.8, 44.6) | (32.0, 39.8) | 33.2 | (19.6, 45.2) | (32.1, 39.8) |
| 60 | 21.6 | (15.2, 31.0) | (27.7, 38.6) | 22.1 | (15.4, 31.7) | (27.6, 38.5) |
| 70 | 15.4 | (11.1, 22.3) | (26.8, 36.9) | 15.7 | (11.3, 22.7) | (26.6, 37.1) |
| 80 | 11.7 | (8.6, 17.1) | (25.2, 31.3) | 11.9 | (8.7, 17.4) | (25.7, 31.9) |

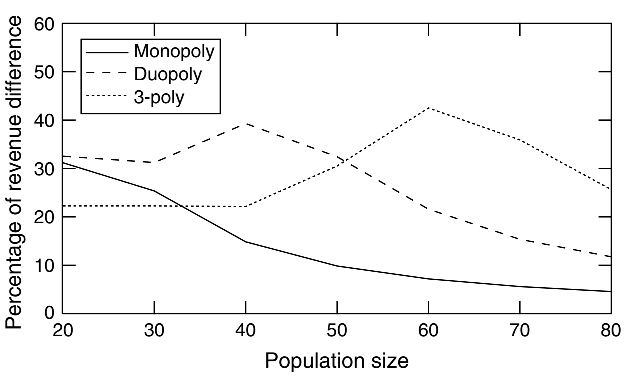
divide the market. The strategic effect declines for $N$ larger than 50 because the competition among consumers for a limited product supply reduces their expected utility. A possible explanation for the smaller effect when $N$ is less than 40 is a generally lower level of prices and revenues because of more intense competition between the companies (so that there is a lower marginal decrease in revenues once the consumers become strategic).

### 7.2. Interplay of Strategic Behavior with the Level of Competition

To further study the interplay of strategic behavior with the level of competition, we compare symmetric equilibria for the cases of monopoly, duopoly, and three-firm oligopoly under the multiple consumer choice assumption. The percentage revenue difference for each population size $N$ in the set $\{20, 30, \ldots, 80\}$ is given in Figure 1 (equilibrium expected revenues used to produce these graphs are given in Figure EC.1 in the e-companion). We observe that strategic consumer behavior leads to lower revenues, and this effect is aggravated by the presence of competitors (the largest difference in revenues is just above 30% in the case of monopoly, about 40% in the case of duopoly, and more than 40% in the case of three-firm oligopoly). In competitive situations, the largest differences occur when the firms divide the market (for the population size of 40 in the case of duopoly and 60 in the case of oligopoly with three firms). The percentage of revenue difference increases because there is a noticeable increase in revenues when consumers are myopic and the market size is increased proportionally with the number of firms. Although an increase in revenues is also observed under similar changes in a market with strategic consumers, the absolute value of this increase is lower than in the myopic case. Increases are made possible by the stochastic nature of the market because the firms have more opportunities to set higher prices when they face a market of proportionally larger size despite the presence of competitors. Thus, this result is dependent on the stochastic nature of the market in our model. Information availability also affects this result because, under incomplete information, it would be harder for the firms to take advantage of particular supply-demand ratio configurations. The firms generally receive higher expected revenues as the population size increases, but reaching a comparable value generally requires larger $N$ in the strategic case.

Significant effects of strategic behavior persist even if some consumers are myopic. An experiment with two market segments of varied sizes, one with fully strategic consumers ($\beta = 1$) and another with myopic consumers ($\beta = 0$), is described in §EC.5.3 of the e-companion.

**Figure 1** Effect of Strategic Behavior on Equilibrium Revenues Under Different Levels of Competition as a Function of Population Size




### 7.3. Losses from Deviating from Equilibrium by Wrongly Assuming That the Consumers Are Myopic

Next, we examine the effects of deviation of one firm from equilibrium in the case of duopoly if the firm assumes that the consumers are myopic when, in fact, they are fully strategic. In this experiment, we use the multiple choice model, and the other firm continues to treat consumers as strategic (consumers and the other firm continue to react to the current market information in the same way as in the original equilibrium). We examine both symmetric ($a_1 = a_2 = 1$, $Y_1 = Y_2 = 20$) and asymmetric ($a_1 = 2$, $a_2 = 1$, $Y_1 = Y_2 = 20$) equilibrium cases and three situations: deviation of one firm from a symmetric equilibrium (labeled "symmetric"), deviation of the firm providing a lower quality product in an asymmetric equilibrium (labeled "weak"), and of the firm providing a higher quality product in an asymmetric equilibrium (labeled "strong"). As an additional benchmark, we also examine a similar type of deviation in a monopoly (labeled "monopoly"). The relative revenue loss incurred by the deviating firm is expressed as a percentage decrease in expected revenues resulting from deviation versus the expected revenues found in a correct equilibrium.

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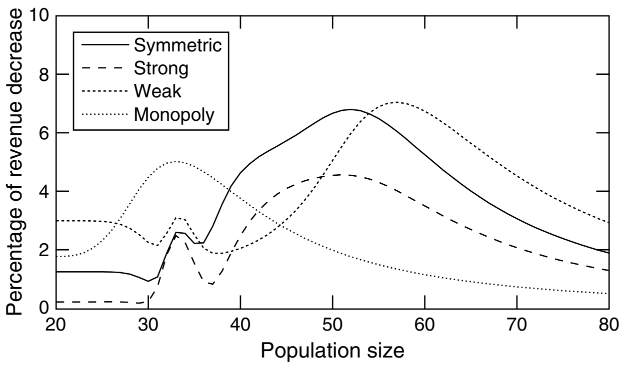
**Figure 2** **Relative Revenue Loss for Incorrect Assumption of Myopic Consumer Behavior When It Is Fully Strategic, as a Function of Population Size**




The losses for the four scenarios are presented in Figure 2 as functions of $N \in \{20, \ldots, 80\}$. The main conclusion is that the strong firm is generally less affected by deviation than the weak firm or the firm in the symmetric scenario. The impact of the loss, which may exceed 7% for the weak case, and almost reaches 7% for the symmetric case, is a direct reduction in profits compared to the case where a correct consumer behavior model is used. The maximum loss in monopoly is comparable to that of the strong firm (about 5%), but significant losses in monopoly occur over a narrower range of market size. The losses finally decrease when $N$ increases and there is more intense competition between the consumers. However, the losses for all duopoly cases exceed 1% even for $N = 80$—much higher than in the monopoly.

## 8. Conclusions and Directions for Future Research

This paper presents a new, inherently stochastic, dynamic pricing model for an oligopoly selling to a finite population of strategic consumers. To our knowledge, it generalizes strategic consumer behavior in a manner not previously achieved. We demonstrate the existence of a subgame-perfect equilibrium pricing policy under reasonable assumptions, provide optimality conditions, and explore some of the structural properties of solutions. We also discuss the effect of strategic consumer behavior on the performance of pricing policies. This model permits computational analysis of dynamics of pricing policies under various market settings. Models of this type can enable managers and business analysts to explore, offline, the complex strategic interactions of pricing and inventory policies within competitive industries and across multiple consumer segments.

One of the most restrictive assumptions of our model is "information availability." In reality, neither firms, nor consumers know all characteristics of the market precisely and in real time. A research direction that could further improve understanding of strategic interactions in the market is incorporating the elements of *learning* into the model. This will allow firms to learn characteristics of the market as time progresses and continuously adjust the optimal pricing policy. Incorporation of consumer learning will provide a more realistic representation of consumer behavior.

## 9. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

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# Electronic Companion—"Dynamic Pricing in the Presence of Strategic Consumers and Oligopolistic Competition" by Yuri Levin, Jeff McGill, and Mikhail Nediak, *Management Science*, DOI 10.1287/mnsc.1080.0936.

# Electronic Companion

## EC.1. Supplemental discussion of modelling assumptions

### EC.1.1. Deterministic but unknown valuations as an alternative modelling approach

An alternative to our specifications of consumer product choice would be to consider deterministic but unknown valuations. Such models have indeed been considered in economics and RM literature using a mechanism design approach for the monopoly case (see Harris and Raviv (1981) and Gallien (2006)). They assume that a seller controls how sales proceed (e.g., full control of a demand fill rate up to the individual consumer level). Along with Gallego and Hu (2006) and Lin and Sibdari (2007) who model sales as a stochastic process, we do not assume that the firms can explicitly control the demand fill rate. However, a competitive mechanism design approach has not been examined in RM and presents an interesting open problem.

### EC.1.2. Example of specifications of the basics model elements

Consider two identical firms and two distinct segments: business (segment 1) and leisure (segment 2) travelers. Business consumers are myopic ($\beta_1 = 0$) and, at time $t$, their valuations of the two products are independent and distributed exponentially with equal means that increase over the course of the selling season. That is $B_{t1j}$, $j = 1, 2$ are exponentially distributed with equal means $a_{t1j} = \mu_1^0 + \mu_1^1 t/T$, $j = 1, 2$ where $\mu_1^1 > 0$, Leisure consumers are fully strategic ($\beta_2 = 1$) and their valuations are independent and exponentially distributed with constant mean. That is $B_{t2j}$, $j = 1, 2$ are exponentially distributed with equal means $a_{t2j} = \mu_2^0$, $j = 1, 2$. The distribution of $\mathbf{B}_{tr}$ captures random preferences (or uncertain perceptions of value) of segment $r$ consumers at time $t$. These preferences change over time in a deterministic fashion.

### EC.1.3. Discussion of the perfect information assumption in a practical context

Contexts where the perfect information assumption is relevant are becoming increasingly common with expanding capabilities in internet and other information technologies. For example, in modern online booking systems for airlines and other travel services it has become commonplace to present consumers with layouts or counts of the remaining availability of assets, and firms can keep track of the number of unique hits to their web-sites. Other examples of such systems include online ticket sales for sports and entertainment events, and online retailers. In many cases, third-party brokers (for example, travel agents) can impart knowledge about inventory levels and historical pricing patterns to consumers, and market size to companies. In addition, such agents often work with consumers individually and have a reasonable understanding of consumers' valuation levels, preferences, and purchasing behavior, and can supply this information as well. There is also the recent emergence of 'price advisor' web-sites that track and report the patterns of company pricing and other policies. This raises the interesting possibility that such price advisors could become sufficiently large and technically sophisticated to deploy 'purchasing cost management' systems to counter firms' RM systems.

## EC.2. The case of high product supply

Here, we discuss the 'high product supply' situation, in which each firm has sufficient capacity to satisfy the purchasing requirements of all consumers. That is, capacity is no longer a constraint on available resources, and the only limited resource is the consumer population. We also consider only one consumer segment ($s = 1$) and omit the consumer segment index in our notation. The situation discussed here provides a limiting case most favorable for strategic consumers: there is a plentiful product supply, and there is no interplay between market segments that could be exploited by the firms. All results, except for the asymptotic one, hold for either choice model.

The first result shows that the game at each time $t$ and state $(\mathbf{y}, n)$ essentially decomposes into $n$ identical games played simultaneously between the firms and each of the remaining consumers individually.

PROPOSITION EC.1. *Let $s = 1$ and suppose that there exists a unique equilibrium in pure strategies. For any $t$ and $(\mathbf{y}, n)$ such that $y_j \geq n$, $j = 1, \ldots, m$ the following hold:*

$$U(t, \mathbf{y}, n) = U(t, \mathbf{1}, 1), \tag{EC.1}$$

$$R_j(t, \mathbf{y}, n) = nR_j(t, \mathbf{1}, 1),\ j = 1, \ldots, m, \tag{EC.2}$$

$$D_j(t,\mathbf{y},n,\mathbf{p}) = nD_j(t,\mathbf{1},1,\mathbf{p}), \textit{ for all } \mathbf{p},\ j=1,\ldots,m, \tag{EC.3}$$

$$p_j^*(t,\mathbf{y},n) = p_j^*(t,\mathbf{1},1),\ j=1,\ldots,m, \tag{EC.4}$$

*where* $\mathbf{1} = (1,1,\ldots,1)^\top$.

We can view the game as decomposed into plays of firms in the 1-consumer markets corresponding to individual consumers because the expected utilities do not depend on $n$ (see (EC.1)) and the expected revenues are additive over the consumers (see (EC.2)). This result also shows that each consumer gains so much market power in this setting that the prices are insensitive to the remaining market size $n$.

A corollary of the above proposition is a particularly simple expression for the expected utility:

Corollary EC.1. *For any $t$ and $(\mathbf{y},n)$ such that $y_j \geq n$, $j=1,\ldots,m$ we have*

$$U(t,\mathbf{y},n,\mathbf{p}) = \bar{\lambda} E_{\mathbf{B}_t}\left[\mathbf{x}^*(t,\mathbf{y},n,\mathbf{p},\mathbf{B}_t)^\top \left(\mathbf{B}_t - \mathbf{p} - \mathbf{1}\beta U(t+1,\mathbf{y},n)\right)\right] + \beta U(t+1,\mathbf{1},1). \tag{EC.5}$$

This results underscores the independent role of model parameters $\bar{\lambda}$ and $\beta$. The maximum shopping intensity $\bar{\lambda}$ determines how quickly consumers can build up expected utility, while $1-\beta$ determines the speed of its deterioration.

Expression (EC.5) immediately implies natural monotonic properties for the expected utility (decreasing in time) and the demand intensity (increasing in time, given that prices are at the same level) when consumers are fully strategic:

Corollary EC.2. *If $\beta = 1$, then for any $t$ and $(\mathbf{y},n)$ such that $y_j \geq n$, $j=1,\ldots,m$ we have $U(t,\mathbf{y},n) \geq U(t+1,\mathbf{y},n)$ and $D_j(t,\mathbf{y},n,\mathbf{p}) \leq D_j(t+1,\mathbf{y},n,\mathbf{p})$, $j=1,\ldots,m$.*

Finally, we obtain a uniform bound on the expected utility of a consumer with limited strategic behavior $\beta < 1$ in the stationary valuation distribution case:

Proposition EC.2. *Let $\beta < 1$ and suppose that the distribution of $\mathbf{B}_t$ does not depend on time. Then, for any $t$ and $(\mathbf{y},n)$ such that $y_j \geq n$, $j=1,\ldots,m$:*

$$U(t,\mathbf{y},n,\mathbf{p}) \leq \bar{U} = \frac{\bar{\lambda}\sum_{j=1}^m E[B_j]}{1-\beta}. \tag{EC.6}$$

Parameter $\bar{\lambda}$ determines how quickly an eager consumer can complete a purchase, while $1-\beta$ measures a relative loss of utility per decision period due to waiting. Thus, when $\beta < 1$, the ratio $\bar{\lambda}/(1-\beta)$ measures the effect of procrastination by consumers in the context of their strategic behavior. A consumer with a high ratio (less procrastination) is more likely to obtain a high utility value within his/her effective planning horizon which is determined by the magnitude of discount factor $\beta$. A consumer whose effective planning horizon is too short to encounter a significant number of genuine purchase opportunities is, in effect, myopic. As a consequence, the above proposition can be used to evaluate the potential importance of consumer strategic behavior for given distributions and parameter values. Recall that certainty equivalent $Q(t,\mathbf{y},n)$ of a future purchase by a strategic customer appears inside the comparison of the form $B_j \geq p_j + Q(t,\mathbf{y},n)$ in the purchase probability $D_j(t,\mathbf{y},n,\mathbf{p})$. Thus, if the magnitude of $Q(t,\mathbf{y},n)$ is comparable to 'typical values' of $B_j$, then the effects of strategic behavior cannot be ignored. If consumer valuations of different products are identically distributed, then, if we use $E[B]$ as a typical valuation value, the condition under which strategic behavior is important can be expressed as

$$E[B] \leq \beta \bar{U} = \frac{\beta \bar{\lambda} m E[B]}{1-\beta},$$

or, equivalently, as

$$\beta \geq \frac{1}{1+\bar{\lambda} m}.$$

The condition has the form of a lower bound on $\beta$ which increases as $\bar{\lambda} m$ decreases. This can be interpreted as follows: since $\bar{\lambda} m$ is an upper bound on the probability that a particular consumer purchases a unit of any of the products in the given time period, $\beta$ has to be sufficiently large for an appreciable utility to be accumulated during an effective planning horizon of a consumer.

**Asymptotic analysis of a symmetric oligopolistic equilibrium**

We now examine the effect of a long time horizon in relation to the number of firms $m$. This analysis applies to situation in which the market consists of a single segment, each firm can supply the entire market, the firms are identical, $\beta < 1$, consumers behave in agreement with the multiple choice model, and the distribution of the valuations is stationary exponential: $f_t(b) = \frac{1}{a}e^{-\frac{b}{a}}$. With stationarity, longer time horizons can be described either by increasing $T$ or by letting $t$ be negative (and go to $-\infty$ in the limit). To simplify notation, we use the second possibility. Based on the results of Proposition EC.1, it is enough to consider a situation with $N = Y_1 = \ldots = Y_m = 1$. Thus, to simplify notation we also drop $n$ and $\mathbf{y}$. Moreover, due to the symmetry of the equilibrium, we can drop the index $j$ from the equilibrium prices and revenues. By Proposition EC.2, $U(t) \le \bar{U}$. Since $\Delta_j R_j(t+1, \mathbf{y}, n) = R(t+1, \mathbf{1}, 1) = R(t+1) \ge 0$, the equilibrium prices can be expressed as $p^*(t) = a + R(t+1) + c$, $j = 1, \ldots, m$, where $c$ is the unit cost (identical for all firms). Let us now turn to the equilibrium revenues, which can be expressed, after substituting the equilibrium prices, as

$$R(t) = R(t+1) + \bar{\lambda} e^{-\frac{a+R(t+1)+c+\beta U(t+1)}{a}}(a - (m-1)R(t+1)).$$

The following proposition summarizes the asymptotic results.

PROPOSITION EC.3. *For the case $m > 1$, under the conditions of this subsection, the sequence $R(t)$ is decreasing in time and $R(t) \le \frac{a}{m-1}$. Moreover $\lim_{t\to-\infty} R(t) = \frac{a}{m-1}$. The equilibrium prices $p^*(t)$ are also decreasing in time and $p^*(t) \le \frac{ma}{m-1} + c$. Moreover, $\lim_{t\to-\infty} p^*(t) = \frac{ma}{m-1} + c$. For the case $m = 1$, while $R(t)$ and $p^*(t)$ are decreasing, we also have $R(t) \to \infty$ and $p^*(t) \to \infty$ as $t \to -\infty$.*

The above proposition shows that monopoly and competitive situations result in markedly different asymptotic behavior. Given a sufficiently long selling horizon, a monopolist firm can start with much higher prices than firms in an oligopoly. This result also shows that strategic behavior by consumers who have limited foresight ($\beta < 1$) becomes less important as the length of a selling season increases.

## EC.3. Proofs

**Proof of Proposition 2** According to "perfect foresight" assumption, consumers can compute the probability of every possible event in the current information state and their expected utility in the corresponding states. It then is a simple matter to compute the expected utility in the current information state. Let

$$f_{tr}^{j}(\mathbf{b}) = \begin{cases} \frac{f_{tr}(\mathbf{b})}{P(\mathbf{B}_{tr} \in A_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p}))} & \text{if } \mathbf{b} \in A_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p}) \\ 0 & \text{otherwise} \end{cases}$$

be the conditional valuation density for a segment $r$ consumer at time $t$ given that this consumer prefers an item $j$. According to the assumption that "consumer perceptions are conditional on purchase", consumers update their valuation distribution to this density if a purchase does occur. Given that a consumer prefers an item $j$ and acquires it in the current decision period, he/she derives the conditional expected utility

$$\int_{\mathbf{b}\in\mathbb{R}^m} (b_j - p_j) f_{tr}^{j}(\mathbf{b}) d\mathbf{b} = \frac{1}{P(\mathbf{B}_{tr} \in A_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p}))} \int_{\mathbf{b}\in A_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p})} (b_j - p_j) f_{tr}(\mathbf{b}) d\mathbf{b}.$$

Recall that $A_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p})$ is, by definition (2), the set of all valuation vectors for which the purchase of product $j$ is preferred to a purchase of any other product or a purchase delay by a segment $r$ consumer. The outcome of the customer purchase of item $j$ occurs with probability $\lambda_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p})$ given by (3).

Other possible events include a sale of item $j$ to a different consumer of some segment $r'$ (perhaps coinciding with $r$) with probability $\lambda_{r'j}(t,\mathbf{y},\mathbf{n},\mathbf{p})$. In this case, the expected present value of utility of a segment $r$ consumer at time $t+1$ is $\beta_r U_r(t+1,(y_j-1,\mathbf{y}_{-j}),(n_{r'}-1,\mathbf{n}_{-r'}))$. It is also possible that no sale occurs, and the expected present value at time $t+1$ in this case is $\beta_r U_r(t+1,\mathbf{y},\mathbf{n})$. Adding over all these possibilities, we find the expected utility in the current information state

$$\begin{aligned} U_r(t,\mathbf{y},\mathbf{n},\mathbf{p}) = \bar{\lambda} \sum_{j=1}^{m} & \int_{\mathbf{b}\in A_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p})} (b_j - p_j) f_{tr}(\mathbf{b}) d\mathbf{b} \\ & + \beta_r \sum_{r'=1}^{s} (n_{r'} - I(r=r')) \sum_{j=1}^{m} \lambda_{r'j}(t,\mathbf{y},\mathbf{n},\mathbf{p}) U_r(t+1,(y_j-1,\mathbf{y}_{-j}),(n_{r'}-1,\mathbf{n}_{-r'})) \\ & \quad + \beta_r \Big(1 - \lambda_{rj}(t,\mathbf{y},\mathbf{n},\mathbf{p}) - \sum_{r'=1}^{s} (n_{r'} - I(r=r')) \sum_{j=1}^{m} \lambda_{r'j}(t,\mathbf{y},\mathbf{n},\mathbf{p})\Big) U_r(t+1,\mathbf{y},\mathbf{n}), \end{aligned}$$

where $I(A)=1$ when $A$ holds and 0 otherwise. After collecting terms corresponding to the same pairs $(r',j)$, we obtain the recursive relation (4) for the expected utility.

**Proof of Theorem 1.** The proof is by backward induction on $t$. The payoffs are uniquely defined for $T$ due to the boundary conditions. Suppose that for any $(\mathbf{y},\mathbf{n})$ the payoffs of firms, $R_j(t+1,\mathbf{y},\mathbf{n})$, $j=1,\ldots,m$, and expected utilities of consumers, $U_r(t+1,\mathbf{y},\mathbf{n})$, in the subsequent decision period $t+1$ are uniquely defined. We can assume that the consumers respond as described in Proposition 1, and they need not be treated explicitly in this reasoning. Thus, the pure strategy payoffs in $\mathcal{G}(t,\mathbf{y},\mathbf{n})$ are defined by (7). Moreover, expression (7) is continuous in $\mathbf{p}$ and the strategy sets of game $\mathcal{G}(t,\mathbf{y},\mathbf{n})$ are compact due to assumption (A) (closed intervals of the form $[0,\bar{p}]$). We next draw upon Theorem 1.3 on page 35 in Fudenberg and Tirole (1991) which is restated here for completeness:

THEOREM EC.1. *Consider a strategic-form game whose strategy spaces are nonempty compact subsets of a metric space. If the payoff functions are continuous, then there exists a Nash equilibrium in mixed strategies.*

By the above theorem, game $\mathcal{G}(t,\mathbf{y},\mathbf{n})$ has an equilibrium in mixed strategies. By assumption (B), even if there are multiple equilibria, only one of them can be implemented. Thus, the resulting equilibrium payoffs in $\mathcal{G}(t,\mathbf{y},\mathbf{n})$ are uniquely defined.

**Proof of Proposition 3.** Recall that, by assumption, consumers average their shopping intensity and the expected utility over their valuation distribution. Suppose that a consumer in segment $r$, in addition to the state information, $(\mathbf{y},\mathbf{n},\mathbf{p})$ also knows the valuation vector $\mathbf{B}_{tr}$. Let the expected utility for this consumer be then denoted as $U_r(t,\mathbf{y},\mathbf{n},\mathbf{p},\mathbf{B}_{tr})$. The other consumers follow equilibrium responses denoted as $\boldsymbol{\lambda}_{r'}(t,\mathbf{y},\mathbf{n},\mathbf{p})$. While the responses of other consumers are not observable, the consumer under consideration can assume that other consumers act in the same way as he does, given their segment, and that their valuations follow known probability distributions. The optimal expected utility in case of known valuation is then computed by taking expectations over all possible purchase events:

$$U_r(t,\mathbf{y},\mathbf{n},\mathbf{p},\mathbf{B}_{tr}) = \max_{\mathbf{x}\geq\mathbf{0},\,\|\mathbf{x}\|_q\leq\bar{\lambda}} \Bigg\{ \bar{\lambda}\sum_{j=1}^{m} x_j(B_{trj}-p_j)$$

$$+\beta_r \sum_{r'=1}^{s} (n_{r'} - I(r=r')) \sum_{j=1}^{m} \lambda_{r'j}(t,\mathbf{y},\mathbf{n},\mathbf{p}) U_r(t+1,(y_j-1,\mathbf{y}_{-j}),(n_{r'}-1,\mathbf{n}_{-r'}))$$
$$+\beta_r \Big(1-\bar\lambda \sum_{j=1}^{m} x_j - \sum_{r'=1}^{s}(n_{r'}-I(r=r'))\sum_{j=1}^{m}\lambda_{r'j}(t,\mathbf{y},\mathbf{n},\mathbf{p})\Big) U_r(t+1,\mathbf{y},\mathbf{n})\Big\}.$$

Since the expression under the max is linear in the strategies of all consumers, we can collect the terms depending on the consumer's strategy and take all other terms outside the maximization operation resulting in

$$U_r(t,\mathbf{y},\mathbf{n},\mathbf{p},\mathbf{B}_{tr}) = \bar\lambda \max_{\mathbf{x}\ge\mathbf{0},\,\|\mathbf{x}\|_q\le\bar\lambda} \left\{\mathbf{x}^\top\left(\mathbf{B}_{tr} - \mathbf{p} - \mathbf{1}\beta_r U_r(t+1,\mathbf{y},\mathbf{n})\right)\right\}$$
$$-\beta_r \sum_{r'=1}^{s} (n_{r'} - I(r=r'))\boldsymbol{\lambda}_{r'}(t,\mathbf{y},\mathbf{n},\mathbf{p})^\top \boldsymbol{\Delta} U_r(t+1,\mathbf{y},\mathbf{n}) + \beta_r U_r(t+1,\mathbf{y},\mathbf{n}), \quad \text{(EC.7)}$$

where we change to vector notation instead of summation over $j$. We see that the maximizer (the optimal intensity for given valuation vector) does not depend on the responses of other consumers, and is given by (8) for $q=\infty$ as a solution vector for a linear maximization problem with simple bound constraints. For $q=1$, the solution is given by (9) in all information states where $\operatorname{Argmax}\{\tilde b_j - p_j : j = 1,\ldots,m\}$ is a singleton. The set of states where $\operatorname{Argmax}\{B_{trj} - p_j : j = 1,\ldots,m\}$ is not a singleton has measure 0. While the optimal $\mathbf{x}$ may not be unique due to ties in the coefficients of the linear objective, the expected values do not depend on this nonuniqueness because the distribution of valuations is continuous. The expression (10) for the expected utility is obtained by taking the expected value of (EC.7) over $\mathbf{B}_{tr}$.

**Proof of Theorem 2.** When $q=\infty$, the expected future payoff of firm $j$ given by the expression (7) is separable in the components of the price vector $\mathbf{p}$. The equilibrium strategy of firm $j$ is to maximize the quantity

$$g_j(p_j) = \sum_{r=1}^{s} D_{rj}^M(t,\mathbf{y},\mathbf{n},p_j)(p_j - \Delta_{rj} R_j(t+1,\mathbf{y},\mathbf{n}) - c_j) \quad \text{(EC.8)}$$

with respect to $p_j$ and is independent of the strategies of other firms at the same time instance. Since $g_j(p_j)$ is a continuous function on a closed interval $[0,\bar p]$, it attains its maximum value. Thus, there is a pure strategy equilibrium in $\mathcal{G}(t,\mathbf{y},\mathbf{n})$ and the resulting equilibrium payoffs are uniquely defined.

**Proof of Theorem 3.** We first establish the following lemma that describes the equilibrium strategies of the firms and the corresponding equilibrium payoffs in the multiple choice case:

LEMMA EC.1. *Let $q=\infty$ and $s=1$, and consider decision period $t$. Suppose that there exists a unique equilibrium in all subsequent periods $t+1,t+2,\dots,T-1$, and the distribution of valuations $\mathbf{B}_t$ satisfies the logconcavity and regularity assumptions (C) and (D). Then there exists an equilibrium in $\mathcal{G}(t,\mathbf{y},n)$ such that firm $j$ strategy is*

$$p_j^*(t,\mathbf{y},n)=\arg\max_{p_j\geq 0} D_j^M(t,\mathbf{y},n,p_j)(p_j-\Delta_j R_j(t+1,\mathbf{y},n)-c_j). \tag{EC.9}$$

*This strategy selection is unique if there is $p_j>\Delta_j R_j(t+1,\mathbf{y},n)+c_j$ such that $D_j^M(t,\mathbf{y},n,p_j)>0$. Moreover, the equilibrium payoffs of firms are defined uniquely by $R_j(t,\mathbf{y},n)=R_j(t,\mathbf{y},n,\mathbf{p}^*(t,\mathbf{y},n))$.*

The condition of the lemma that there is a $p_j>\Delta_j R_j(t+1,\mathbf{y},n)+c_j$ such that $D_j^M(t,\mathbf{y},n,p_j)>0$ means that either there is no price $\bar{p}_j$ (typically called a *shutdown price*) such that $D_j^M(t,\mathbf{y},n,\bar{p}_j)=0$, or $\bar{p}_j$ exceeds $\Delta_j R_j(t+1,\mathbf{y},n)+c_j$.

When $q=\infty$ and $s=1$ the equilibrium strategy of each firm is to maximize the quantity $g_j(p_j)=D_j^M(t,\mathbf{y},n,p_j)(p_j-\Delta_j R_j(t+1,\mathbf{y},n)-c_j)$, which is a special case of (EC.8). Note that no firm will set its price below $p_j=\Delta_j R_j(t+1,\mathbf{y},n)+c_j$. Because of the regularity condition, $\lim_{p_j\to\infty} g_j(p_j)=0$ and, consequently, there exists a maximizer of $g_j(p_j)$ on $[0,+\infty)$. Note also that $g_j(p_j)$ is a logconcave function of $p_j$ for $p_j\geq\Delta_j R_j(t+1,\mathbf{y},n)+c_j$ as a product of logconcave functions. Moreover, it is strictly logconcave where it is positive. If there is a $p_j>\Delta_j R_j(t+1,\mathbf{y},n)+c_j$ such that $D_j^M(t,\mathbf{y},n,p_j)>0$, then $g_j(p_j)$ will have a unique maximizer.

Now, combining Propositions 3 and EC.1 with the fact that payoffs are uniquely defined as 0 in all 'terminal' states, we obtain the claim of Theorem 3.

**Proof of Proposition EC.1.** The proof is by backward induction on $t$. Relations (EC.1) and (EC.2) hold for $t=T$. Suppose that (EC.1)-(EC.2) hold for $t+1$ and all $(\mathbf{y},n)$ such that

$y_j \geq n$, $j = 1, \ldots, m$. We now prove that (EC.1)-(EC.4) for $t$. Relation (EC.1) for $t+1$ implies that $\boldsymbol{\Delta} U(t+1, \mathbf{y}, n) = 0$, resulting in

$$U(t, \mathbf{y}, n, \mathbf{p}) = E_{\mathbf{B}_t}\left[\mathbf{x}^*(t, \mathbf{y}, n, \mathbf{p}, \mathbf{B}_t)^\top \left(\mathbf{B}_t - \mathbf{p} - \mathbf{1}\beta U(t+1, \mathbf{y}, n)\right)\right] + \beta U(t+1, \mathbf{y}, n)$$

for all $\mathbf{y}, n$ such that $y_j \geq n$, $j = 1, \ldots, m$. Moreover, since $U(t+1, \mathbf{y}, n) = U(t+1, \mathbf{1}, 1)$, we also have

$$\mathbf{x}^*(t, \mathbf{y}, n, \mathbf{p}, \mathbf{B}_t) = \mathbf{x}^*(t, \mathbf{1}, 1, \mathbf{p}, \mathbf{B}_t), \tag{EC.10}$$

and it follows that

$$U(t, \mathbf{y}, n, \mathbf{p}) = \bar{\lambda} E_{\mathbf{B}_t}\left[\mathbf{x}^*(t, \mathbf{1}, 1, \mathbf{p}, \mathbf{B}_t)^\top \left(\mathbf{B}_t - \mathbf{p} - \mathbf{1}\beta U(t+1, \mathbf{1}, 1)\right)\right] + \beta U(t+1, \mathbf{1}, 1) = U(t, \mathbf{1}, 1).$$

From (EC.10) we also conclude that (EC.3) holds. The relation $R_j(t+1, \mathbf{y}, n) = nR_j(t+1, \mathbf{1}, 1)$ for all $\mathbf{y}, n$ such that $y_j \geq n$, implies that, for any $j'$, $\Delta_{j'} R_j(t+1, \mathbf{y}, n) = R_j(t+1, \mathbf{1}, 1) = \Delta_{j'} R_j(t+1, \mathbf{1}, 1)$. From (7) and the above, we get

$$\begin{aligned} R_j(t, \mathbf{y}, n, \mathbf{p}) &= nD_j(t, \mathbf{1}, 1, \mathbf{p})(p_j - \Delta_j R_j(t+1, \mathbf{1}, 1) - c_j) \\ &\quad - \sum_{j' \neq j} nD_{j'}(t, \mathbf{1}, 1, \mathbf{p}) \Delta_{j'} R_j(t+1, \mathbf{1}, 1) + nR(t+1, \mathbf{1}, 1) = nR(t, \mathbf{1}, 1, \mathbf{p}), \end{aligned}$$

and (EC.2) follows.

**Proof of Proposition EC.2.** Because of (EC.1), it is enough to prove the statement for $\mathbf{y} = \mathbf{1}$ and $n = 1$. Observe that, under either choice model, $x_j^*(t, \mathbf{1}, 1, \mathbf{p}, \mathbf{b}) \leq I(b_j - p_j \geq \beta U(t+1, \mathbf{1}, 1))$. It follows that

$$U(t, \mathbf{1}, 1, \mathbf{p}) \leq \bar{\lambda} \sum_{j=1}^{m} E[(B_{tj} - p_j - \beta U(t+1, \mathbf{1}, 1))^+] + \beta U(t+1, \mathbf{1}, 1).$$

Since $(B_{tj} - p_j - \beta U(t+1, \mathbf{1}, 1))^+ \leq B_{tj}$, we have

$$U(t, \mathbf{1}, 1, \mathbf{p}) \leq \bar{\lambda} \sum_{j=1}^{m} E[B_{tj}] + \beta U(t+1, \mathbf{1}, 1).$$

Recall now that the distribution of $\mathbf{B}_t$ is stationary, and consider the quantity $\bar{U}$ defined in the statement. We can prove by backward induction on time that $U(t, \mathbf{1}, 1, \mathbf{p}) \le \bar{U}$. Indeed, the statement holds for $t = T$ since $U(T, \mathbf{1}, 1, \mathbf{p}) = 0$. Suppose it holds for $t+1$. Then we have

$$U(t, \mathbf{1}, 1, \mathbf{p}) \le \bar{\lambda} \sum_{j=1}^{m} E[B_j] + \beta \frac{\bar{\lambda} \sum_{j=1}^{m} E[B_j]}{1-\beta} = \bar{U}.$$

Since this inequality holds for any prices, including the equilibrium ones, it follows that the equilibrium utility satisfies $U(t, \mathbf{1}, 1) \le \bar{U}$.

**Proof of Proposition EC.3.** Indeed, $R(T) = 0$, therefore the bound holds for $t = T$ and $R(T-1) > R(T)$ since $a - (m-1)R(T) > 0$. Similarly, if $R(t+1) < \frac{a}{m-1}$, then $a - (m-1)R(t+1) > 0$ and $R(t) > R(t+1)$. On the other hand, since $(m-1)\bar{\lambda} < m\bar{\lambda} \le 1$, and $e^{-\frac{a+R(t+1)+c+\beta U(t+1)}{a}} < 1$, it follows that

$$R(t) < \left(\frac{a}{m-1} - R(t+1)\right) + R(t+1) < \frac{a}{m-1}.$$

Finally, since $R(t+1)$ and $U(t+1)$ are bounded from above, the quantity $e^{-\frac{a+R(t+1)+c+\beta U(t+1)}{a}}$ is bounded from below by some value $v$. Thus, $R(t)$ is closer to $\frac{a}{m-1}$ than $R(t+1)$ by no less than a fraction $\bar{\lambda}v$ of $\frac{a}{m-1} - R(t+1)$. Therefore, $\lim_{t\to-\infty} R(t) = \frac{a}{m-1}$. The implication for the equilibrium price is that $\lim_{t\to-\infty} p^*(t) = \frac{ma}{m-1} + c$. In the monopoly case, the recursion $R(t) = \bar{\lambda}e^{-\frac{a+R(t+1)+c+\beta U(t+1)}{a}} a + R(t+1)$ leads to $R(t) \to \infty$ and $p^*(t) \to \infty$ as $t \to -\infty$.

## EC.4. Demand functions and best-response mappings for the exponential valuation distribution and numerical issues

The choice of valuation distributions in the exponential form results in a very efficient numerical implementation (both for the multiple and the specific choice cases).

The purchase probabilities (demand functions) in the specific and multiple choice cases are obtained by substituting the exponential density (12) into expressions (6) and (11), respectively. The multiple consumer choice model results in segment $r$ demand for product $j$ of the form:

$$D_{rj}^{M}(t, \mathbf{y}, \mathbf{n}, p_j) = n_r \bar{\lambda} e^{-\frac{p_j + Q_r(t, \mathbf{y}, \mathbf{n})}{a_{rj}}}.$$

For the case of a one-segment market, the resulting equilibrium prices are then given by the closed form expression $p_j^*(t,\mathbf{y},n) = \max\{a_j + \Delta_j R_j(t+1,\mathbf{y},n),\, 0\}$. This result can be obtained immediately by applying the first-order optimality conditions in expression (EC.9) established in Lemma EC.1. This solution is in closed form and can be computed very efficiently. In the multiple-segment case, a numerical maximization of the objective

$$\sum_{r=1}^{s} n_r \bar{\lambda} e^{-\frac{p_j+Q_r(t,\mathbf{y},\mathbf{n})}{a_{rj}}} (p_j - \Delta_j R_j(t+1,\mathbf{y},\mathbf{n}))$$

with respect to $p_j$ is required. However, this can also be done efficiently using local search methods (like Newton-Armijo) from multiple starting points since the problem is one-dimensional. A simple search over sufficiently fine mesh also provides an acceptable approximate alternative.

For the specific consumer choice model we can use conditioning on the value of $B_{tj} = b_j$ in expression (6) to obtain (since we treat the case of $s=1$, we can omit the $r$ index) the demand functions of the form:

$$D_j^S(t,\mathbf{y},n,\mathbf{p}) = n\bar{\lambda} \int_{p_j+Q(t,\mathbf{y},n)}^{+\infty} \frac{1}{a_j} e^{-\frac{b}{a_j}} \prod_{j'\neq j} \left(1 - e^{-\frac{b-p_j+p_{j'}}{a_{j'}}}\right) db. \tag{EC.11}$$

Consider the duopoly case and let $\tilde{a} = \left(\frac{1}{a_1} + \frac{1}{a_2}\right)^{-1}$. The expression in the integral (EC.11) simplifies to

$$\begin{aligned}
D_j^S(t,\mathbf{y},n,\mathbf{p}) &= n\bar{\lambda} \int_{p_j+Q(t,\mathbf{y},n)}^{+\infty} \frac{1}{a_j} e^{-\frac{b}{a_j}} \left(1 - e^{-\frac{b-p_j+p_{j'}}{a_{j'}}}\right) db \\
&= \frac{n\bar{\lambda}}{a_j} \int_{p_j+Q(t,\mathbf{y},n)}^{+\infty} \left(e^{-\frac{b}{a_j}} - e^{-\frac{b}{\tilde{a}}} e^{\frac{p_j-p_{j'}}{a_{j'}}}\right) db \\
&= \frac{n\bar{\lambda}}{a_j} \left(a_j e^{-\frac{p_j+Q(t,\mathbf{y},n)}{a_j}} - \tilde{a} e^{-(p_j+Q(t,\mathbf{y},n))\left(\frac{1}{a_{j'}}+\frac{1}{a_j}\right)} e^{\frac{p_j-p_{j'}}{a_{j'}}}\right) \\
&= n\bar{\lambda} e^{-\frac{p_j+Q(t,\mathbf{y},n)}{a_j}} \left(1 - \frac{a_{j'}}{a_j+a_{j'}} e^{-\frac{p_{j'}+Q(t,\mathbf{y},n)}{a_{j'}}}\right),\ j'\neq j.
\end{aligned}$$

We now turn to derivation of the best response mapping in state $(t,\mathbf{y},n)$. The expected profits of firm $j$ are obtained by substituting the derived demand functions into expression (7):

$$R_j(t,\mathbf{y},n,\mathbf{p}) = n\bar{\lambda} e^{-\frac{p_j+Q(t,\mathbf{y},n)}{a_j}} \left(1 - \frac{a_{j'}}{a_j+a_{j'}} e^{-\frac{p_{j'}+Q(t,\mathbf{y},n)}{a_{j'}}}\right) (p_j - \Delta_j R_j(t+1,\mathbf{y},n) - c_j)$$

$$-n\bar{\lambda}e^{-\frac{p_{j'}+Q(t,\mathbf{y},n)}{a_{j'}}}\left(1-\frac{a_j}{a_j+a_{j'}}e^{-\frac{p_j+Q(t,\mathbf{y},n)}{a_j}}\right)\Delta_{j'}R_j(t+1,\mathbf{y},n)+R(t+1,\mathbf{y},n).$$

To find the best response of firm $j$ (the maximizer of this expression with respect to $p_j$ for a given $p_{j'}$) we apply the first-order optimality condition. The derivative of the above expression with respect to $p_j$ is equal to

$$\frac{\partial R_j}{\partial p_j}=n\bar{\lambda}e^{-\frac{p_j+Q(t,\mathbf{y},n)}{a_j}}\left(1-\frac{a_{j'}}{a_j+a_{j'}}e^{-\frac{p_{j'}+Q(t,\mathbf{y},n)}{a_{j'}}}\right)\left(1-\frac{p_j-\Delta_jR_j(t+1,\mathbf{y},n)-c_j}{a_j}\right)$$
$$-n\bar{\lambda}e^{-\frac{p_{j'}+Q(t,\mathbf{y},n)}{a_{j'}}}\frac{1}{a_j+a_{j'}}e^{-\frac{p_j+Q(t,\mathbf{y},n)}{a_j}}\Delta_{j'}R_j(t+1,\mathbf{y},n).$$

Setting this expression equal to 0 and dividing by

$$\frac{n\bar{\lambda}}{a_j}e^{-\frac{p_j+Q(t,\mathbf{y},n)}{a_j}}\left(1-\frac{a_{j'}}{a_j+a_{j'}}e^{-\frac{p_{j'}+Q(t,\mathbf{y},n)}{a_{j'}}}\right)$$

(a positive quantity), we obtain

$$-p_j+a_j+\Delta_jR_j(t+1,\mathbf{y},n)+c_j-\frac{\frac{a_j}{a_j+a_{j'}}e^{-\frac{p_{j'}+Q(t,\mathbf{y},n)}{a_{j'}}}\Delta_{j'}R_j(t+1,\mathbf{y},n)}{1-\frac{a_{j'}}{a_j+a_{j'}}e^{-\frac{p_{j'}+Q(t+1,\mathbf{y},n)}{a_{j'}}}}=0.$$

The solution of this equation in terms of $p_j$ exists and is unique. The left-hand-side is positive (negative) for values of $p_j$ less (more) than the solution value. Note also that if this solution is negative then the left-hand-side is negative for $p_j=0$. Since the sign of the left-hand-side coincides with the sign of the derivative of $R_j(t,\mathbf{y},n,\mathbf{p})$ with respect to $p_j$, a negative solution means that the derivative is negative for all $p_j\geq 0$. We conclude that the best response of firm $j$ has the form

$$p_j(p_{j'})=\max\left\{a_j+\Delta_jR_j(t+1,\mathbf{y},n)-\frac{a_j\Delta_{j'}R_j(t+1,\mathbf{y},n)}{(a_j+a_{j'})e^{(p_{j'}+Q(t,\mathbf{y},n))/a_{j'}}-a_{j'}},0\right\},\ j'\neq j.$$

With best-response mappings in hand, we can use best-response iteration to find the equilibrium (it converged in our numerical experiments).

To solve for equilibrium we apply the dynamic programming principle and start in decision period $T-1$ (expected utilities and revenues are given as 0 at time $T$ by the boundary conditions). Once the equilibrium prices for stage $t$ are found, it is a matter of a simple application of equations (7) and (10) (where we substitute equilibrium prices for $\mathbf{p}$) to propagate calculations to stage $t-1$. CPU times for a single equilibrium calculation of the scale discussed in our numerical section are of the order of a few seconds to a few minutes on a modern workstation.

## EC.5. Additional numerical illustrations

### EC.5.1. Effects of asymmetry in consumer willingness to pay and firms' capacity on equilibrium

When we examine the effect of asymmetry in $a_j$ in Table EC.1, we see that the first firm (whose product is, on average, valued by the consumers twice as much as the competing product) roughly doubles its revenues in the case of myopic consumers and more than doubles in the case of strategic consumers. In contrast, the second firm is affected only slightly by a better competitor product in the myopic consumer case, but it can be seen to suffer a significant loss in the strategic consumer case. This suggests that competition in quality could intensify in a market with strategic consumers. We can also see that higher capacity can have either positive or negative effects. When the population size $N \geq 50$ and the consumers are myopic, the second firm benefits from having a higher capacity. However, in the case of strategic consumers, the benefit only shows at $N = 70$ and 80, and it is much smaller than the benefit in the myopic case in relative terms. We can explain this as a negation of the positive effects of higher capacity by an increase in strategic behavior of consumers. The first firm (with a smaller capacity) usually suffers from the higher capacity of the competitor. The negative effects on its revenues are more pronounced (in relative terms) in the case of strategic consumers.

### EC.5.2. Effects of strategic behavior on prices

Since the most profound differences in equilibrium are observed in the asymmetric situation of $a_1 = 2$, $a_2 = 1$, we conducted a pricing policy simulation to understand how this asymmetry affects the policies. Figure EC.3 shows representative price sample paths, and Figure EC.4 shows the averages of price sample paths over 10,000 simulations of the equilibrium policy. The presence of strategic consumers leads to less variability in prices of both firms – flatter graphs of prices over time. Moreover, the weak firm is more affected by this phenomenon and consistently avoids price increases. The pricing strategy of the strong firm is more flexible, but, in the strategic case, its price exhibits significant variability later in the selling season than in the myopic case. This is explained by the uncertainty in demand. In the strategic case, the strong firm may have an option of increasing the price in the end of the season, but only if its sales proceeded well up to that point. The weak firm does not have many opportunities to increase prices and usually carries too much product in the end of the season. (After one or two seasons like this, a weak firm would presumably lower production.)

### EC.5.3. Effects of strategic behavior when only some consumers are strategic

Significant effects of strategic behavior persist even if some consumers are myopic. To model such situations we use two market segments: one with fully strategic consumers ($\beta = 1$) and another with myopic consumers ($\beta = 0$). The segments are otherwise identical. The contours of the percentage difference in revenues between the partially strategic and completely myopic cases are shown in Figure EC.2 for symmetric duopoly with $Y_1 = Y_2 = 20$ and monopoly with initial capacity $Y_1 = 40$. Again, we see that the effect of strategic behavior is strongest when the firms divide the market ($N = 40$). Given the fraction of myopic consumers, the effect of strategic behavior in the monopoly is independent of population size if $N \leq Y_1 = 40$. This occurs because the monopolist with a higher capacity than the market cannot induce competition between strategic consumers (while it is possible to do so in a duopoly for a lower-capacity firm in information states with highly asymmetric capacity).

### EC.5.4. Losses from deviating from equilibrium by assuming wrong competitor capacity

Our model assumes the ideal case of perfect information. It is also important to understand how departures from this assumption might affect revenues. Thus, we examine the effect of deviation from equilibrium in duopoly if one firm wrongly estimates competitor capacity. The capacity of a competitor may be the hardest to know exactly in practice. The same will likely be true for consumers; however, to focus on the consequences of a firm's estimation error, ceteris paribus, we continue to allow full information for consumers. Thus, we take the same base cases in terms of $a_j$'s (and with $Y_1 = Y_2 = 20$) and compute the expected revenues in situations when one of the firms consistently underestimates or overestimates the current remaining capacity of the competitor by 5 units (25% of the initial capacity value). In this experiment, we use a multiple choice model. We present the loss in revenue relative to the equilibrium value as a function of the population size in Figure EC.5. The figure contains four plots corresponding to each possible combination of myopic/fully strategic consumers and underestimation/overestimation of the competitor capacity. The symmetric, weak and strong scenarios have the same meaning as in the previous subsection. In the case of myopic consumers, the relative loss in all three scenarios is identical. In the case of underestimation, the maximum loss occurs for $N = 30$ and reaches 5%. In the case of overestimation, the maximum loss occurs for $N = 40$ and is only 1.5%. On the other hand, we see a significant difference in losses when the consumers are strategic. The strong firm is the least affected by either under- or overestimation (the loss is well under 0.5% for all values of $N$), while the weak firm is the most affected (the loss of almost 2% in the worst case). We see that in case of error, it is somewhat better to underestimate than to overestimate when the population size equals or exceeds the combined capacity of the firms (40). In contrast, we see that overestimation is less serious than underestimation in the case of lower population size. Thus, ceteris paribus, if firms do not know competitor capacities exactly, they should prefer to overestimate when the population size is small and underestimate when it is large.

### EC.5.5. The effects of strategic behavior on equilibrium in the case of two market segments

In this subsection, we illustrate the model in the case of symmetric duopoly with $Y_1 = Y_2 = 20$ and two equally sized market segments. In the reference case, segment 1 has $a_{1j} = 1$ and segment 2 – $a_{2j} = 2$. The following scenarios are studied: both segments are myopic, segment 2 (high valuation) is strategic while segment 1 (low valuation) is myopic, segment 2 is myopic while segment 1 is strategic, and both segments are strategic. Figure EC.6 shows the effect of segment size on equilibrium prices in the initial state, the strategic behavior effect on equilibrium revenues (% revenue difference), and the relative utility gain/loss for consumers of each segment when another segment becomes strategic. The top plot shows the graphs of initial price for each of the four scenarios. As the segment sizes grow, the initial prices start to increase in all four situations. However, this occurs for larger segment sizes when the high-valuation segment is strategic, and the prices are much lower when the high-valuation segment (or both) are strategic. When only low valuation consumers are strategic and the segment sizes are small, the initial price is higher than in the completely myopic case. This can be explained by attempts of the firms to initially price low-valuation consumers out of the market. On the second plot, the effects of high-valuation segment strategic behavior is much stronger across all market sizes than those of low-valuation segment strategic behavior, and the revenue drop is the largest for $N_1 = N_2 = 20$ which is consistent with our results for one-segment symmetric duopoly (where the ratio is the lowest for $N = 40$). Finally, the third plot shows that high-valuation consumers always gain when low-valuation consumers become strategic while low valuation consumers gain only if the segment sizes are sufficiently large.

In the final experiment, we study the properties of equilibrium policies for two distinct market segments in a symmetric duopoly by means of simulation. We examine the same four scenarios in terms of strategic behavior as above. Figure EC.7 shows plus/minus one standard deviation ranges around the averages for the price sample paths over 10,000 simulations for the case of $a_{1j} = 1$, $a_{2j} = 2$. A fundamental difference in policy realizations between the scenarios of strategic and myopic high valuation consumers is that the prices lose variability and become much flatter overall when the high valuation consumers are strategic. An additional reduction in variability can be seen when both segments are strategic. If high valuation consumers are myopic, we see a decreasing trend of an average price realization. An early deviation from the downward trend when only low valuation consumers are strategic can be explained by the attempts of firms to sell primarily to high valuation myopic consumers in the beginning of the selling season. The low valuation consumers are priced out of the market until the second half of the selling season when their behavior resemble myopic behavior (because of the lower expected utility).

# EC.6. Tables and Figures

| | | Equilibrium with multiple choice | | | Equilibrium with specific choice | | |
|---|---|---|---|---|---|---|---|
| $N$ | $\beta$ | Symmetric | $a_1 = 2a_2$ | $Y_1 = Y_2/2$ | Symmetric | $a_1 = 2a_2$ | $Y_1 = Y_2/2$ |
| 20 | 0 | 17.4 | ( 34.7, 17.4)* | ( 17.4, 17.4) | 16.1 | ( 33.2, 15.5) | ( 16.1, 16.1) |
| 20 | 1 | 11.7 | ( 31.7, 7.3) | ( 11.7, 11.7) | 11.3 | ( 31.3, 6.7) | ( 11.3, 11.3) |
| 30 | 0 | 25.5 | ( 51.0, 25.5) | ( 25.9, 25.7) | 22.2 | ( 48.8, 21.1) | ( 23.4, 23.6) |
| 30 | 1 | 17.6 | ( 47.9, 11.0) | ( 17.6, 17.6) | 16.9 | ( 47.4, 10.2) | ( 16.9, 16.9) |
| 40 | 0 | 39.8 | ( 79.6, 39.8) | ( 39.1, 38.7) | 38.3 | ( 78.0, 37.8) | ( 38.4, 37.3) |
| 40 | 1 | 24.2 | ( 66.3, 18.0) | ( 24.1, 23.7) | 23.3 | ( 65.9, 17.3) | ( 23.3, 22.8) |
| 50 | 0 | 50.4 | (100.8, 50.4) | ( 45.9, 54.4) | 49.6 | ( 99.8, 49.4) | ( 45.0, 52.9) |
| 50 | 1 | 34.1 | ( 80.8, 27.9) | ( 31.2, 32.7) | 33.2 | ( 80.2, 27.1) | ( 30.6, 31.9) |
| 60 | 0 | 56.9 | (113.8, 56.9) | ( 50.8, 69.9) | 56.3 | (113.0, 56.2) | ( 49.8, 68.4) |
| 60 | 1 | 44.6 | ( 96.5, 39.2) | ( 36.7, 43.0) | 43.9 | ( 95.6, 38.4) | ( 36.1, 42.1) |
| 70 | 0 | 61.6 | (123.3, 61.6) | ( 56.1, 84.7) | 61.2 | (122.7, 61.1) | ( 55.0, 83.5) |
| 70 | 1 | 52.2 | (109.6, 47.9) | ( 41.1, 53.4) | 51.6 | (108.9, 47.2) | ( 40.3, 52.5) |
| 80 | 0 | 65.4 | (130.9, 65.4) | ( 61.1, 95.6) | 65.1 | (130.4, 65.0) | ( 60.2, 94.7) |
| 80 | 1 | 57.7 | (119.6, 54.2) | ( 45.7, 65.6) | 57.3 | (119.1, 53.7) | ( 44.7, 64.4) |

**Table EC.1 Expected revenues for duopoly with multiple and specific consumer choice in the cases of symmetric ($a_1 = a_2 = 1$, $Y_1 = Y_2 = 20$) and asymmetric ($a_1 = 2$ and $Y_2 = 40$) equilibria (note: $^*$ – (firm1, firm2))**

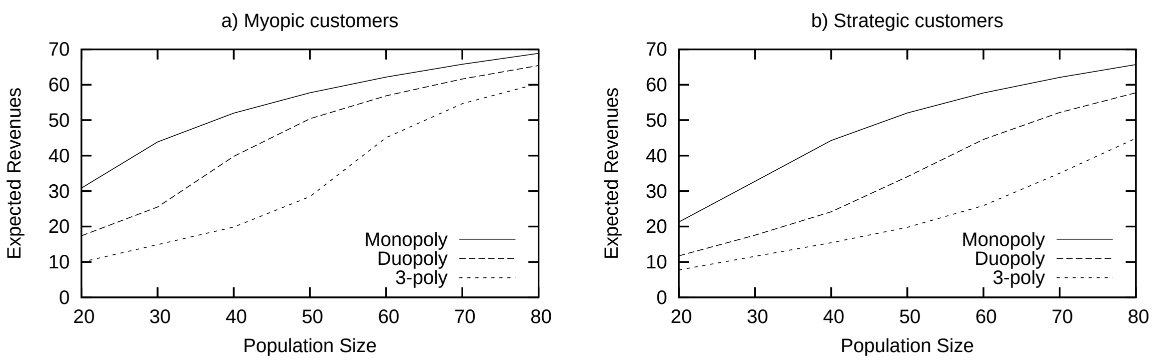


**Figure EC.1 Equilibrium revenues under myopic and strategic consumer behavior and different levels of competition as a function of population size**

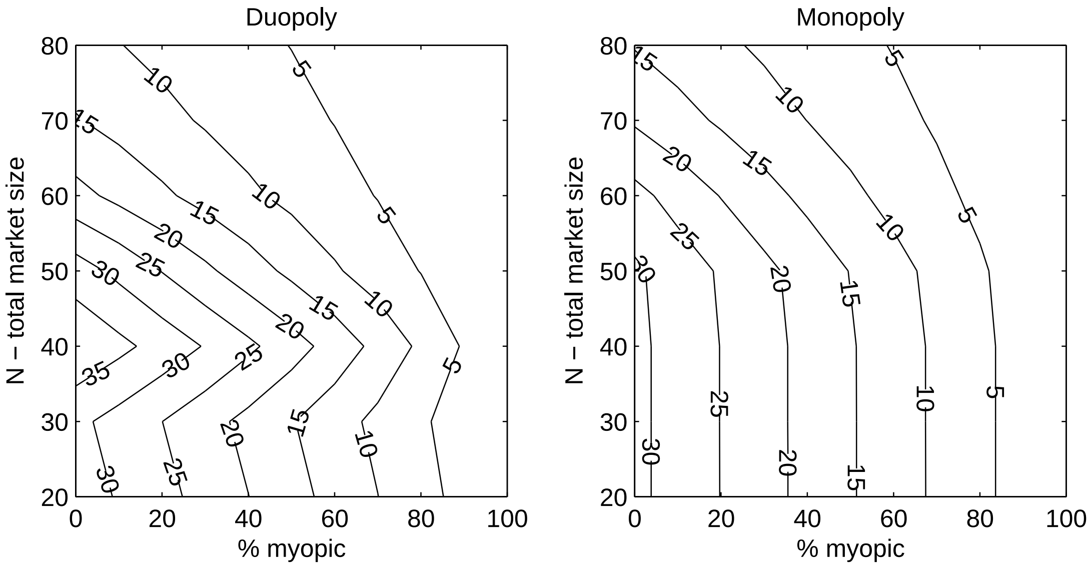


**Figure EC.2** Levels of equal % revenue difference as a function of population size and the percentage of myopic consumers for symmetric duopoly and monopoly

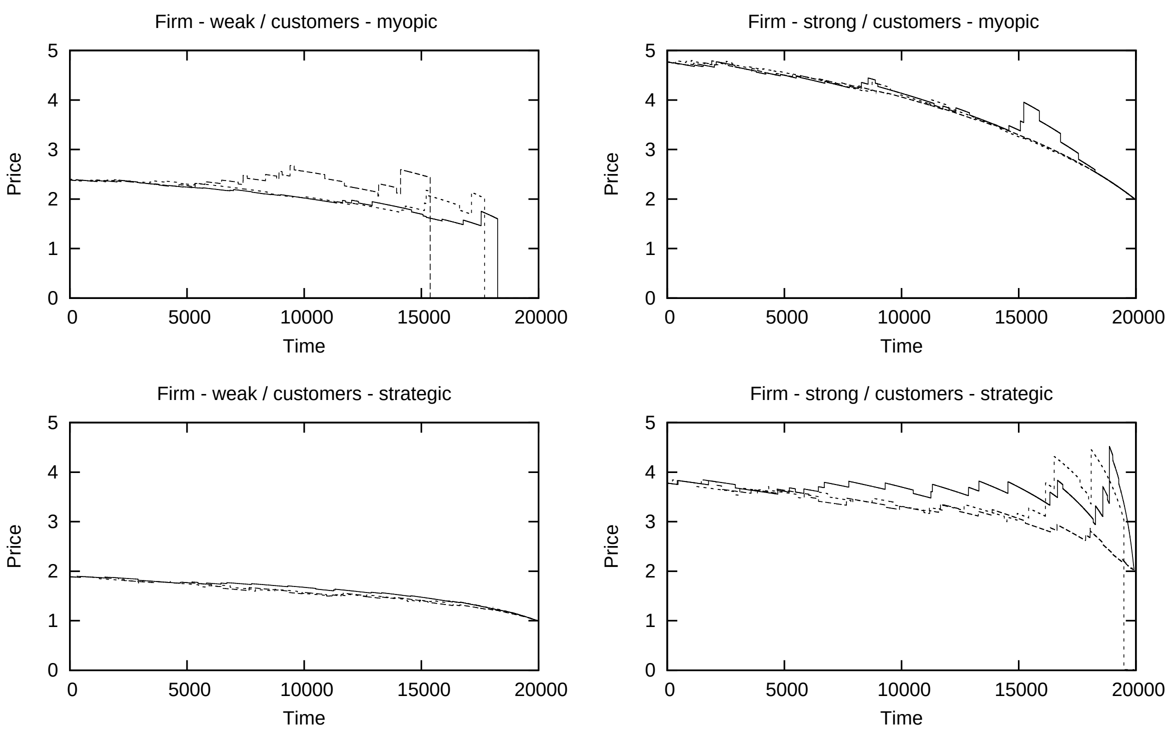


**Figure EC.3** Representative simulated price paths under asymmetric ($a_1 = 1$, $a_2 = 2$) equilibrium pricing policies for $N = 40$

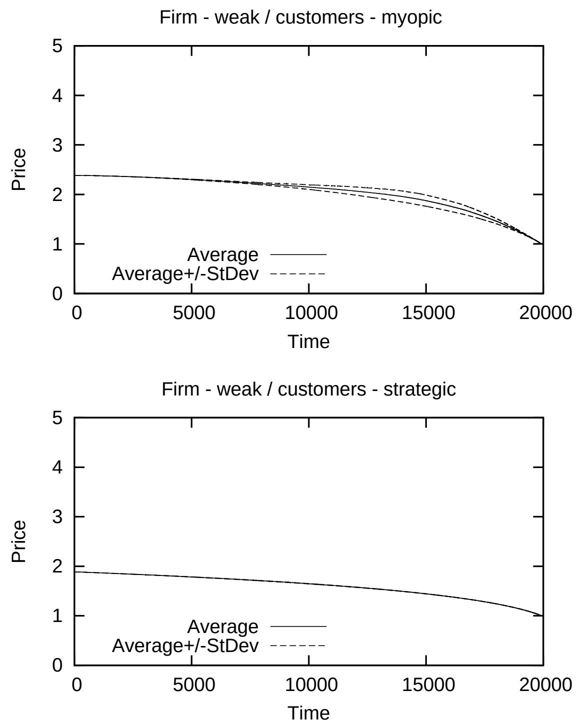


**Figure EC.4** **Averages of simulated price paths under asymmetric ($a_1 = 1$, $a_2 = 2$) equilibrium pricing policies for $N = 40$**

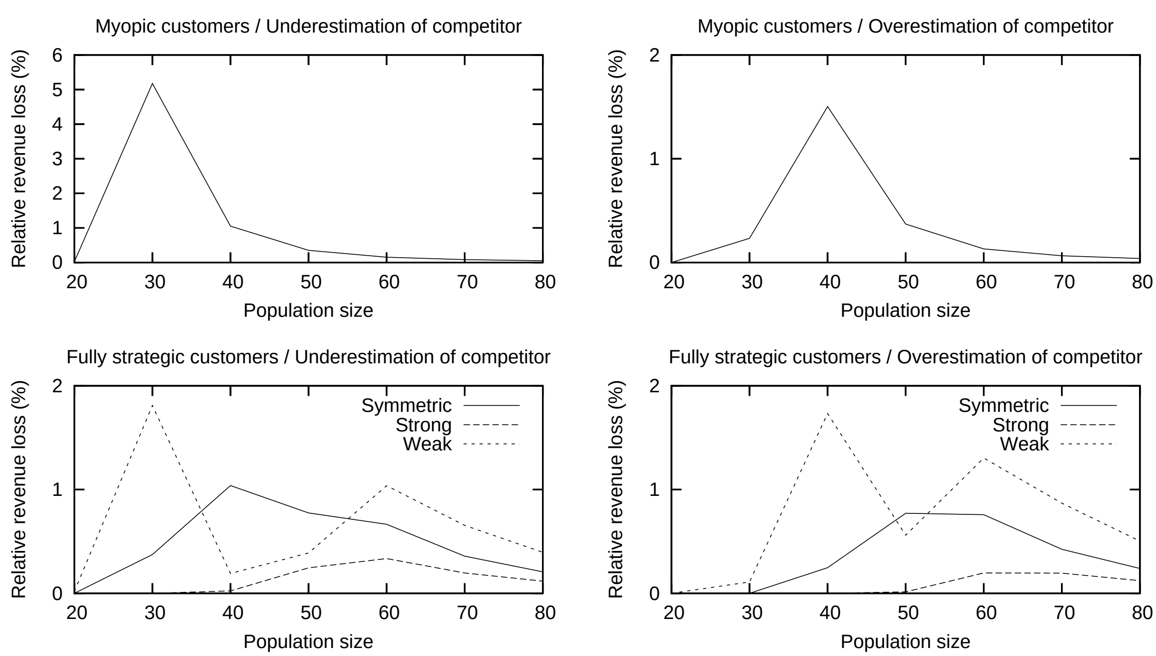


**Figure EC.5** **Relative revenue loss for deviation from equilibrium by assuming wrong competitor capacity, as a function of the population size**

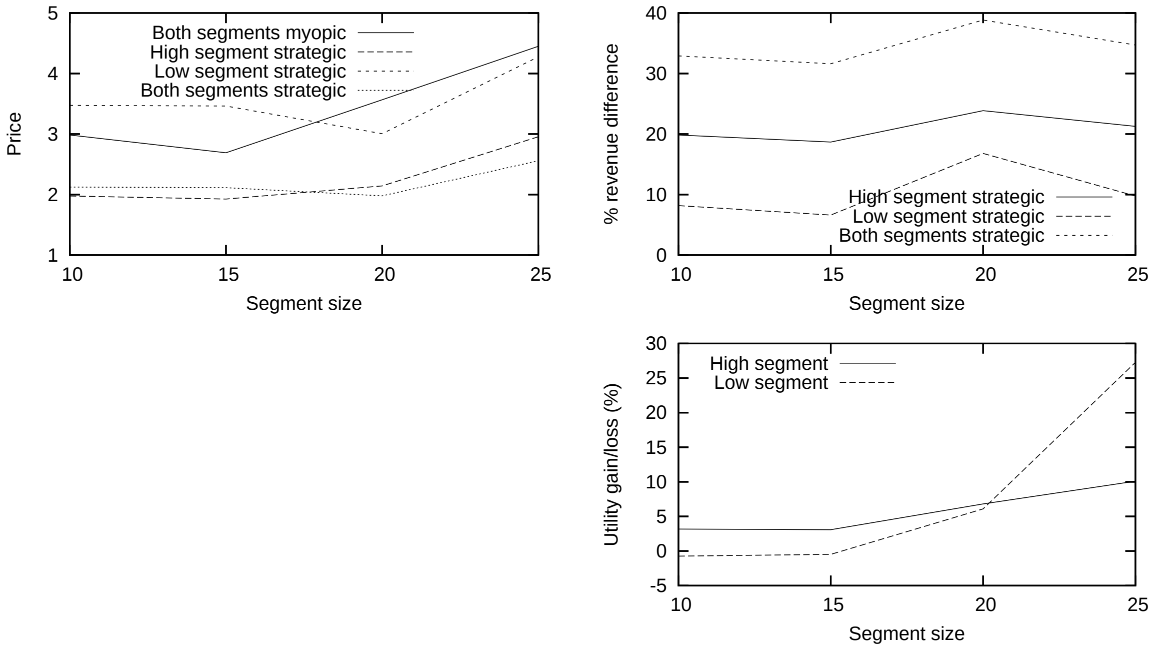


**Figure EC.6 Initial prices, effect of strategic behavior on revenues and utility gain/loss from strategic behavior of another segment in the case of symmetric duopoly and equally-sized segments with $a_{1j} = 1$ and $a_{2j} = 2$, as functions of segment size**

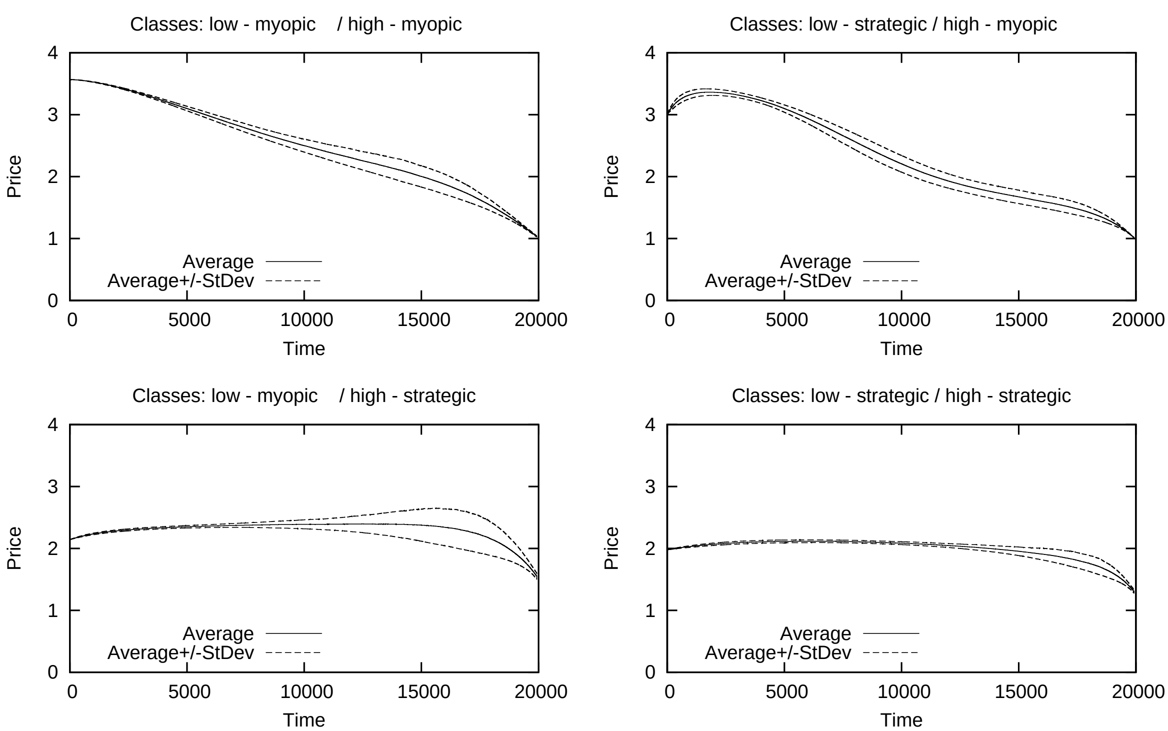


**Figure EC.7 Averages of simulated price paths under two segment ($a_{1j} = 1$, $a_{2j} = 2$) symmetric equilibrium pricing policies for $N_1 = N_2 = 20$**