QUANTITY COMPETITION IN THE PRESENCE OF STRATEGIC CONSUMERS AND RETAIL PRICE MAINTENANCE

ANDREI BAZHANOV, YURI LEVIN AND MIKHAIL NEDIAK

Abstract. Oligopolistic retailers decide on the initial inventories of an undifferentiated limited-lifetime product offered to strategic consumers. A manufacturer sets the first-period (full) price, while the second-period (clearance) price is determined by a market clearing process. The resulting symmetric pure-strategy equilibria may lead to no sales in the first or second period (Cournot outcome versus collusion), and sales in both periods with the clearance price above or at the salvage value. The equilibria possess a comprehensive set of monotonic properties. In particular, increasing strategic behavior can benefit retailers, hurt consumers, push a high-cost retailer out of the market, and lead to increasing inventory of a low-cost retailer. Increasing competition may harm the local economy, and high levels of strategic behavior may insure against oversupply that leads to clearance sales at the salvage value. The welfare-optimal number of retailers can lead to the above-cost clearance price.

Keywords: oligopoly, strategic consumers, Nash-Cournot equilibrium, consumer behavior, rational expectations

1. Introduction

In the current global economy, it is common for major transnational manufacturers to introduce a new version of a product in local markets. Characteristic examples include a December 23, 2013 launch of Samsung Galaxy Grand 2 in India, an introduction of a new model of the Official Match Ball of the FIFA World Cup by Adidas every four years, called “Brazuca” in 2014, and a 2013 introduction of six new Ford models in China. The rapid pace of fashion, innovation, and technological progress limits the lifetime of products, making them obsolete within a relatively short time. At the time of entry, the manufacturer may wield considerable market power and an ability to control the initial price but, eventually, competing innovation or fashion takes its course and reduces consumer willingness to pay for the product. A local market for such limited-lifetime product may have an arbitrary number of retailers that initially sell it at the manufacturer-controlled price, but eventually engage in clearance sales to liquidate the remaining inventory.

A similar process takes place in local markets for transportation services. Cargo forwarders and aggregator companies have to negotiate seasonal allotment contracts with carriers that ultimately provide the underlying capacity. The predominant fixed-commitment contracts (see Pompeo and Sapountzis (2002)) force forwarders to offer discounts for any unused capacity with the approach of the departure time. In their roles, a carrier is similar to a manufacturer, while forwarders and aggregators are similar to retailers.

A major operational decision faced by retailers and cargo forwarders is determining the quantity of the product that they are going to supply to the market. At the strategic level, this decision involves more than just procuring a certain inventory of the product; it can include choosing which retail outlets carry the product or even opening new outlets, allocating the warehouse capacity, making shipping arrangements, sizing the sales staff, and making other marketing and operational decisions. All these complicating aspects contribute to product cost and supply inflexibility. The
same factors increase the importance of the product quantity decision which, in isolation, is relatively easy to formalize.

The main selling season of the product is often characterized by resale price maintenance (RPM) through which the manufacturer exerts price controls over the market. The history of RPM traces back to the nineteenth century and “has been one of the most controversial antitrust topics ever since” (Orbach (2008)). Available evidence indicates that the scale of RPM use is essentially underestimated and the effects of this phenomenon on economy require close attention. As Marvel and McCafferty (1985) put it, “the absence of significant empirical evidence is surely the greatest remaining impediment to a comprehensive analysis of RPM.” When RPM was illegal in the USA, Butz (1996) quoted antitrust authorities arguing that RPM is “ubiquitous” and “endemic”, “but based upon ‘winks and nods’ rather than written agreements that could be used in court.” Butz concludes that “manufacturers have many, many instruments” to punish or reward retailers in order to control the retail price “and to some extent will do so whether or not the law permits it.”

In 2007, the Supreme Court determined that all manufacturer-imposed vertical price fixing should be evaluated using a rule of reason approach. According to the Court, “Absent vertical price restraints, the retail services that enhance interbrand competition might be underprovided. This is because discounting retailers can free ride on retailers who furnish services and then capture some of the increased demand those services generate” (www.ftc.gov, accessed 5 August, 2015). Using the data before and after the Court decision, MacKay and Smith (2014) show that, after 2007, RPM became even more common. The explicit evidence mentioned in the literature included “manufacturers and suppliers of childcare and maternity gear, light fixtures and home accessories, pet food and supplies, and rental cars. Sony has publicly used minimum RPM on electronics such as camcorders and video game consoles, and as of mid-2012, Sony and Samsung began enforcing minimum RPM on their televisions. Other retailers do not comment on whether or not they enter minimum RPM agreements, perhaps due to negative consumer sentiment associated with higher prices.” The study concludes that “In states where RPM contracts are treated under the more relaxed rule-of-reason standard, prices increased.” Ding et al. (2015) show that for a major area of consumer valuations and search costs, a manufacturer, a demand-generating retailer (vs. online free-rider), and consumers would prefer RPM rather than minimum advertised price policy.

On the consumer side of the market, we see a population that is accustomed to quick changes in fashion, the emergence of new models of the products, and their limited life cycles. Consumers are familiar with typical price trajectories resulting from intertemporal price discrimination by the sellers, and may engage in strategic, or forward-looking, shopping behavior by delaying the purchase until the period of price reductions. In doing so, consumers realize that delaying the purchase may reduce the sense of novelty and their enjoyment of the product, but they still make this calculated trade-off. Sitting between a manufacturer with near-monopoly power and strategic consumers, retailers must make their best of the situation while aggressively competing for their market shares. Competition typically forces retailers to increase their supply to the market at the cost of decrease in their profits. On the other hand, the literature shows that profit-maximizing retailers shrink the inventory in response to increasing strategic consumer behavior, which we confirm in our setting. This intriguing interplay between the opposing phenomena brings the following research questions. First and foremost, what are the effects of strategic consumer behavior on retailer inventory decisions and profits? A common view is that strategic consumer behavior is detrimental for retailers, but is this necessarily true under competition? Does the speed of reduction in product value play a substantial role in these effects? Better yet, do consumers themselves necessarily benefit from their strategic behavior? The answer is not obvious because consumer behavior drives competitive responses from the retailers. Finally, do the increases in strategic consumer behavior and retailer competition benefit the local economy?
In addressing these questions, we turn our attention to a generic local market in which a transnational manufacturer engages into RPM essentially setting the first-period list price or manufacturer suggested retail price (MSRP) for a limited-lifetime product. Currently, there is no single theory explaining pervasiveness of RPM use by manufacturers and retailer acceptance. For example, collusive retailers can force a manufacturer to declare a desirable first-period price (Orbach (2008)). Retailers may follow the suggested price under repeated interactions even when this price is non-binding since the manufacturer uses it to communicate private information on marginal cost and consumer demand to the retailers (Buehler and Gärtner (2013)). Given a variety of possible scenarios, the focus is on the effects of strategic consumer behavior on oligopolistic markets with RPM rather than incentives of retailers to comply.

We consider all possible outcomes for any reasonable value of the first-period price and interaction between oligopolistic retailers and strategic consumers in the corresponding regimes allowing, e.g., to find a profit-maximizing list price for the manufacturer. If the manufacturer operates only in a single market, this value of the list price in combination with other parameters, including the number of retailers and the level of strategic behavior, will determine the structure of equilibrium. As a rule, however, transnational manufacturers operate in multiple markets with notably different valuations for the same product whereas, to maintain a consistent brand image, to combat cross-border/market consumer diversion due to online sales, or, possibly, to comply with anti-dumping regulations\footnote{According to World Trade Organization (2015), “if a company exports a product at a price lower than the price it normally charges on its own home market, it is said to be ‘dumping’ the product.” Moreover, Bulow et al. (1985), indicate that “[t]he broadest definition of dumping includes any case where a firm price discriminates between two markets.”}, prices are often set to be comparable when converted to local currencies. As an example, Table 1 indicates prices of Apple Watch (Sport, 38mm model) in GBP across different launch regions. The average price without tax is £240.71 with a standard deviation of £4.71, which is approximately 2\% of the average. Consequently, comparable currency-denominated MSRP values may substantially vary across the markets when they are expressed in terms of the maximum consumer valuation, leading to different outcomes.

We consider a stylized two-period model where retailers compete in quantities by making inflexible first-period supply decisions. Since the main intention of first-period operational decisions, associated with the quantity decisions, is to increase first-period sales, we assume that the first-period demand and the resulting sales are non-decreasing in the initial order quantities. In the first period, regular consumers plan their purchases according to their expectations of the second-period price. Because of the capacity commitments of the retailers, the second-period (clearance) sales are described by the Nash-Cournot model. The lower bound on the second-period price is provided by

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<th>Launch region</th>
<th>Price (£)</th>
<th>Tax rate</th>
<th>Price without tax</th>
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<tr>
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<td>249.17</td>
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<tr>
<td>US (California)</td>
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<td>235.78</td>
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<tr>
<td>US (Montana)</td>
<td>236</td>
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<td>280</td>
<td>15%</td>
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<td>Canada (Alberta)</td>
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<td>258</td>
<td>10%</td>
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<td>8%</td>
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<tr>
<td>China</td>
<td>282</td>
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the salvage value since there is usually a large number of bargain-hunter consumers who are ready to absorb the excess supply at a sufficiently low price.

We answer the research questions by analyzing a game among retailers where the information set is determined by the manufacturer and consumer behavior characteristics. We derive a closed-form solution for the rational expectations symmetric equilibrium (RESE) in pure strategies for the proposed generalized Nash-Cournot model. This analytical tractability is a distinguishing feature of our approach to an otherwise unwieldy problem. The equilibrium permits a complete characterization and takes one of the following forms:

1. When the list price is sufficiently high relative to consumer valuations, all consumers delay their purchases until the second period, effectively turning the market into a one-period Nash-Cournot.
2. When the list price is relatively low and the relative decrease in valuations between the two periods is large (the product is not very durable), the market reduces to the first period only, because retailers limit the amount of product they supply to the market. This is essentially an indirect collusive outcome facilitated by the manufacturer.
3. For intermediate values of the first-period price and a sufficiently low salvage value, RESE results in sales during both periods, as well as a second-period price higher than the salvage value.
4. In the same range of the first-period price as form 3, and with a sufficiently high salvage value, RESE still results in sales in both periods. However, the second-period sales take place at the salvage value. This “salvaging” outcome is not attractive to the retailers because they incur a large loss in the second period due to product oversupply.

Thus, possible equilibria describe situations where retailers fully follow the suggested price (RESE2), sell only part of inventory at MSRP (RESE3 or 4), and do not sell at MSRP (RESE1). Within each type, the equilibrium is unique. Across all types, the unique equilibrium always exists under the conditions of RESE1 and 2, but it may not be unique in the complementary case. For the latter, we provide a sufficient condition that guarantees that RESE3 exists and is unique. This condition requires the unit cost to be high compared to the salvage value. The equilibria exhibit stability in terms of consumer expectations and the strategies used by the individual retailers, which is manifested by convergence of realized expectations or retailer actions to equilibrium under a linear adjustment process.

Besides intuitive monotonic properties, the equilibria reveal several novel insights. It is intuitive that, when the number of retailers increases, the total supply of the product does not decrease, the resulting second-period price falls, the total profit of retailers decreases, and the total surplus of consumers increases. However, it is not always true that the aggregate welfare (the sum of the total profit and consumer surplus) increases with the level of competition. For example, when a relative decrease in consumer valuations for the product between two periods is very small, the aggregate welfare may be increasing, decreasing, or it may even attain an internal maximum. From a regulator’s point of view, the corresponding optimal market structure would involve, respectively, a monopolistic retailer, a perfect competition, or an oligopoly. For the third form of RESE, the maximum of the aggregate welfare with respect to the level of competition results in a clearance price above the unit cost.

The response of equilibrium to changes in the level of strategic consumer behavior is more complex. The total quantity supplied to the market never increases with an increase in strategic behavior. That is, at the aggregate level, retailers always respond to strategic behavior by reducing supply despite competitive pressures. As a result, retailers may capitalize on strategic behavior since the total profit may be non-monotonic. Typically, total profit decreases as consumers become more strategic, e.g., when the relative decrease in valuations between the two periods is large or
in a monopoly. However, there are two distinct cases leading to profit gains resulting from the equilibrium response of retailers to strategic consumer behavior:

- the “continuous gain” is characterized by continuously increasing profit in the level of strategic behavior; this gain may happen when the second-period sales are either profitable or at loss, but only when the relative decrease in valuations is small and the level of strategic behavior is high;
- the “discontinuous gain” occurs at various levels of strategic behavior and the relative decrease in valuations, but only when the difference between the unit cost and the salvage value is relatively small; profit increases because retailers reduce inventories in response to increased strategic behavior, which leads to the switch from RESE4 to RESE3.

The most pronounced form of these phenomena is the “boundary-value gain”, i.e., profit with myopic consumers is less than with fully strategic consumers. This may occur only when a strong first-period inventory competition leads to the second-period sales below cost.

Since RESE4 is unfavorable for the retailers, they would generally prefer to avoid it whenever possible. It is then particularly noteworthy that an increase in strategic consumer behavior may prevent salvaging equilibrium from taking place. We provide a sufficient condition to rule out RESE4 in the form of a lower bound on the level of strategic behavior. On the other hand, as shown in §5.3, the most beneficial markets for the manufacturer have intermediate values of the ratio of MSRP to the highest consumer valuation and RESE4 is possible.

The total consumer surplus is not generally monotonic and may attain maximum at an intermediate level of strategic behavior. Thus, the consumer population as a whole does not necessarily benefit from becoming more strategic, and may, in fact, lose by being “too strategic.” Similarly, the aggregate welfare is generally non-monotonic: it may attain a maximum that tends to arise for high levels of retailer competition and a small relative decrease in valuations. Non-monotonicity of the aggregate welfare is characterized in closed form for the case of salvaging equilibrium.

We present a review of related literature in §2, describe the model in §3, and state the characterization of equilibrium as well as a sufficient condition for its existence and uniqueness in §4. We analyze equilibrium properties and some extensions of the model in §5 and the properties of consumer surplus and the aggregate welfare in §6. The extension of the basic model for different costs is studied in §7. Finally, §8 provides a summary of monotonic properties and outlines several possibilities for extending and applying the proposed model. All mathematical proofs are presented in the online appendix.

2. Quantity decisions and strategic consumers

One of the first to recognize the importance of strategic consumer behavior is Lazear (1986) who studies a monopoly pricing problem with fixed inventory. Among a variety of two-period settings, Lazear considers a given population of strategic buyers whose valuations for a fashion good decrease by a fixed factor in the second period.

Coase (1972) has initiated a study of strategic buyer behavior in an intertemporal pricing problem faced by a durable good monopolist. The essence of his famous conjecture is that “the competitive outcome may be achieved even if there is but a single supplier”. As one of the possible solutions to this problem, Coase proposes to restrict the quantity of the good supplied to the market through contractual or other arrangements.

These early studies have led to further research in consumer behavior in the context of intertemporal pricing. Shen and Su (2007) survey results involving strategic consumer models, and Aviv et al. (2009) review the research on the mitigation of strategic consumer behavior. We focus our attention on results where quantity-based decisions of sellers affect strategic consumers. For a monopolistic retailer, Cachon and Swinney (2009) consider a two-period model with uncertain demand and find that the optimal choice of the initial inventory and subsequent markdown is better than
committing to a price even in the presence of strategic consumers. Moreover, an opportunity to replenish the inventory at the beginning of the second period is much more valuable for the retailer in the presence of strategic consumers than when all consumers are myopic.

Su (2007) considers a deterministic model of monopolistic pricing and rationing policy for a fixed inventory of a limited-lifetime product. The market consists of four segments characterized by one of the two fixed valuation levels (high- or low-valuation consumer types) and one of the two given values of waiting costs (patient or impatient consumers). Su shows that market heterogeneity may lead to profit gains from the increased strategic behavior of low-valuation consumers when high-valuation consumers are myopic (impatient). In this case, the retailer sells the product at a high price to the arriving high-valuation consumers, while the arriving low-valuation patient (fully strategic) consumers are waiting for clearance. When the market of low-valuation consumers becomes large enough, the monopolist drops the price, effectively exploiting a price discrimination scheme. This effect relies on the threat of stockouts for high-valuation consumers, which increases their willingness to pay, and on the proportional rationing rule used in the model.

Liu and van Ryzin (2008) concur that “capacity decisions can be even more important than price in terms of influencing strategic consumer behavior”; they study the effects of capacity decision when prices are fixed while consumers have full information and can be risk-averse. The decision is expressed in terms of consumer rationing risk. Liu and van Ryzin find that capacity rationing can mitigate strategic consumer behavior, but it is not profitable for risk-neutral consumers. Under competition, the effectiveness of capacity rationing is reduced, and there exists a critical number of firms beyond which rationing never occurs in equilibrium.

These studies suggest that retailers are most challenged by strategic consumer behavior when there is a large number of competitors, consumers are risk-neutral, and the market is homogeneous with respect to the level of strategic behavior. Moreover, when consumers do not know the total supply of the product, it is impossible to use strategic rationing to control their behavior. Our study fills the gap in the existing results for this challenging setting.

Without strategic consumers or two-period demand, quantity competition has been considered by Sherali et al. (1983) for a homogeneous product in a leader-follower framework, and by Farahat and Perakis (2011) and Kluberg and Perakis (2012) for differentiated products. Kreps and Scheinkman (1983) argued that the first-stage capacity commitment by duopolistic firms selling an undifferentiated product yields a Cournot outcome even if the equilibrium production and prices are determined by price competition in the second stage. Among other settings, Cachon (2003) considered, in §6.5, newsvendor competition with a fixed first-period price and demand proportional to inventory.

3. Model description

We consider a two-period market for a limited-lifetime product with an arbitrary number of identical retailers. All the retailers have the same unit cost $c$ and offer the product to regular consumers at the same first-period price $p_1 > c$. This assumption is not unusual “in a competitive retail market, where retailers frequently stock identical products, sell them at the same suggested retail prices, and at nearly identical costs from manufacturers” (Liu and van Ryzin (2008)). A relaxation of the “same cost” assumption is discussed in §7. An example of resale price maintenance is provided by the Microsoft’s Xbox One game console (Kinect model) that was introduced in the USA in November 2013 at $499 and continuously remained at that level as of June 2015 at any US retailer. The prices are also exogenously fixed in the model of Netessine and Zhang (2005).

If there is some product remaining after the first period, retailers engage in clearance sales in the second (clearance) period. Since the product offerings are undifferentiated, the retailers lower their prices until all remaining inventory is cleared, that is, second-period price $p_2$ (identical for all retailers) is sufficiently low for the total clearance demand to equal the total remaining inventory. Similarly to Cachon and Swinney (2009), we assume that, in the second period, there is an infinite
number of bargain-hunting consumers who can buy any remaining product at per-unit salvage value \( s < c \). As a result, \( p_2 \) never goes below \( s \). The salvage value also allows for the possibility of inventory buy-back contracts of retailers with the manufacturer, or the availability of alternative sales channels for the retailers.

Each retailer maximizes its profit by selecting the initial inventory level. The resulting game among the retailers is similar to the classical Nash-Cournot model, but with a substantially distinct two-period structure and a special role played by MSRP in the first period.

We now describe the market dynamics. Let retailers be indexed by set \( I \) of size \( n = |I| \), and retailer \( i \in I \) product supply and sales in the first period be \( y_i^1 \) and \( q_i^1 \). Since the second-period market is cleared, each retailer’s second-period supply and sales are equal to \( y_i^1 - q_i^1 \). Denote the total first-period product supply and sales as \( Y = \sum_{i \in I} y_i^1 \) and \( Q = \sum_{i \in I} q_i^1 \), respectively. Then the total second-period supply is \( Y - Q \) and the retailer \( i \) profit is

\[
r_i^1 = -c y_i^1 + p_1 q_i^1 + p_2 (y_i^1 - q_i^1). \tag{1}
\]

First-period sales \( q_i^1 \) are determined based on a consumer decision model.

3.1. **Consumer decision model.** The consumer decision model describes two aspects: demand allocation between two periods and among the retailers. We will start with the first one.

3.1.1. **Demand allocation between two periods.** We normalize the number of regular consumers to one, and let first-period valuations \( v \) be drawn from a uniform distribution on the interval \([0, 1]\). Normalization of valuations effectively expresses revenue and inventory as “unitless” quantities and MSRP as a share of maximum valuation. In order to capture a typical decrease in valuations for seasonal and limited-lifetime products, we introduce factor \( \beta \in [0, 1] \): if the consumer’s first-period valuation is \( v \), the second-period valuation becomes \( \beta v \). Two logical restrictions ensure non-trivial equilibrium results. First, inequality \( \beta > c \) guarantees that the highest-valuation consumer is prepared to pay more than the unit cost in the second period. If this restriction does not hold, the clearance price can never be above the unit cost. We also suppose that \( p_1 > \frac{s}{\beta} \) to ensure that salvage value \( s \) is less than the highest-second-period valuation \( \beta p_1 \) of regular consumers who are forced to delay their purchases by MSRP. Similarly to \( \beta > c \), this restriction supports a non-trivial second-period outcome in an equilibrium with a substantial role of regular consumers.

The availability of information about total supply of the product varies among the markets. Some markets, such as land or real estate, have nearly perfect information, an assumption used in Liu and van Ryzin (2008). In many other markets, total system-wide inventory is unobservable, which reduces the ability of retailers to use rationing as a tool for stimulating first-period demand from strategic consumers. When consumers do not observe total supply, they cannot infer exact price \( p_2 \) and availability \( \alpha \in \{0, 1\} \) that the product is available in the second period.

**Assumption 1.** Consumers do not know the total product supply and form expectations: (a) expected availability \( \bar{\alpha} \in \{0, 1\} \) of the product in the second-period and (b) expected second-period price \( \bar{p}_2 \).

Given these expectations, consumers decide whether a first or second-period purchase maximizes their intertemporal surplus, which is similar to Lazear (1986), Su (2007), and Cachon and Swinney (2009):

**Assumption 2.** In addition to their expectations, consumers know only their private valuations \( v \), list price \( p_1 \), product durability \( \beta \), and the second-period surplus discount factor \( \rho \in [0, 1] \). When the product is available, a consumer with valuation \( v \) buys in the first period if the first-period surplus \( \sigma_1 \triangleq v - p_1 \) is not less than the expected second-period surplus \( \sigma_2 \triangleq \bar{\alpha} \rho (\beta v - \bar{p}_2) \).

Since \( \sigma_2 \geq 0 \), consumers with \( v < p_1 \) never buy in the first period because such a purchase would result in a negative surplus. The proposition below describes the first-period demand, where we use the notation \( a \lor b \triangleq \max\{a, b\} \), \( a \land b \triangleq \min\{a, b\} \).
Lemma 1. Given consumer expectations, surplus-maximizing behavior is to buy in the first period if \( v \geq v^{\min} \), where the unique valuation threshold is given by \( v^{\min} = p_1 \vee \left( \frac{p_{-1 - \alpha \rho \beta}}{1 - \alpha \rho \beta} \wedge 1 \right) \). The resulting total first-period demand is \( D = 1 - v^{\min} \).

Undervaluation of the surplus from delaying a purchase means that even for a product that does not depreciate much by the second period, i.e., \( \beta \) is near one, consumers with any valuation may myopically ignore the second period during the first-period deliberations, i.e., have \( \rho = 0 \). The value of \( \rho \) may depend on the market targeted by the product, e.g., for age- or culture-oriented products, and on the consumer confidence in the stability of the financial situation. As \( \rho \) increases, consumers place more emphasis on the second period in their wait-or-buy decisions. Thus, unlike \( \beta \), which models an objective decrease in valuations, \( \rho \) is a subjective parameter of the consumers describing the level of their strategic behavior. The essence of the distinct roles of \( \beta \) and \( \rho \) has been succinctly captured by Pigou (1932): “Everybody prefers present [i.e., \( \rho < 1 \)] pleasures or satisfaction of given magnitude to future pleasures and satisfaction of equal magnitude [i.e., \( \beta = 1 \)], even when the latter are perfectly certain to occur.” Consumer discount rates can be estimated in practice, as Busse et al. (2013) illustrate in the context of car purchases.

3.1.2. Demand allocation among retailers. Since consumers have no preferences among the retailers, the marketing and distribution efforts are the only differentiating aspect. It is recognized both in practice and in research (e.g., Balakrishnan et al. (2004)), that typical consumer behavior results in larger sales of a particular retailer if the product is presented to consumers at a larger number of retail outlets, in larger quantities on store shelves, and in more ads. We consider attraction \( a^i(y^i) \) as a measure of retailer \( i \) efforts that are not decreasing in the retailer’s inventory.

Assumption 3. The function \( a^i(y^i) \) is continuous, not decreasing in \( y^i \), and \( a^i(0) = 0 \). Consumers do not know the functional form of \( a^i(y^i) \) and react only to the resulting vector of attraction values.

Identical retailers operate under alike conditions and use similar recipes for creating the firm’s attractions, i.e., \( a^i(y^i) = a(y^i) \) for all \( i \in I \). Moreover, any two identical retailers with the same attraction have equal market shares, and the market share of any retailer decreases by the same amount if the attraction of any other retailer is increased by a particular amount. These properties, complemented by a simple assumption that zero attraction leads to zero market share, satisfy the conditions of the market share theorem of Bell et al. (1975), which claims that the functional form of the market share of retailer \( i \), in this case, is \( a(y^i)/\sum_{j \in I} a(y^j) \).

The first-period demand \( d^i \) of retailer \( i \), determined by its market share, depends not only on \( y^i \) via \( a(y^i) \) but also, inversely, on \( y^{-i} \) – the vector of inventories of the others. Assumption 1 implies that demand \( d^i \) is homogeneous of degree zero: \( d^i(ky^i, ky^{-i}) = d^i(y^i, y^{-i}) \) for any \( k > 0 \), i.e., total supply cannot influence \( d^i \) when the ratios \( y^i/Y \), \( i \in I \) remain the same. Since the total first-period demand \( D = \sum_{i \in I} d^i(y^i, y^{-i}) \) is also homogeneous of degree zero, retailer \( i \) market share \( d^i(y^i, y^{-i})/D \), is homogeneous of degree zero. The following lemma specifies the functional form of \( a(y^i) \).

Lemma 2. If retailer \( i \) market share is homogeneous of degree zero and has functional form \( a(y^i)/\sum_{j \in I} a(y^j) \), where \( a(y) \) is continuous in \( y \), \( a(y) \) has the unique functional form \( a(y) = a(1)y^{\gamma} \).

By choosing the scale of attraction so that \( a(1) = 1 \), we obtain the functional form for demand \( d^i \) :

\[
d^i(y^i, y^{-i}) = D \frac{(y^i)^\gamma}{\sum_{j \in I} (y^j)^\gamma}, i \in I,
\]

where \( \gamma \in [0, 1] \) is the inventory elasticity of attraction or inventory elasticity of demand, normalized by the market share of other retailers (Online Appendix B). Function (2) is a symmetric form of a widely used general attraction model (e.g., Monahan (1987), Gallego et al. (2006)). An empirical
study of Naert and Weverbergh (1981) concludes that the attraction model is “more than just a theoretically interesting specification.” This model “may have a significantly better prediction power than the more classic market share specifications.” This conclusion is supported by later studies (e.g., Klapper and Herwartz (2000)). The case \( \gamma = 0 \) means that a retailer’s attraction does not depend on \( y^i \), and \( d^i = \frac{D}{n} \) for any \( y^i > 0 \) and \( i \in I \). (Even though attractions are not continuous at 0 in this case, we demonstrate that the analysis is still possible.) This case was used in §4.4 of Liu and van Ryzin (2008) to study the effect of rationing on strategic behavior of risk-averse consumers. Cachon (2003), in §6.5, considers a newsvendor competition model where retail demand is “divided between the \( n \) firms proportional to their stocking quantity” which matches the case of \( \gamma = 1 \) in our model. This case can be viewed as a fluid limit of the following simple randomized allocation model. Suppose all retailers pool their (discrete) inventory into an urn (one may think of different retailers’ inventory being identified by different colors). Each customer randomly picks an item from the urn (without replacement), and the retailer to whom the item belongs is credited for the sale. In such allocation model, the case of intermediate \( 0 < \gamma < 1 \) corresponds to pooling of attractions rather than inventories.

Since product is undifferentiated and the retailers are identical, consumers buy from any retailer with available product. If the combined supply of retailers is insufficient to satisfy the combined demand, one of the rationing rules can be used. For example, according to the surplus-maximizing rule (see Tirole (1988)), consumers buy in the order of their valuations. The following lemma shows that retailers have no stockouts independently of the rationing rule.

**Lemma 3.** Consider any \( \bar{Y} \geq 1 - v_{\min} \), symmetric inventory profile \( (\frac{\bar{Y}}{n}, \ldots, \frac{\bar{Y}}{n}) \in \mathbb{R}_+^n \), and any behavior of consumers under stockouts in the first period. For any \( i \in I \), let \( y^{-i} = (\frac{\bar{Y}}{n}, \ldots, \frac{\bar{Y}}{n}) \in \mathbb{R}_+^{n-1} \). The following claims hold: (I) any profit-maximizing response of retailer \( i \) to \( y^{-i} \) must satisfy \( y^i \geq \bar{y}^i \), where \( \bar{y}^i \) is the unique positive solution to \( \bar{y}^i = d^i(\bar{y}^i, y^{-i}) \); (II) for any \( y^i \geq \bar{y}^i \), (a) stockouts are impossible, (b) the total first-period sales are \( Q = 1 - v_{\min} \), the individual first-period sales are \( q^i = d^i(y^i, y^{-i}) \), the resulting second-period inventories are \( y^i - q^i, i \in I \) and (c) the second-period price is

\[
\bar{p}_2 = s \vee [\beta(1 - \bar{Y})].
\] (3)

The timing of main events in the market and the corresponding inputs are outlined in Figure 1.

### 3.2. Rational expectations equilibrium

Lemma 1 identifies rational consumer behavior for given expectations, list price \( p_1 \), and behavioral parameters \( \rho, \beta \) which are the only inputs known to consumers according to Assumption 2. In particular, it specifies valuation levels of consumers who purchase in the first period. However, these results are insufficient to identify how consumer expectations form. While it is possible to look for equilibrium behavior of retailers for given expectations, our ultimate goal is to find equilibria that can be sustained in the long run. Therefore, we need to close the loop by identifying expectations that are rational. That is, the equilibrium inventory levels of the retailers must lead to precisely the same observed product availability and clearance prices as expected by the consumers.

When a completely new technology, such as the very first personal computer or cell phone, is introduced to the market, consumers may not be able to form rational expectations about the appearance of subsequent versions of the product and the resulting pricing policies. However, in mature markets, manufacturers regularly launch similar products, or new models of the same product, and consumers, getting accustomed to price-drop patterns, adjust their expectations about future pricing policies to closely match their observations. Adjustments are no longer needed if the expectations coincide with the eventual observations. On the other hand, retailers regularly conduct market research to estimate current consumer expectations. Thus, we assume that retailers operate under complete information.
Retailer game

Manufacturer determines $p_1, c$

Total demand: $D = D(p_1, p_2, \alpha, \beta, \rho)$

Retailers supply $y^i, i \in I$

Retailer $i$ demand: $d^i = d^i(y^i, y^{-i})$

Regular sales

Clearance

Market clearing at $p_2 \geq s$

Using this notion of rationality, the rational expectations symmetric Cournot-Nash equilibrium (RESE) in pure strategies is defined as follows:

1. Given consumer expectations and $y^{-i}$, let the best response of retailer $i$ be $BR_i(y^{-i}, \bar{p}_2, \bar{\alpha}) = \arg \max_{y^i} r_i(y^i, y^{-i}, \bar{p}_2, \bar{\alpha})$.

2. For given consumer expectations, let $\hat{y} = \hat{y}(\bar{p}_2, \bar{\alpha})$ denote a symmetric Nash-Cournot equilibrium inventory level in the retailer game, i.e., $\hat{y}(\bar{p}_2, \bar{\alpha}) = BR_i([\hat{y}, \ldots, \hat{y}], \bar{p}_2, \bar{\alpha})$, where $(\hat{y}, \ldots, \hat{y}) \in \mathbb{R}^{n-1}_+$, and $\hat{y}(\bar{p}_2, \bar{\alpha}) = n\hat{y}(\bar{p}_2, \bar{\alpha})$ be the corresponding total inventory.

3. The tuple $(Y^*, p_2^*, \alpha^*)$ is a RESE if $Y^* = \hat{Y}(p_2^*, \alpha^*), p_2^* = s \lor [\beta(1 - Y^*)]$, and either $\alpha^* = 0$, if $Y^* = 1 - v^*$, or $\alpha^* = 1$, if $Y^* > 1 - v^*$ where $v^*$ is the equilibrium value of $v^{\text{min}}$.

From now on, $r^*$ denotes the equilibrium profit of a retailer. Equilibrium values may be specified for the type of RESE, e.g., $r^*3$ or $Y^{*,1}$ if necessary.

3.3. Discussion of model assumptions. We conclude this section with the discussion of specific implications of model assumptions. Some of the assumptions are quite common and well understood. For example, consumers are modeled as homogeneous in the level of strategic behavior $\rho$ and relative valuation decrease $\beta$. This assumption is applicable to any products targeting specific market segments. The value of $\rho$ may also be tied to the average time value of money (rate of return), which is relatively homogeneous for all consumers. The assumption of retailer symmetry is common for qualitative models of competition, when retailers do not differ in their cost structure or brand value. We relax this assumption in §7.

The information structure of the model is rather general. Indeed, it is relatively rare for the total product supply in the market to be visible to consumers while the market share effort, such as the number of outlets, does signal to consumers the relative market power of the retailers. Generally, consumer expectations about the second-period price and rationing risk may or may not be probabilistic in this context but, for pure strategy (deterministic) equilibria, deterministic expectations are consistent with retailer behavior. As we show below, there is a considerable amount of insight even from the pure-strategy case.
The following sections focus on the case of $\gamma = 1$ in the first-period demand (2) as most results hold for any $\gamma \in (0, 1]$, and this case is more reader-friendly than for intermediate values of $\gamma$. Some of the effects weaken when $\gamma$ goes to zero and disappear for $\gamma = 0$. The robustness of the main results with respect to changes in $\gamma$, including the closed-form analysis for $\gamma = 0$, is shown in the online appendix B.

4. Characterization of RESE

The restriction of rationality for consumer expectations immediately implies the following conclusions about the equilibrium.

**Lemma 4.** In any rational expectations equilibrium, (1) $p_2 < \beta p_1$ if there are sales in the second period; (2) $Y \geq 1 - p_1$, which holds as an equality only if there are no sales in the second period; (3) $\rho \beta Y < 1 - p_1$ if there are sales in both periods and $p_2 > s$; $\rho \beta Y \geq 1 - p_1$ and $p_2 \geq c$ if there are sales only in the second period; and (4) $v^\text{min} = p_1$ if and only if $\alpha = 0$ or $\rho = 0$.

Since $v^\text{min} \geq p_1$, part (2) of Lemma 4 justifies the assumption of Lemma 3 for a RESE.

4.1. No-salvaging RESE. We start by providing closed-form expressions for three of the possible equilibrium cases.

**Theorem 1.** A unique RESE with the stated structure exists if and only if the respective conditions hold:

- RESE1 (No sales in the first period): $v^* = 1$, $\alpha^* = 1$, $p_2^* = c + \frac{\beta - c}{n+1}, Y^* = \frac{n}{n+1} (1 - c/\beta)$, and $r^* = \frac{(\beta - c)^2}{(n+1)^2 \beta}$ under condition $p_1 \geq 1 - \frac{n^2}{n+1} \rho (\beta - c) \Delta P_1$.
- RESE2 (No sales in the second period): $v^* = p_1$, $\alpha^* = 0$, $Y^* = 1 - p_1$, and $r^* = \frac{1}{n} (p_1 - c)(1 - p_1)$ under condition $p_1 \leq \frac{nc}{n-1+\beta} \Delta P_2$.
- RESE3 (Sales in both periods, $p_2^* > s$): $v^* = \frac{p_1 - \rho (1 - Y^*)}{1 - \rho \beta}$, $\alpha^* = 1$, $p_2^* = \beta (1 - Y^*)$, where $Y^*$ is the larger root of a quadratic equation, and $r^* = \frac{1}{n} [(p_1 - c)(1 - v^*) + (p_2^* - c)(Y^* - 1 + v^*)]$, under condition $P_2 < p_1 < P_1$ and one of the following:
  - (a) $\frac{n-1}{n} (p_1 - s)(1 - v^*) Y^* \leq (c - s)(1 - s/\beta)^2$, or
  - (b) condition (a) does not hold, $v^* < 1 - \frac{\hat{r}^i}{\beta}$, and $r^* \geq \hat{r}^i i \Delta P_2 \leq \frac{\sqrt{(p_1 - s)(1 - v^*)} - \sqrt{\frac{n-1}{n} Y^*(c - s)}}{2}$, where $\hat{r}^i$ is the maximum profit of a firm deviating from this RESE in such a way that $p_2 = s$ (the total inventory is greater than $1 - \frac{\hat{r}^i}{\beta}$).

The equilibrium characteristics $Y^*$, $v^*$, and $r^*$ are continuous on the boundaries between these forms of RESE. Moreover, in RESE3, $Y^* \geq \frac{n}{n+1} (1 - c/\beta)$.

If the initial consumer expectations of the second-period price are such that $p_2^0 < p_2^*$, the game is repeated, and expectations follow a linear adjustment process, then the sequence of games converges to $p_2 = p_2^*$ for any sufficiently small speed of adjustment.

Market-share competing retailers may have incentives to deviate from MSRP but, according to Federal Trade Commission (www.ftc.gov, accessed 5 August, 2015), “[a] manufacturer ... may stop dealing with a retailer that does not follow its resale price policy.” In some states and countries vertical price fixing is illegal and manufacturers can indirectly influence the price by offering to the retailers who follow MSRP various cost-reducing benefits (e.g., Butz (1996), Buehler and Gärtner (2013)). Proposition below provides an upper bound on the profit of a deviating retailer assuming that this retailer obtains the entire first-period demand by setting the first-period price below MSRP. This retailer has an additional fixed cost $K$ and its unit cost is $c^H > c$. 

Proposition 1. The profit $r^i$ of retailer $i$ with the first-period price below $p_1$ is upper-bounded by $UB^i = -K + \frac{1}{4}(1-c^H)^2$ if $\frac{\beta(1-2Y^{-i})}{2-\beta} < c^H \leq 2p_1 - 1$ or $UB^i = -K + (p_1 - c^H)(1-p_1)$ if $c^H > \max\{2p_1 - 1, \beta(p_1 - Y^{-i})\}$.

The proof shows, in particular, that when the $c^H$-range in the first case is not empty, the $c^H$-range in the second case is $c^H > 2p_1 - 1$. The following corollary provides conditions on $K$ and $c^H$ guaranteeing that retailers do not deviate from MSRP under RESE1 where $p_1$ is the greatest among RESE and the incentive for retailers to decrease the first-period price is quite strong.

Corollary 1. Under RESE1, the first-period price below MSRP is unprofitable for a retailer if $\frac{(\beta-n)\beta+2(n-1)c}{(2-\beta)(n+1)} < c^H \leq 2p_1 - 1$, and $K \geq \frac{1}{4}(1-c^H)^2 - \frac{(\beta-c)^2}{(n+1)^2\beta}$ or $c^H > \max\{2p_1 - 1, \beta p_1 - \frac{n-1}{n+1}(\beta-c)\}$ and $K \geq (p_1 - c^H)(1-p_1) - \frac{(\beta-c)^2}{(n+1)^2\beta}$.

For example, if $n = 2, \beta = 1$, and $\frac{1+2c}{3} < c^H \leq 2p_1 - 1$, where $p_1 \geq 1 - \frac{2}{3}\rho(1-c)$ (RESE1 exists) retailers have no incentive to decrease price below $p_1$ because, by Corollary 1, the sufficient "no-deviation" condition becomes $K \geq \frac{1}{4}(1-c^H)^2 - \frac{(1-c)^2}{9} = \frac{1-c^H - c^2}{9}$, which holds for any $K \geq 0$ after $1-c^H \geq \frac{1}{2}$.

We assume below that retailers follow MSRP in the first period. In practice, market outcomes corresponding to RESE1 can be identified by very small first-period sales primarily arising, e.g., from slight heterogeneity in the level of strategic behavior. Therefore, RESE1 describes practical scenarios where overwhelming majority of consumers wait for clearance sales. This scenario is common in retail because it describes, e.g., MSRP use solely to make discounts seem more dramatic than they actually are since the retailers effectively avoid using MSRP in the first period.

RESE2 is the opposite to RESE1: $p_1$ is low (high-valuation market), all consumers whose valuations are higher than $p_1$ buy in the first period, and there are no sales in the second period. Condition $p_1 \leq p_2$ implies that the existence of this RESE does not depend on $\rho$ because $\hat{a} = 0$ — consumers do not expect second-period sales and, by Lemma 1, the equilibrium valuation threshold of the first-period buyers is $v^* = p_1$ regardless of $\rho$. Also, RESE2 input area shrinks in $\beta$ and $n$, disappearing for $\beta = 1$ and $n \to \infty$. The "$\beta$-effect" results from increasing attractiveness of the second-period market when retailers can gain from two-period price discrimination, and the "$n$-effect" results from increasing quantity competition for the market share that may force retailers to procure more inventory than just for the first period. The input area increases in $c$ because the second-period profit approaches zero in $c$ faster than the first-period profit, decreasing the relative attractiveness of the second-period sales. Retailers divide the profit associated with the total supply that is just enough to cover the first-period market. Since the supply is determined by an externally set MSRP, retailer competition is reduced to market sharing and we can interpret this outcome as an MSRP-facilitated collusion. In either of the first two equilibria, the intertemporal effect of competition is (locally) eliminated and, consequently, $Y^*$ and $r^*$ do not depend on $\rho$.

RESE3 describes scenarios with intermediate $p_1$ leading to sales in both periods achieving price discrimination between high and low valuation consumers. It provides a bridge between the opposites: a competitive Cournot outcome of RESE1 and an MSRP-enabled collusion of RESE2. Conditions (a) and (b) correspond to different attractiveness of salvage-value sales for a potential
deviator from RESE3 that increases inventory. Condition (a) means that the deviator profit monotonically decreases, i.e., for the inputs that satisfy this condition, RESE3 is stable with respect to small parameter deviations given that $p_1$ is sufficiently far from the boundary. Under condition (b), deviator profit has a local maximum with $p_2 = s$ but this maximum does not exceed the profit under RESE3. The inputs for which (b) holds are near the boundary of RESE3 existence where this equilibrium may be unstable with respect to parameter misestimation.

In reality, consumer expectations may deviate from actual market outcome. The last paragraph of Theorem 1 provides a simple sufficient condition of convergence for possibly misaligned second-period price expectations when similar products (with the same $\beta$) are repeatedly introduced to the market with the same parameters. This condition assumes that retailers can accurately identify consumer expectations. If consumers incorrectly assume no sales in the second period, the equilibrium outcome, by Lemma 1, is the same as with myopic consumers ($\rho = 0$). If consumers assume there are sales in the second period but make mistake about the second-period price, the structure of RESE1 remains the same because it does not depend on expectations whenever $v^* = 1$. As to RESE3, the following result implies the upper bound on change in $\hat{Y}$ (the total symmetric equilibrium inventory with $\bar{p}_2 \neq p_2$) due to the effect of expectation errors.

**Corollary 2.** Under RESE3, $\frac{\partial \hat{Y}}{\partial p_1} \leq \frac{\rho}{1-\rho\beta} \max \left\{1, \frac{p_1 - c}{\beta - \rho} \right\}$.

For a monopolist, RESE3 takes a simpler form described in the following corollary. In particular, condition $P_2 < p_1 < P_1$ is necessary and sufficient.

**Corollary 3.** For $n = 1$ and any $c \beta < p_1 < 1 - \frac{c}{2} (\beta - c)$, RESE is $v^* = \frac{2p_1 - \rho c}{2 - \rho \beta}$, $\alpha^* = 1$, $p_2^* = \frac{\beta v^* + c}{2}$, $Y^* = 1 - \frac{1}{2} \left( \frac{c}{\beta} + v^* \right)$.

Since price and quantity decisions are equivalent for a monopoly, this corollary provides a characterization of the price-skimming policy when the first-period price is externally regulated. The second-period price for a monopolist in our model always exceeds the unit cost (because $v^* \geq p_1 > \frac{c}{\beta}$ in RESE1 and 3). On the other hand, increasing competition may drive the second-period price below cost, which we demonstrate in a market for a durable good with myopic consumers and some $n > 2$. The second-period price in this case remains above cost in a duopoly.

**Corollary 4.** For $\beta = 1$, $\rho = 0$, and $c < p_1 < 1$, RESE1 and RESE2 cannot be realized and, in RESE3, the second-period price is below cost if and only if $n \geq 2 + \frac{p_1 - c}{1 - p_1}$.

Increasing competition not only decreases the second-period price below cost, but undermines the very existence of RESE3. Indeed, condition (a) in RESE3 holds for any $n \geq 1$ only if $s$ is sufficiently low. On the contrary, for large $s$, condition (a) may not hold. Moreover, one can show that the condition $r^* > \bar{r}_1$ will then be violated for all sufficiently large $n$ (this is the case presented below in Corollary 5). This means that growing competition provides an incentive for a retailer to deviate from this form of RESE by increasing supply beyond the point where $p_2 = s$. Despite the resulting losses in the second period, this deviation can be profitable because the first-period market share of the deviating retailer is dramatically higher. Hence, growing competition may result in the non-existence of RESE3 even though condition $P_2 < p_1 < P_1$ holds.

**Corollary 5.** If condition $P_2 < p_1 < P_1$ holds and condition (a) of RESE3 existence is violated in the limit of $n \to \infty$, RESE3 does not exist for all sufficiently large $n$.

This result shows that we need to refine our understanding of the equilibrium and conditions for its existence. For monopoly ($n = 1$), Theorem 1 exhaustively covers all feasible parameter values. Starting from duopoly, condition $P_2 < p_1 < P_1$ may not guarantee the existence of RESE3. The result presented below shows that, in the same $p_1$-range, there may exist one more form of RESE with sales in both periods and $p_2^* = s$. 
4.2. Salvaging RESE. The best response in the retailer game depends on $Y^{-i} \triangleq Y - y^i$ — total inventory less the inventory of retailer $i$. If $Y^{-i} < 1 - \frac{s}{\beta}$, retailer $i$ has control over the second-period price. Namely, $p_2 > s$ if $y^i < 1 - \frac{s}{\beta} - Y^{-i}$ (no salvaging) or $p_2 = s$ if $y^i \geq 1 - \frac{s}{\beta} - Y^{-i}$ (salvaging). If $Y^{-i} \geq 1 - \frac{s}{\beta}$, salvaging is forced on retailer $i$, i.e., $p_2 = s$ regardless of supply $y^i$. Condition $Y^{-i} < 1 - \frac{s}{\beta}$ is used in a symmetric form with $Y^{-i} = \frac{n-1}{n}Y^*$ in the following characterization of the last equilibrium form further referred to as RESE4.

**Theorem 2** ("Salvaging" RESE4: sales in both periods, $p_2^* = s$). RESE with $\alpha^* = 1, p_2^* = s, v^* = \frac{p_2^* - \rho s}{1 - \rho^2}, Y^* = \frac{n-1}{n}p_2^*s(1 - v^*)$, and $r^* = \frac{p_2^* - s}{ns^2}(1 - v^*)$ exists if and only if one of the following mutually exclusive conditions holds:

- (a) salvaging is forced on retailers, i.e., $\frac{n-1}{n}Y^* \geq 1 - \frac{s}{\beta}$;
- (b) condition (a) does not hold, and $\left(\beta \left(1 - \frac{s}{\beta}\right)^2 + (p_1 - \beta)(1 - v^*)\right)\frac{n-1}{n}Y^* > \left(\frac{1}{\beta} - \frac{1}{\beta^2}\right);$
- (c) conditions (a) and (b) do not hold, $Y^* > 1 - \frac{s}{\beta}$, and there are no real roots of the equation

$$2Y^3 - \left(2 - v^* - \frac{c}{\beta} + \frac{n-1}{n}Y^*\right)Y^2 + \left(1 - \frac{p_1}{\beta}\right)(1 - v^*)\frac{n-1}{n}Y^* = 0$$

in the interval $(1 - v^*, 1 - \frac{s}{\beta})$, or $r^* \geq \hat{\beta}^2(\hat{Y})$, where $\hat{\beta}^2(\hat{Y})$ is the maximum profit of a firm deviating from this RESE in such a way that $p_2 > s$, and $\hat{Y}$ is the only root of (4) in the interval $(1 - v^*, 1 - \frac{s}{\beta})$.

If the initial consumer expectations of the second-period price are such that $\rho > s$, the game is repeated, and expectations follow a linear adjustment process, then the sequence of games converges to $\hat{p} = s$ for any sufficiently small speed of adjustment.

One practical realization of this equilibrium outcome is a retailer use of liquidation channels for excess inventory (such as www.salvagesale.com, liquidations.walmart.com, and www.liquidationchannel.com). RESE4 provides an example of an overinvestment (in this case, in inventory) which is a known phenomenon in economics. Unlike RESE1-3, RESE4 cannot exist for $n = 1$ since a monopolist would not have an incentive to overinvest in this setting. This can be seen, e.g., from the expression for $Y^*$. The larger $n$ is, the easier retailers find themselves in RESE with $p_2^* = s$. Similar to RESE3, conditions (b) and (c) correspond to different attractiveness of a higher second-period price for a potential deviator from RESE4 that decreases inventory. Condition (b) means that the deviator profit monotonically increases in inventory, i.e., for the inputs that satisfy (b), RESE4 is stable with respect to small parameter changes when $p_1$ is sufficiently far from the boundary. The first part of condition (c) — no real roots of (4) in the interval $(1 - v^*, 1 - \frac{s}{\beta})$ — means that the deviator profit has no local maximum with $p_2 > s$, whereas inequality $r^* \geq \hat{\beta}^2(\hat{Y})$ requires that when this maximum exists at $y^i = \hat{Y} - \frac{n-1}{n}Y^*$, it does not exceed the profit under RESE4. The inputs where RESE4 exists only by the second part of (c) are close to the boundary of RESE4 existence where this equilibrium may be unstable with respect to parameter misestimation. Conditions (a)-(c) hold if $c - s$ is sufficiently small, i.e., the cost is largely compensated by salvaging any excess units, which makes this outcome attractive for the retailers. This form of equilibrium results in market overcapacity and salvage value sales of a significant portion of the total supply.

The last paragraph of Theorem 2, similar to the one in Theorem 1, provides a sufficient convergence condition for misaligned second-period price expectations when a new version of the product (with the same $\beta$) is repeatedly introduced to the market with the same parameters, and retailers do not make mistakes about consumer expectations. The effect of expectation error on the total inventory is limited by the following result.

**Corollary 6.** Under RESE4, $\frac{\partial \hat{Y}}{\partial p_2} = \frac{\rho}{1 - \rho^2} \frac{n-1}{n}p_2 - s/c - s$. 

Theorem 2 implies a necessary condition \( v^* < 1 \), which means that there is positive demand in the first period. This condition is equivalent to the upper bound \( p_1 < 1 - \rho(\beta - s) \) \( \equiv P_4 \) signifying that a relatively high MSRP precludes salvaging outcome. Alternatively, this condition represents an upper bound on the level of strategic behavior:
\[
\rho < \frac{1 - p_1}{\beta - s}.
\]

As long as the product is durable enough for \( 1 - p_1 < \beta - s \) to hold, highly strategic (with \( \rho \) near one) consumers guarantee that the salvaging outcome is impossible. Since \( P_4 < P_1 \) (the bound that separates RESE1 and 3), \( P_4 \) separates RESE4 and 3.

We now turn to the question of equilibrium uniqueness. By Theorem 1, RESE1, 2, and 3 are mutually exclusive since the corresponding \( p_1 \)-ranges do not intersect. The result below shows that RESE1, 2, and 4 are also mutually exclusive. Moreover, part (b) guarantees that condition (a) of Theorem 1 holds for \( p_1 \)-range of RESE3 and, at the same time, RESE4 cannot exist.

**Proposition 2.** A unique RESE exists and is of the form stated if any of the following conditions hold: (a) RESE1 if \( p_1 \geq P_1 \), or RESE2 if \( p_1 \leq P_2 \), or RESE3 if (b.1) \( P_2 < p_1 < P_1 \) and (b.2) \( \frac{n-1}{n} (p_1 - s)(1 - p_1) \leq (c - s) \left( 1 - \frac{s}{\beta} \right) \).

Condition (b.2) trivially holds for \( n = 1 \). In general, it has the form of a lower bound on \( c - s \), i.e., the unit cost is sufficiently high compared to the salvage value. The condition holds for any \( n > 1 \) and \( p_1 \), if it holds for \( n \to \infty \) and \( p_1 = \frac{1}{2}(1 + s) \) (\( p_1 \) maximizing the left-hand side). The resulting stronger inequality is \( c - s \geq \frac{(1-s)^2}{4(1-s/\beta)} \), which holds, e.g., for \( c = 0.25 \) and \( s = 0 \). Thus, when the unit cost is relatively high, retailers avoid the unfavorable “salvaging” outcome.

The analysis of this section leaves a possibility that RESE does not exist. This is indeed the case, but the fraction of model inputs where this may occur is very small. Combining all conditions in Theorems 1 and 2, we can determine which of the four types of equilibria exist (if any) for any given set of inputs \((n, \rho, \beta, c, s, p_1)\) satisfying the feasibility conditions \( 0 \leq \rho < 1 \), \( 0 \leq s < c < \beta \leq 1 \), and \( \frac{s}{\beta} \lor c < p_1 \leq 1 \). We have performed this analysis for 1,000,000 randomly (according to uniform distribution) sampled feasible model inputs for different values of \( 1 \leq n \leq 1,000 \). The results are presented in Figure 2. Subgraph (a) is an area plot that shows the fractions of inputs resulting in a particular equilibrium structure (RESE1, 2, or 3 only, both RESE3 and 4, RESE4 only) as the heights of the respective shaded areas for each \( n \). As \( n \) increases, RESE2 disappears and the prevalence of RESE1 and 4 grows with RESE4 reaching more than 50% of model inputs.
Subgraph (b) shows the fractions of inputs resulting in both RESE3 and 4 as well as non-existence of equilibrium. The fraction of inputs where both RESE3 and 4 exist is 4% for a duopoly and considerably less for other levels of competition. The fraction of inputs where no RESE exists is at most 0.191% (reached for $n = 5$).

5. Properties of RESE

The results of previous sections can be used, e.g., by a manufacturer or retailer to estimate possible outcomes of entering the market. These outcomes depend on the current levels of competition, strategic behavior, and other parameters. For an existing market, the effects of changes in these parameters can be more relevant in order to anticipate possible market alterations. As to changes in consumer strategic behavior, one of their drivers is macroeconomic. When the economy is expanding, more consumers prefer to buy now than wait, and vice versa – an average consumer is more inclined to delay the purchase when the economy shrinks. For example, a study of a Fortune 500 retailer sales by Allenby et al. (1996) shows that even “fashion-forward consumers who purchase apparel early in the season are more sensitive to economic conditions and expectations than previously believed.”

Various forms of Consumer Confidence Indicators report on changes in consumer behavior. For example, the Index of Consumer Confidence is defined by the Conference Board of Canada website as “a crucial indicator of near-term sales for companies in the consumer products sector... Data is collected on each respondent’s age, sex, marital status, and geographic location of residence.” Using these data and other macroeconomic variables, a retailer and/or manufacturer can estimate possible changes in $\rho$ and in market outcomes, respectively, given that the current situation is known. Lemmens et al. (2005), in an empirical study of the European markets, conclude that “the Consumer Confidence Indicators become much more homogeneous as the planning horizon is extended.” This homogeneity emerges inside of regions, and is determined by cultural, economic and geographic differences.

This section, on one hand, supports previous studies showing that equilibrium total supply increases in $n$ and decreases in $\rho$. Both trends typically decrease retailers’ profits. On the other hand, we specify two distinct cases when these opposing trends “compensate” each other leading to increasing profit in $\rho$. Increasing $\rho$ also has different effects on consumer second-period surplus and total second-period sales depending on the consumer valuation and market situation, respectively.

5.1. RESE1-3 (no salvaging). The analysis below takes into account possible switches between different forms of RESE. The requirement of a unique RESE existence in some statements can be guaranteed, e.g., by Proposition 2.

5.1.1. Switches between RESE forms. When RESE is unique, $p_1$-ranges indicated in Theorem 1 provide a unique mapping between input parameter values and different forms of RESE. Figure 3 illustrates how these ranges change with $n$ and $\rho$:

(a) the bounds on $p_1$ that separate RESE3 from RESE1 and 2 are decreasing in $n$;
(b) the upper bound on $p_1$ in RESE3 is decreasing, and the lower bound is constant in $\rho$; and
(c) the lowest possible value of $p_1$ that leads to RESE1 is strictly above the highest possible value that leads to RESE2.

These observations are summarized as follows:

**Proposition 3** (Changes in RESE structure). For RESE1-3, the following claims hold:

1. (From 2 to 3 in $n$) If $p_1 \leq \frac{c}{\beta}$, there exists $n_2 \triangleq \frac{p_1(1-\beta)}{p_1-c} \geq 1$ such that RESE can only be realized with sales only in the first period (RESE2) for any $n \leq n_2$, and with sales in both periods and $p_2 > s$ (RESE3) for any $n > n_2$. 

Figure 3. Typical boundaries between RESE1-3 in $(n, p_1)$ and $(\rho, p_1)$ (example for $c = 0.3, s = 0.05, p_1 = 0.5, \beta = 0.6$, and, for left plot, $\rho = 0.6$)

(2) (From 3 to 1 in $n$) For any $\rho \in (0,1)$, if $1 - \rho(\beta - c) < p_1 < 1 - \frac{1}{2}\rho(\beta - c)$, there exists $n_1 \triangleq \frac{1 - p_1}{p_1 - 1 + \rho(\beta - c)} \geq 1$ such that RESE can only be realized with sales in both periods and $p^*_2 > s$ (RESE3) for any $n < n_1$, and with sales only in the second period (RESE1) for any $n \geq n_1$.

(3) (From 3 to 1 in $\rho$) For any $n \in [1, \infty)$, if $1 - \frac{n}{n+1}(\beta - c) < p_1 < 1$, there exists $\rho_1 \triangleq \frac{n + 1 - p_1}{n + 1 - \beta - c}$ such that RESE can only be realized with sales in both periods and $p^*_2 > s$ (RESE3) for any $\rho < \rho_1$ and with sales only in the second period (RESE1) for any $\rho \geq \rho_1$.

(4) (No switches) If $\frac{\rho}{\beta} < p_1 \leq 1 - \beta + c$, RESE can only be realized with sales in both periods and $p^*_2 > s$ (RESE3) for any $n \geq 1$ and $\rho \in [0, 1)$.

The changes in equilibrium structure generally lead to shifts in sales to the second period as the levels of competition or strategic behavior increase. Next, we examine changes in the quantitative characteristics of equilibrium.

5.1.2. Monotonicity of $Y^*, v^*,$ and $nr^*$. We now examine the monotonicity of $v^*, Y^*$, and $nr^*$ in $n$ and $\rho$ within RESE1-3 and, by continuity, between these forms of RESE.

Proposition 4. For RESE described in Theorem 1, the following claims hold:

(1) The equilibrium total supply $Y^*$ is non-decreasing in $n$ (constant for RESE2; increasing for RESE1 and 3) and non-increasing in $\rho$ (decreasing for RESE3; constant for RESE1 and 2).

(2) $v^*$ is non-decreasing in $n$ (constant for RESE1, 2, and RESE3 with $\rho = 0$; increasing for RESE3 with $\rho > 0$) and non-decreasing in $\rho$ (increasing for RESE3; constant for RESE1 and 2).

(3) The total equilibrium profit of all retailers $nr^*$ is non-increasing in $n$ (constant for RESE2; decreasing for RESE1 and 3), decreasing in $\rho$ for RESE3 with $p_1 \geq \beta - \frac{n}{2(n+1)}(\beta - c)$ or $n = 1$, and constant in $\rho$ for RESE1 and 2.

Monotonicity of the total supply and the total profit in the level of competition agree with the theory of oligopoly and can be viewed as a sanity test for the model. On the other hand, monotonicity in the level of strategic behavior (represented by $\rho$) is a much finer result. The new insights of this paper are connected to the following non-trivial interaction between firms and consumers while $\rho$ is increasing. Part (2) of Proposition 4 states that $v^*$ is increasing in $\rho$ when
there are sales in both periods (RESE3) and retailers effectively engage in intertemporal price discrimination. Increasing \( v^* \) means that more consumers delay their purchases, even though total supply \( Y^* \) is decreasing in \( \rho \) (by part (1)), resulting in a decreasing total number of purchases and increasing second-period price. The nature and consequences of this interaction are considered below in more detail. Part (3) of Proposition 4, for the monopoly, agrees with the existing literature that strategic consumer behavior reduces profits. We generalize this effect to the case of oligopoly when the product is not very durable (i.e., \( \beta \) is sufficiently low) or MSRP is relatively high.

In RESE3, oligopolistic retailers counteract additional consumer delays in purchase resulting from increasing \( \rho \) by increasing the equilibrium second-period price \( p^*_2 = \beta (1 - Y^*) \). As a result, the expected surplus of waiting \( \sigma_2 = \rho [\beta v - \beta (1 - Y^*)] \) may not be increasing in \( \rho \). Indeed, its derivative in \( \rho \) is \( \frac{\partial \sigma_2}{\partial \rho} = \frac{\sigma_2}{\rho} + \rho \frac{\partial Y^*}{\partial \rho} \). The first term in the RHS is the realized second-period surplus, which is non-negative for the consumers who buy in the second period. The second term reflects the equilibrium response of the oligopolistic retailers. By part (1) of Proposition 4, this term is negative for any \( \rho > 0 \). For RESE3, the following corollary shows that increasing \( \rho \) has a different effect on \( \sigma_2 \) depending on the consumer valuation.

**Corollary 7.** For RESE1 and 3, expected surplus \( \sigma_2 \) of waiting is (1) increasing in \( n \) for any \( v \in [0, 1] \), and (2) increasing in \( \rho \) for \( v^0 < v \leq 1 \), and decreasing in \( \rho \) for \( 0 \leq v < v^0 \), where \( v^0 = \frac{1}{\beta} \left( p^*_2 + \rho \frac{\partial p^*_2}{\partial \rho} \right) = 1 - Y^* - \rho \frac{\partial Y^*}{\partial \rho} \in \left[ \frac{p^*_2}{\beta}, v^* \right] \) is such that \( \frac{\partial \sigma_2}{\partial \rho} \bigg|_{v=v^0} = 0 \).

For consumers with high valuations, \( \sigma_2 \) is increasing in \( \rho \). In particular, for the consumers with \( v = v^* \), the purchase in the second period is becoming more attractive than in the first period, which means that \( v^* \) is increasing in \( \rho \) (part (2) of Proposition 4). In contrast, \( \sigma_2 \) is decreasing for consumers with low valuations. For example, for the second-period buyers with the lowest valuation \( v = \frac{p^*_2}{\beta} \), the second-period surplus is becoming negative, leading to a decrease in the total number of purchases. For myopic (\( \rho = 0 \)) consumers, \( v^0 = \frac{p^*_2}{\beta} \) and \( \frac{\partial \sigma_2}{\partial \rho} > 0 \) for all second-period buyers.

An increase in \( \rho \) leads to either an increase or decrease in the total equilibrium second-period sales \( Q^*_2 = Y^* - (1 - v^*) \), depending on the parameters:

**Corollary 8.** For RESE3, (1) \( \frac{\partial Q^*_2}{\partial \rho} > 0 \) when \( n = 1 \); and (2) \( \frac{\partial Q^*_2}{\partial \rho} < 0 \) when \( n \to \infty, \rho = 0, \beta < 1 \) and \( p_1 = 1 \).

Proposition 4 claims that \( nr^* \) is decreasing in \( \rho \) for monopoly and oligopoly with a not very durable good. To show that \( nr^* \) may increase in \( \rho \), we consider a limiting case of a durable (within the time frame of the problem) product and consumers with the maximum level of strategic behavior. In this case, the consumer choice of purchase time is determined only by price.

**Proposition 5.** Let \( \bar{n} \triangleq \frac{1 - p_1}{p_1 - c} \). For \( \beta = 1 \) and \( \rho \to 1 \), RESE2 and 4 do not exist and the equilibrium has the form

1. RESE1 with \( v^* \big|_{\rho \to 1} = Y^* \big|_{\rho \to 1} = \frac{n}{n+1} (1 - c) \), \( p^*_2 \big|_{\rho \to 1} = \frac{nc+1}{n+1} < p_1 \), and \( nr^* \big|_{\rho \to 1} = \frac{n(1 - c)^2}{(n + 1)^2} \)

2. RESE3 with \( v^* \big|_{\rho \to 1} = p_1 + n(p_1 - c) \), \( Y^* \big|_{\rho \to 1} = 1 - p_1 \), \( p^*_2 \big|_{\rho \to 1} = p_1 \), and \( nr^* \big|_{\rho \to 1} = (p_1 - c)(1 - p_1) < \frac{n(1 - c)^2}{(n + 1)^2} \) if \( n < \bar{n} \) and

\[
\frac{n - 1}{n} \frac{(p_1 - s)(1 - p_1 - n(p_1 - c)) (1 - p_1)}{(c - s)(1 - s)^2} < 1. \tag{6}
\]

Moreover, when \( n = \bar{n} \),

3. the limiting cases (1) and (2) coincide, and

4. (boundary-value gain) for all \( p_1 \) and \( c \) such that \( \bar{n} \geq 3 \), we have \( nr^* \big|_{\rho \to 1} > nr^* \big|_{\rho = 0} \).
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(a) $\beta = 1$ (b) $\beta = 0.9$

Figure 4. The total profit for RESE3 with $c = 0.45$, $s = 0.05$, $p_1 = 0.5$

In both limiting scenarios, sales occur at a single price: the one-period Nash-Cournot price in part (1) and $p_1$ in both periods in part (2). In part (1), representing a high level of competition, no sales occur at $p_1$, i.e., $v^* = 1$, and price decreases to Nash-Cournot in the second period. The Nash-Cournot supply level in this case exceeds the MSRP-determined lower bound $1 - p_1$ of Lemma 4. In part (2), representing a low level of competition, retailers counteract strategic behavior by reducing the total supply all the way to $1 - p_1$ which exceeds the Nash-Cournot supply level.

Thus, the extreme level of strategic behavior, in combination with MSRP, forces retailers into a collusive outcome. Part (3) shows that $n = \bar{n} = \frac{1 - p_1}{p_1 - c}$ plays a role of a parameter coordination condition ensuring that the Nash-Cournot price coincides with $p_1$. This condition is critical for understanding part (4) that demonstrates the total profit increase as consumer behavior changes from myopic to completely strategic.

This profit gain may appear counterintuitive because strategic consumer behavior is usually considered detrimental. However, in this case, strategic behavior prevents the second-period sales at a loss. Indeed, with completely strategic consumers, the total sales are equal to $1 - p_1$ and occur at $p_1$. In the case of myopic consumers, the first-period sales are the same, while the second-period sales are at loss for any $n \geq 3$ according to Corollary 4. Since the increase in profit is strict, the effect presented in part (4) is quite robust. Indeed, for each $n \geq 3$, there is a continuum of model instances satisfying the parameter coordination condition. Moreover, by continuity in parameters, increased strategic behavior leads to an increase in profit in a local neighborhood of these instances. The “boundary-value” profit gain described in part (4) results in this case from the “continuous gain” since the entire range $\rho \in [0, 1)$ belongs to the same equilibrium RESE3 (Figure 4).

Example 1. Condition (6) is a limiting version of condition (a) of RESE3 existence, which is less restrictive than sufficient condition (b.2) of Proposition 2. Condition (6) holds for all $p_1$ and $1 \leq n < \bar{n}$ if $c > \frac{1 + 4s}{5}$, e.g., if $s = 0$ and $c > 0.2$.

The numerical example below illustrates the behavior of the total profit in $\rho$. For small $\rho$, the total profit is decreasing in $\rho$ (see Figure 4 (a)). On the other hand, when $\beta = 1$, the total profit is increasing for $\rho$ near one. For all values of $n \leq \bar{n} = 10$ in this example, the total profit attains the limit $(p_1 - c)(1 - p_1)$ established in part (2) of Proposition 5. It is natural to expect that this effect of “durable-good non-monotonicity” is becoming weaker when $\beta < 1$. Indeed, Figure 4 (b) illustrates that, for $\beta = 0.9$, the total profit is decreasing in $\rho$ for $n = 1, 2, 3$ and 5, and the increase of $nr^*$ in $\rho$ is small for $n = 10$. This increase can no longer compensate for the losses in $nr^*$ resulting from increased competition.
5.2. RESE4 (salvaging). The following proposition establishes the monotonic properties of $Y^*$, $v^*$, and $nr^*$ for RESE4. These properties partially match those of RESE1-3.

**Proposition 6.** For RESE4, (1) $v^*$ is constant in $n$ and increasing in $\rho$; (2) $Y^*$ is increasing in $n$ and decreasing in $\rho$; and (3) $nr^*$ is decreasing in $n$ and decreasing in $\rho$.

By part (a) of Proposition 2, RESE4 does not exist under the conditions of RESE1 and 2. However, RESE3 and 4 can both exist under the same inputs. In that case, one needs to resort to focal-point arguments to predict which of the two equilibria will be realized. The example of Figure 5 illustrates this fact.

The inputs for Figure 5 are such that inequality $P_2 < p_1 < P_1$ and condition (a) of Theorem 1 for RESE3 existence hold for $n = 1$ but this condition does not hold for $n = \infty$. By Corollary 5, RESE3 does not exist for sufficiently large $n$. At the same time, by part (b) of Proposition 2 and by rationality, RESE4 does not exist and RESE3 is realized uniquely for $n = 1$. It is also straightforward to check that RESE3 can be realized for $n = 1, \ldots, 11$, while RESE4 can be realized for $n = 8, \ldots, \infty$. For $n = 8, \ldots, 11$ either equilibrium is possible.

In line with the interpretation of rational expectations equilibrium as a structure that is self-sustaining in the long run, a possible focal point is an equilibrium with a structure that is similar to the past. Figure 5 (b) shows that RESE4 is considerably worse for the retailers than RESE3. However, RESE4 may be realized because a single retailer cannot unilaterally benefit from decreasing its market share as long as others expect the RESE4 structure and act accordingly.

Now, what is the effect of changing $\rho$ when there is an overlap in inputs leading to RESE3 and 4? For the data considered above, a minor increase in $\rho$ from 0.5 to 0.6 qualitatively changes the situation because, for $\rho = 0.6$, inequality $P_2 < p_1 < P_1$ and condition (a) of RESE3 existence hold for any $n \geq 1$ and neither of subcases (a)-(c) of Theorem 2 hold. Therefore, RESE4 cannot exist in the scenario considered above, and this increase in $\rho$ leads to a discontinuous profit gain and serves as an insurance against salvaging. Such an increase in $\rho$ works by decreasing capacity in RESE3 at the cost of a slight decrease in profit (compare the solid and dashed lines in Figure 5 (b)). The discontinuous profit gain can lead to the boundary-value gain even when $\beta$ is small (Figure 6 b) and can be combined with the continuous gain when $\beta$ is near one. As a result, equilibrium profit can have up to three local maxima in $\rho$ (Figure 6 a).

An increase in $\rho$ cannot always prevent retailers from realizing RESE4, but it does reduce the fraction of inputs leading to it. Figure 7 shows that for $\rho = 0.999$ the maximum fraction of model
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(a) $\beta = 1, n = 5, p_1 = 0.4$

(b) $\beta = 0.5, n = 10, p_1 = 0.8$

Figure 6. Equilibrium profit in $\rho$ for $c = 0.1, s = 0.05$

(a) Prevalence of RESE structures

(b) Prevalence of multiple or no RESE

Figure 7. Fractions of model inputs resulting in a particular RESE structure for $\rho = 0.999$ and given $n$

inputs leading to RESE4 reduces to 37.2% compared to more than 50% in Figure 2, where $\rho$ is unrestricted. Moreover, the area of inputs leading to both RESE3 and 4 shrinks to less than 1.25%.

5.3. Equilibrium inventory and $p_1$. As mentioned in the introduction, the paper primarily focuses on exogenous $p_1$, e.g., when $p_1$ is specified by the manufacturer-retailer agreement (Orbach (2008)). Manufacturers often operate in multiple markets with notably different valuations for the same product, but MSRP may have to be comparable when converted to local currencies for strategic reasons (e.g., maintaining brand image). In this case, the ratio of MSRP to the highest valuation on a specific market can take any value from the range $(c, 1]$ and lead to any type of RESE considered above.

However, a product may target only one specific market, or valuations on several markets might almost be the same. In this case, the manufacturer can try to negotiate $p_1$ to improve its profit. When all other parameters are constant, the manufacturer profit in the local market is directly proportional to the total sales $Y^*$ in this market. Thus, we consider $p_1$ maximizing the total equilibrium retailer inventory $Y^*$.

The simplest “benchmark” case is RESE2 where $p_1$ is relatively low and $Y^* = 1 - p_1$. The supremum of the manufacturer’s sales in RESE2 is obtained as $p_1$ tends to $c$. In practice, this supremum cannot be achieved because retailer profits must be positive and consumer valuations
Proposition 7. When the corresponding RESE exists, (1) \( Y^* < 1 - c \); (2) the unique maximum of \( Y^{*,4} \) in \( p_1 \) is \( Y^{*,4} = \frac{(n-1)(\rho_1 - s)^2}{n(1-\rho_2)(c-s)} \) at \( p_1 = \tilde{p}_1 = \frac{1}{2}(P_4 + s) \); \( Y^{*,4} \geq 1 - c \) if and only if \( c - s \leq \frac{n-1}{n} \tilde{p}_1 - s \frac{1 - \rho_1 - \rho(\beta - s)}{1 - \rho_2 \rho^3} \); (3) \( Y^{*,3} < 1 - c \) for \( n = 1 \); for \( n \to \infty \) and \( p_1 \to P_2 = c, Y^{*,3} \to 1 - c \) and, if \( \rho = 0, \frac{\partial Y^{*,3}}{\partial p_1} \bigg|_{p_1 = P_2 + 0} > 0 \).

The proposition implies that RESE2 is the best for the manufacturer in a market with a single retailer and \( \beta < 1 \) (RESE2 must exist). However, using simulation we find that for most feasible combinations of \( \rho, \beta, c, s \) and \( n > 1 \) (by volume in the space of all feasible combinations of these parameters) the manufacturer, who varies \( p_1 \) while other parameters are fixed, would prefer a value that achieves RESE4, which is the worst for the retailers. Consistently with Proposition 4, Figure 8 shows that the fractions of RESE3 and 4 instances, where \( Y^{*,3} \) and \( Y^{*,4} \) are greater than \( 1 - c \), are increasing in \( n \). For RESE3, this fraction is zero at \( n = 1 \) and remains below 40% for \( n > 1 \), while it is at least 95% for RESE4. Therefore, the manufacturer may prefer markets with many retailers, where the ratio of MSRP to the highest valuation takes intermediate values and RESE4 can be realized. For Figure 8, we used the same simulation approach as for Figure 2. Typical qualitative behavior of \( Y^* \) in \( p_1 \) is illustrated in Figure 9.

Maximization of manufacturer sales with respect to \( p_1 \) does not invalidate the claim of Proposition 5 about the possibility of profit gains when consumers shift from myopic to fully strategic behavior. In practice, there is a minimum first-period price \( p_1^{\text{min}} > c \) that separates retailer margins from zero. By Proposition 5, \( Y^{*,3} \big|_{\rho \to 1} = 1 - p_1 \) so the maximum of \( Y^{*,3} \big|_{\rho \to 1} \) is attained at \( p_1^{\text{min}} \). For any \( p_1^{\text{min}} = p_1 \in (c, \frac{1 + 3c}{4}] \), the value \( \bar{n} \) from Proposition 5 is greater or equal to 3. Thus, part (4) still holds for \( n = \bar{n} \) implying the boundary value gain \( nr^{*,3} \big|_{\rho \to 1} > nr^{*,3} \big|_{\rho \to 0} \). This profit gain disappears only with \( p_1^{\text{min}} = c \) leading to \( nr^{*,3} \big|_{\rho \to 1} = nr^{*,3} \big|_{\rho \to 0} = 0 \). However, the case \( p_1^{\text{min}} = c \) is infeasible and implausible in this problem.

This subsection illustrates a non-trivial nature of manufacturer-retailer interactions under oligopoly with strategic consumers. The properties of possible outcomes described in the above sections can be used to study these interactions in a two-tier supply chain framework. Such analysis includes a
distinct set of research questions, e.g., the comparison of supply chain efficiency under centralized and decentralized settings with various types of contracts (see Su and Zhang (2008) for monopoly), and deserves a separate consideration.

5.4. Retailer’s discount. Lazear (1986) (p. 25) showed that the discounted second-period profit leads to decreasing prices, which typically corresponds to increasing sales. Our setting leads to a similar result in terms of inventory. The proposition below shows that when retailers solve a non-degenerate two-period profit-maximization problem, the equilibrium inventory increases if a discount factor becomes less than one. We call a two-period problem degenerate if it reduces to one period, which happens for RESE1, 2, and for a monopolist in RESE3 since, for n = 1, the first-period demand does not depend on inventory.

Proposition 8. If retailer i’s profit is \( r^i = (p_1 - c)q^i + \lambda(p_2 - c)(y^i - q^i) \), \( \lambda \in (0, 1) \), equilibrium total inventory \( Y^* \) decreases in \( \lambda \) for RESE4, RESE3 with \( n > 1 \) and constant for RESE1, 2, and 3 with \( n = 1 \). If \( \lambda = (1 + \delta)^{-1} \), where \( \delta \) is the interest rate between two periods, the relative increase in \( Y^{*,4} \) from introducing \( \lambda < 1 \) is \( \left( Y^{*,4} - Y^{*,4} \right) / Y^{*,4} = \frac{p_1 - \delta}{p_1 - s} \delta < \delta \).

For example, if \( \lambda = 1, p_1 = 0.5, n = 10, \beta = 0.75, c = 0.1, s = 0.05 \), and \( \rho = 0 \), then by condition (a) of Theorem 2, RESE4 is realized with \( Y^{*,4} = 4.05 \). If, for the same data, retailers consider a 2% interest rate between periods, \( Y^{*,4} = 4.122 \), which is around 1.8% greater than \( Y^{*,4} \). For the same data, but \( \rho = 0.7 \), RESE3 is realized by condition (a) of Theorem 1 with \( Y^{*,3} = 0.85276 \). The same 2% interest rate yields \( Y^{*,3} = 0.85346 \), which is only about 0.08% greater than \( Y^{*,3} \).

5.5. RESE stability. An equilibrium is more likely to emerge in practice if it is (a) asyptotically locally stable, i.e., when the initial retailers’ inventories are close to an equilibrium, they converge to the equilibrium values, or (b) globally stable, i.e., when any initial inventories converge to an equilibrium when it is unique. In our setting, by Theorem 1 and Proposition 2, RESE is unique for any inputs except for a small fraction where both RESE3 and 4 may exist (Figure 2). In the latter case, however, the feasible inventory ranges for RESE3 and 4 are separated by a non-empty interval (Figure 9).

RESE1, 3, and 4, for \( n \geq 2 \), represent a non-degenerate game between retailers that can be reformulated as a one-period game with retailer \( i \)'s payoff function \( \pi^i(g^i, Y^{-i}) = y^i P(g^i, Y^{-i}) - C_i(y^i) \). Then using, e.g., Theorem 3 in al Nowaihi and Levine (1985), the following result holds.

Proposition 9. For any inputs where RESE1, 3, or 4 exist in an open neighborhood of \( Y^* \), a RESE is locally asymptotically stable.
As to global stability, a seminal work of Theocharis (1960) showed that for a linear demand and constant per unit cost, the best-response discrete adjustment process \( y_{t+1}^i = BR^i(Y_{t}^{-i}) \), \( t = 0, 1, \ldots, i \in I \), converges for \( n = 2 \) and any \( y_0^i, y_0^j \). This process means that each retailer observes rivals’ inventories at some time \( t \) and makes a payoff-maximizing inventory decision for \( t+1 \). Further studies refined this result for slower adjustment processes \( y_t^i = y_{t-1}^i + k_i[BR^i(Y_{t-1}^{-i}) - y_{t-1}^i] \) or \( y_t^i = y_{t-1}^i + k_i \partial \pi^i / \partial y^i \) where \( k_i \in (0,1) \) is the speed of adjustment. In particular, according to Fisher (1961), “given the number of sellers, it is always possible to find [slow enough] speeds of adjustment such that the system is stable.”

6. Total consumer surplus and aggregate welfare

In this section, we examine the effects of strategic consumer behavior and retailer competition on the consumers and the local economy. In a two-period problem, the total two-period (realized) consumer surplus is \( \Sigma^* \equiv \Sigma_1 + \Sigma_2 \), where \( \Sigma_1 \) is the total surplus of consumers buying in the first period and \( \Sigma_2 \) in the second. The second-period surplus is not discounted by \( \rho \) since \( \rho \) is a subjective behavioral parameter and such a discount would not reflect the actual surplus. In the extreme case of \( \rho = 0 \), such discounting would completely disregard the second-period surplus of myopic consumers. The expression for \( \Sigma^* \) is given by the following:

**Lemma 5.** For a RESE with valuation threshold \( v^* \) and second-period price \( p_2^* \), total consumer surplus is \( \Sigma^* = (1 - v^*) \left[ \frac{1+v^*}{2} - p_1 \right] + \frac{(\beta v^* - p_1)^2}{2 \beta} \), where the first term is \( \Sigma_1 \) and the second is \( \Sigma_2 \).

Effects on the local economy can be measured in terms of the aggregate welfare of consumers and retailers defined as \( W^* \equiv \Sigma^* + n v^* \). The main structural result for \( \Sigma^* \) and \( W^* \) is

**Proposition 10.** Under the conditions of Theorems 1 and 2,

1. Total consumer surplus \( \Sigma^* \) is non-decreasing in \( n \) (constant for RESE2 and 4 and increasing for RESE1 and 3), constant in \( \rho \) for RESE1 and 2, and increasing in \( \rho \) for RESE4;
2. Aggregate welfare \( W^* \) is
   1. increasing in \( n \) for RESE1, constant for RESE2, and decreasing for RESE4;
   2. constant in \( \rho \) for RESE1 and 2, and, for RESE4, increasing in \( \rho \) for \( \rho < \rho^+ \) and decreasing for \( \rho > \rho^+ \) where \( \rho^+ \equiv \left( 1 - \frac{1}{n} \frac{p_2 - s}{p_1 \beta - s} \right) / \left( 1 - \frac{\beta}{n} \frac{p_2 - s}{p_1 \beta - s} \right) \) if \( n > \frac{p_2 - s}{p_1 \beta - s} \), and \( \rho^+ \equiv 0 \) otherwise.

This proposition implies that the consumer population as a whole benefits from an increase in competition. On the other hand, \( \Sigma^* \) may not be globally monotonic in \( \rho \) for RESE3. The non-monotonicity is established below for the case of \( \beta = 1 \):

**Corollary 9.** Under the conditions of RESE3, \( \beta = 1 \) implies: (1) for all \( n \geq 1 \) and \( \rho \) sufficiently close to one, \( \partial \Sigma^* / \partial \rho < 0 \); and (2) for \( n = 1 \) or \( n \to \infty \) and \( \rho = 0 \), \( \partial \Sigma^* / \partial \rho > 0 \).

Corollary 9 and continuity of \( \Sigma^* \) in \( \beta \) imply that \( \Sigma^* \) has a maximum in \( \rho \) if \( \beta \) is sufficiently large (Figure 10 (a)). Non-monotonic behavior is less pronounced for smaller \( \beta \) (Figure 10 (b)).

Along with the monotonicity of \( \Sigma^* \), Proposition 10 describes certain settings with monotonic aggregate welfare. The direction of monotonicity in a particular parameter varies depending on the equilibrium structure and other inputs. For example, \( W^* \) is increasing in \( n \) for RESE1 (Nash-Cournot outcome), which matches increasing welfare results for a standard one-period Nash-Cournot equilibrium corresponding to our model. However, in other quantity competition settings, welfare may not be increasing in the level of competition. For example, Bulow et al. (1985) (§VI, Example E) claim that welfare may decrease when a retailer with high marginal costs enters a monopoly market. In our model, the aggregate welfare decreases in \( n \) for RESE4 (salvaging outcome) because the resulting increase in product oversupply does not benefit the consumers and
only decreases profits of the retailers. For RESE4, the level of strategic behavior \( \rho^+ \) attains the internal maximum of \( W^* \) as long as \( \beta < 1 \) and the level of competition is sufficiently high, i.e., \( n > \frac{p_1 - s}{p_1 \beta - s} \). For \( n \leq \frac{p_1 - s}{p_1 \beta - s} \), \( W^* \) is decreasing for all \( \rho \). The dependence of \( W^* \) on \( n \) was omitted in Proposition 10 for RESE3 because this case warrants special attention:

**Corollary 10** (Non-monotonicity of \( W^* \) in \( n \)). Treating \( n \) as a continuous variable and \( p^*_2 \) as a function of \( n \) under the conditions of RESE3, the following result holds:

\[
\frac{\partial W^*}{\partial n} \geq 0 \quad \text{if and only if} \quad p^*_2 \geq c \left( \frac{1 - \rho \beta^2}{1 - 2 \rho \beta + \rho^2 \beta} + p_1 - \frac{\rho \beta (1 - \beta)}{1 - 2 \rho \beta + \rho^2 \beta} \right). \tag{7}
\]

The right-hand side of (7) equals \( c \) when \( \rho = 0 \) or \( \beta = 1 \) and strictly greater than \( c \) otherwise.

If there exists \( n^W \) that is strictly within the feasible interval for RESE3 and maximizes \( W^* \), it satisfies (7) as equality. When \( \rho = 0 \) or \( \beta = 1 \), this means that the second-period price corresponding to \( n^W \) equals the unit cost. On the other hand, when \( \rho > 0 \) and \( \beta < 1 \), the corresponding second-period price is strictly greater than the unit cost. Since, by Proposition 4, \( Y^* \) and, therefore, \( p^*_2 \) are strictly monotonic for RESE3, \( n^W \) is unique whenever it exists. From this unique value, we obtain the maximum of the aggregate welfare (the candidates for the integer-valued point of maximum are \([n^W]\) and \(\lceil n^W \rceil\) since, generally, the solution to this equation is real-valued). We illustrate the behavior of \( W^* \) in \( n \) and \( \rho \) in Figures 11 and 12, respectively, for the same set of inputs as our earlier illustrations. For \( \beta = 1 \), Figure 11 (a) demonstrates that the aggregate welfare can be monotonically increasing in \( n \) (for high levels of strategic behavior), and it can also attain the maximum at intermediate values of \( n \) (for lower levels of strategic behavior). The latter illustrates Corollary 10. For \( \beta = 0.9 \), Figure 11 (b) shows that the aggregate welfare may remain monotonically decreasing in the whole range of \( \rho \). In all cases presented in Figure 11 (b), the maximum value of the aggregate welfare is attained by the monopoly. These findings may provide theoretical support for a regulator introducing a policy that affects the number of independent retail chains. Figure 12 indicates that myopic consumer behavior or strategic behavior at an intermediate level may be the best for the local economy in terms of the aggregate welfare. Myopic behavior is the best for low levels of competition, and the optimum level of \( \rho \) tends to increase as \( n \) increases. A smaller value of \( \beta = 0.9 \) leads to the optimality of myopic behavior in a wider range of \( n \).

7. **Different costs**

In this section, we relax the assumption of identical retailers for the case of duopoly. In particular, we examine the impact of the difference in retailer costs on the gains in retailer profits (both
continuous and discontinuous), which may happen when consumers are becoming more strategic. We also provide two additional qualitative effects of strategic consumers on competing retailers. Namely, high levels of strategic behavior in combination with a high first-period price can push a high-cost retailer out of the market. At the same time, equilibrium inventory of the low-cost retailer may increase in \( \rho \).

The latter two effects can be easily illustrated in the equilibrium with second-period sales only. Similar to the symmetric case, this equilibrium (denote it as REE1) exists in a non-trivial form only when consumers are strategic, i.e., when \( \rho \geq \rho_1 = \frac{3 - p_1}{2(\beta - \bar{c})} \), where \( \bar{c} \triangleq \frac{1}{2}(c_L + c_H) \) is the average cost and indices \( L \) and \( H \) denote a low-cost and a high-cost retailers respectively (the effects of different costs on the conditions of equilibria existence are shown in §A.30). The high-cost equilibrium inventory is \( y_{H,*} = \frac{1}{3} \left[ 1 - (2\beta - c_L) / \beta \right] \), which is zero whenever \( c_H \geq \frac{1}{2}(c_L + \beta) \).

For the example presented in Figure 13, only REE3 exists at \( \rho = 0 \) with \( y_{H,*} = 0.1156 \) and \( y_{L,*} = 0.3318 \). Only REE1 exists at \( \rho \geq 0.5 \), ceteris paribus, with \( y_{H,*} = 0 \) and \( y_{L,*} = \frac{1}{3} \left[ 1 - (2\beta - c_L) / \beta \right] = \frac{1}{3} \). The example shows that \( y_{L,*} \) can be increasing in \( \rho \) and even “boundary-value” increasing, i.e., \( y_{L,*} |_{\rho=0} < y_{L,*} |_{\rho=0.5} = y_{L,*} |_{\rho=1} \). This finding refines the results in the existing literature, including Proposition 4 in this paper, that the equilibrium inventory decreases in the level of strategic behavior. In this example, the total equilibrium inventory, indeed, decreases in \( \rho \). The increase in \( y_{L,*} \) is a side-effect of market monopolization: the high-cost retailer inventory

![Figure 11. The aggregate welfare \( W^* \) in \( n \) with \( c = 0.45, s = 0.05, p_1 = 0.5 \)](image1.png)

![Figure 12. The aggregate welfare \( W^* \) in \( \rho \) with \( c = 0.45, s = 0.05, p_1 = 0.5 \)](image2.png)
is less pronounced for a low-cost retailer than for a high-cost one, whose increase in profit may
increase in strategic behavior, described in Proposition 3, that there exists such strategic substitutes:
therefore, it obviously affects whether products are strategic complements or substitutes. Thus, the
determine by increased strategic behavior while in the example of Bulow et al. (1985) the high-
decreases the aggregate welfare. This decrease happens if the (homogeneous) products of both
The continuous gain (Figures 4 and 14 (a)) is possible under REE4, the low-cost retailer always considers the product
For any $1 - \frac{2}{3}(\beta - \bar{c}) < p_1 < 1$, inequality $y^{L,}\rho = 0 < y^{L,}\rho \rightarrow 1$ is equivalent to
inequality $|\rho| = 0$.
In REE1, both retailers consider their products as strategic complements: $\partial^2_{y^L} < 0$. In REE4, the low-cost retailer always considers the products of both firms as strategic complements:
$\partial^2_{y^L} > 0$, while for the high-cost retailer the products are strategic substitutes: $\partial^2_{y^H} < 0$. In REE1, both retailers consider their products as strategic substitutes: $\partial^2_{y^L} = -\beta$. Since an increase in the level of the aggregate welfare (Figure 13 (b)) decreases from $W|_{\rho = 0} = 0.08$ to $W|_{\rho = 0.5} = 0.05$, when the high-cost retailer is pushed out of the market. This example contrasts with the above-mentioned example in Bulow et al. (1985), where the entry of a high-cost retailer to a monopoly market decreases the aggregate welfare. This decrease happens if the (homogeneous) products of both firms (competing in quantities) are considered as strategic substitutes by a low-cost retailer, i.e., $\partial^2_{y^L} < 0$. In our example, this derivative is also negative for both $\rho$. The direction of change in welfare is different in our example because the switch from duopoly to monopoly is endogenously determined by increased strategic behavior while in the example of Bulow et al. (1985) the high-cost retailer entry is exogenous. As shown in Figure 13 (b), $W$ decreases in $\rho$ because both retailer profits $r^L$ and $r^H$ as well as the total consumer surplus $\Sigma^*$ are decreasing when $\rho$ increases from zero to 0.5. This contrast with Bulow et al. (1985) underscores the importance of including strategic consumer behavior in the models of capacity competition.
Finally, we show how the difference in costs affects two types of profit gain due to increased strategic behavior, described in §5. The continuous gain (Figures 4 and 14 (a)) is possible under REE3 and from REE3 to REE1 (given other parameters fixed), it obviously affects whether products are strategic complements or substitutes. Thus, the conclusions of Bulow et al. (1985) potentially depend on the level of strategic behavior.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure13.png}
\caption{REE3 for high and low cost retailers at $c^L = 0.1, c^H = 0.2$, $p_1 = 0.95, \beta = 0.3$, $s = 0$}
\end{figure}
result even in the boundary-value gain, i.e., \( r^H,^* |_{\rho=0} < r^H,^* |_{\rho \to 1} \) (Figure 14 (a)). When the costs are the same, the continuous boundary value gain is possible only for \( n \geq 3 \) (Proposition 5). The discontinuous gain occurring under switches from REE4 to REE3 (Figure 14 (b)) is also more pronounced for the high-cost retailer (in the example shown, the high-cost retailer even experiences the boundary-value gain).

8. Conclusions

Even when consumers are risk-neutral and homogeneous in the level of strategic behavior, retailers can gain from increasingly strategic consumers for any level of competition. There are two distinct cases of this effect: the continuous gain, when the equilibrium profit increases continuously in the level of strategic behavior, and the discontinuous gain, when the profit increases because of the switch from the “salvaging” equilibrium to another two-period equilibrium with a higher second-period price.

The first type of gain occurs only for relatively high levels of strategic behavior and small decreases in valuations. With this gain, retailers use strategic consumer behavior to approach an outcome that is equivalent, in terms of the profit value, to a tacit collusion. The discontinuous gain occurs at various levels of strategic behavior and the relative decrease in valuations, but only when salvage sales are attractive enough, i.e., the salvage value is relatively close to the unit cost. (In case of duopoly, a refinement of the model to the case of asymmetric costs shows that increasing strategic behavior may also push a high-cost retailer out of the market, lead to increasing inventory of a low-cost retailer, as well as increase retailer profits.) For a manufacturer, increasing strategic behavior is always unfavorable because it decreases the total equilibrium inventory procured by the retailers. Both types of retailer profit gains are reversible. When the economy picks up, more consumers buy at the first-period price, becoming less strategic. Then, the incentive for quantity competition increases, and retailers may find themselves in the unfavorable “salvaging” outcome.

We summarize the monotonic properties of equilibrium characteristics with respect to competition level \( n \) and strategic behavior level \( \rho \) in Table 2 using \( \gtrless, \lesseqgtr, \equiv \) to indicate a monotonically increasing, decreasing, or constant property, respectively. The possibility of an internal maximum or minimum is indicated by “max” or “min”, respectively. When multiple symbols are present, it means that different behaviors are possible for different inputs. The monotonicity results with respect to the level of competition are quite conclusive and are obtained in the analytic form. The direction of monotonicity can only vary for the aggregate welfare in REE3. The latter finding may be important for regulatory policies with respect to the level of competition. For REE4, the increasing level of competition is always detrimental for the local economy.
Table 2. Summary of monotonic properties in \( n \) and \( \rho \) by equilibrium form

<table>
<thead>
<tr>
<th>RESE</th>
<th>Monotonicity in ( n )</th>
<th>Monotonicity in ( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1  2  3  4</td>
<td>1  2  3  4</td>
</tr>
<tr>
<td>( Y^* )</td>
<td>≡ ≡ ≡ ≡</td>
<td>≡ ≡ ≡ ≡</td>
</tr>
<tr>
<td>( v^* )</td>
<td>≡ ≡ ≡ ≡</td>
<td>≡ ≡ ≡ ≡</td>
</tr>
<tr>
<td>( nr^* )</td>
<td>≡ ≡ ≡ ≡</td>
<td>≡ ≡ ≡ ≡, min</td>
</tr>
<tr>
<td>( \Sigma^* )</td>
<td>≡ ≡ ≡ ≡</td>
<td>≡ ≡ ≡, max</td>
</tr>
<tr>
<td>( W^* )</td>
<td>≡ ≡ ≡, ( \gamma ), max</td>
<td>≡ ≡ ≡, ( \gamma ), max</td>
</tr>
</tbody>
</table>

Advantages of the presented model include its analytical tractability and natural connections to established oligopoly results. Possible extensions cover a wide range of problems in the study of competition in the presence of strategic consumers, for example: (1) analysis of policy decisions, including taxes and subsidies for the manufacturer, retailers, and/or consumers; (2) study of supply-chain coordination; (3) analysis of competition when advertisement and inventory decisions are decoupled; and (4) study of price-matching contracts as a tool to counteract strategic consumer behavior.

References


**APPENDIX A. MAIN TEXT SUPPLEMENT**

A.1. Proof of Lemma 2 ($a(y) = y^\gamma$). By the conditions of the lemma, equality $a(ky^j)/\sum_{j=1}^n a(ky^j) = a(y^j)/\sum_{j=1}^n a(y^j)$ holds for any $y^j > 0$, $j \in I$, and $k > 0$. Therefore, it holds for $y^i = y > 0$,
\[ y^j = 1, j \neq i, \] and \[ y^l = y, y^j = 1, l \neq i, j \neq l. \] Namely,

\[
\frac{a(ky)}{a(ky) + (n-1)a(k)} = \frac{a(y)}{a(y) + (n-1)a(1)} \quad \text{and} \quad \frac{a(k)}{a(ky) + (n-1)a(k)} = \frac{a(1)}{a(y) + (n-1)a(1)},
\]

which implies \( a(ky)/a(y) = a(k)/a(1) \iff a(ky) = a(k)a(y)/a(1) \). Denoting \( \tilde{k} \triangleq \ln k, \tilde{y} \triangleq \ln y \), and \( \tilde{a}(z) \triangleq \ln(a(e^z)) - \ln a(1) \), the logarithm of the last equation is \( \ln a[\exp(\tilde{k} + \tilde{y})] = \ln a[\exp(\tilde{k})] + \ln a[\exp(\tilde{y})] - \ln a(1) \) or \( \tilde{a}(\tilde{k} + \tilde{y}) + \ln a(1) = \tilde{a}(\tilde{k}) + \ln a(1) + \tilde{a}(\tilde{y}) \iff \tilde{a}(\tilde{k} + \tilde{y}) = \tilde{a}(\tilde{k}) + \tilde{a}(\tilde{y}) \). Since any continuous additive function of one variable is linear with zero intercept, we have \( \tilde{a}(\tilde{y}) = \gamma \tilde{y} \) (note that by the definition of \( \tilde{a}(z) \), \( \tilde{a}(0) \) is, indeed, zero), which implies \( a(y) = a(1) \exp[\gamma \ln y] = a(1)y^\gamma \).

**A.2. Profit function, its properties and inventory decisions for \( \gamma = 1 \).** By Lemma 3, the first-period total sales are \( Q = 1 - v_{\min} \) and retailer \( i \) sales are \( q^i = d^i(y^i, Y^{i-1}) \), which, for \( \gamma = 1 \), is \( y^i Q \). The second-period sales of retailer \( i \) are equal to its second-period inventory \( y^i \left( 1 - \frac{Q}{Y} \right) \).

Then the general expression for retailer \( i \) profit, using (1) and (3), takes the form

\[
\begin{align*}
\text{profit} & = -cy^i + p_1 \frac{y^i}{Y}(1 - v_{\min}) + \left\{ s \vee [\beta (1 - Y)] \right\} \left\{ y^i - \frac{y^i}{Y}(1 - v_{\min}) \right\}. 
\end{align*}
\]

While this expression is continuous in all parameters and inventory \( y^i \), it is generally not globally differentiable. Next, we consider all possible subintervals in terms of \( y^i \). Each subinterval results in a differentiable expression for the profit function and a qualitatively distinct market outcome.

**A.2.1. No sales in the second period.** Formula (1) for profit becomes \( r^i = (p_1 - c)y^i \), which yields a unique profit-maximizing inventory \( y^i = (1 - v_{\min} - Y^{i-1})^+ \), where \( Y^{i-1} = \sum_{j \neq i} y^j \), and the maximum profit \( r^i = (p_1 - c)(1 - v_{\min} - Y^{i-1})^+ \), leading to the following lemma:

**Lemma 6.** For given model inputs and consumer expectations, retailer rationality implies that the effective domain of the inventory decision is \( y^i \geq (1 - v_{\min} - Y^{i-1})^+ \) and \( (p_1 - c)(1 - v_{\min} - Y^{i-1})^+ \) is the lower bound for the optimal profit.

**A.2.2. second-period sales with \( p_2 > s \).** If \( v_{\min} > 1 - Y \) (or \( y^i > 1 - v_{\min} - Y^{i-1} \), there are sales in the second period. If this condition is combined with \( 0 < y^i < 1 - \frac{s}{\beta} - Y^{i-1} \), then \( p_2 > s \) and the profit is given by

\[
\begin{align*}
r^i & = -cy^i + p_1 \frac{y^i}{Y}(1 - v_{\min}) + \beta (1 - Y) y^i \left( 1 - \frac{1 - v_{\min}}{Y} \right) \\
& = y^i \left[ \beta (1 - Y) - c + (p_1 - \beta (1 - Y)) \frac{1 - v_{\min}}{Y} \right] \\
& = y^i \left[ \beta (1 - Y) - c + \beta (1 - v_{\min}) + \frac{(p_1 - \beta)(1 - v_{\min})}{Y} \right]
\end{align*}
\]

with the derivative

\[
\begin{align*}
\frac{\partial r^i}{\partial y^i} & = \beta (1 - Y) - c + [p_1 - \beta (1 - Y)] \frac{1 - v_{\min}}{Y} \\
& + y^i \left[ \beta \left( -1 + \frac{1 - v_{\min}}{Y} \right) - (p_1 - \beta (1 - Y)) \frac{1 - v_{\min}}{Y^2} \right],
\end{align*}
\]

which, using equations \( Y = y^i + Y^{i-1} \) and (10), can be rewritten as

\[
\begin{align*}
\frac{\partial r^i}{\partial y^i} & = \beta (1 - Y^{i-1}) - c + \beta (1 - v_{\min}) - 2\beta y^i + (p_1 - \beta)(1 - v_{\min}) \frac{Y^{i-1}}{Y^2}.
\end{align*}
\]
The second derivative is
\[
\frac{\partial^2 r^i}{\partial (y^i)^2} = -2 \left[ \beta + (p_1 - \beta)(1 - v^{\min}) \frac{Y^{-i}}{Y^2} \right].
\] (13)

A.2.3. second-period sales with \( p_2 = s \). This case is possible only under oligopoly (\( Y^{-i} > 0 \)) because, for a monopolist, any price \( p_2 \leq c \) is not rational. If there are sales in the second period and \( y^i \geq \left( 1 - \frac{s}{\beta} - Y^{-i} \right)^+ \) (or \( Y \geq 1 - \frac{s}{\beta} \)), then \( p_2 = s \) and (8) becomes
\[
r^i = -cy^i + p_1 \frac{y^i}{Y} (1 - v^{\min}) + sy^i \left( 1 - \frac{1 - v^{\min}}{Y} \right)
\]
\[
= -(c - s)y^i + \frac{y^i}{Y} (p_1 - s) (1 - v^{\min})
\] (14)

with the derivative
\[
\frac{\partial r^i}{\partial y^i} = -(c - s) + \frac{Y - y^i}{Y^2} (p_1 - s) (1 - v^{\min})
\]
\[
= -(c - s) + \frac{Y^{-i}}{Y^2} (p_1 - s) (1 - v^{\min}),
\] (15)

which is monotonically strictly decreasing in \( y^i \) when \( v^{\min} < 1 \).

A.2.4. Properties of the profit function. The following lemma provides the properties of retailer \( i \) profit \( r^i \), using the continuity of \( r^i \) in \( y^i \).

Lemma 7. The profit function \( r^i \) is such that

1. If \( 1 - \frac{s}{\beta} - Y^{-i} > 0 \), then
   \[
   (1.1) \quad \frac{\partial r^i}{\partial y^i}|_{y^i=1-\frac{s}{\beta}-Y^{-i}-0} < \frac{\partial r^i}{\partial y^i}|_{y^i=1-\frac{s}{\beta}-Y^{-i}+0}.
   \]

(1.2) \( r^i \left( 1 - \frac{s}{\beta} - Y^{-i} \right) \leq 0 \) if and only if
   \[
   \frac{(p_1 - s) (1 - v^{\min})}{(1 - \frac{s}{\beta})(c - s)} \leq 1; \quad (16)
   \]

(1.3) \( r^i \) is pseudoconcave in \( y^i \) and strictly concave if \( p_1 \geq \beta v^{\min} \) on the interval \( (1 - v^{\min} - Y^{-i})^+ \leq y^i \leq 1 - \frac{s}{\beta} - Y^{-i} \);

(1.4) \( r^i \) is strictly concave on the interval leading to \( p_2 = s \), i.e. \( y^i \geq 1 - \frac{s}{\beta} - Y^{-i} \); and

(1.5) \( r^i \) is pseudoconcave on the interval \( y^i \geq (1 - v^{\min} - Y^{-i})^+ \) if either
   \[
   \frac{\partial r^i}{\partial y^i}|_{y^i=1-\frac{s}{\beta}-Y^{-i}+0} \leq 0 \quad \text{or} \quad \frac{\partial r^i}{\partial y^i}|_{y^i=1-\frac{s}{\beta}-Y^{-i}-0} \geq 0.
   \]

2. If \( 1 - \frac{s}{\beta} - Y^{-i} \leq 0 \), \( r^i \) is strictly concave on its entire domain \( y^i \geq 0 \).

A.3. Proof of Lemma 1 (first-period demand). Recall that, for a first-period buyer with valuation \( v \geq p_1 \), the surpluses of buying in the first period and that of waiting are, respectively, \( s_1 = v - p_1 \) and \( s_2 = \rho \alpha (\beta v - \bar{p}_2)^+ \). Condition \( \sigma_1 \geq 0 \) is equivalent to \( v \geq p_1 \). Condition \( \sigma_1 \geq \sigma_2 \) is equivalent to \( v - p_1 \geq \rho \alpha (\beta v - \bar{p}_2) \Leftrightarrow v \geq \frac{p_1 - \sigma_2}{\rho \alpha (\beta v - \bar{p}_2)} \). Combining these inequalities, we obtain the stated expression for \( v^{\min} \). Since all consumers with \( v \geq v^{\min} \) would buy in the first period, the total demand is \( D = 1 - v^{\min} \).
A.4. **Proof of Lemma 3 (no stockouts).** Part (I). The existence of the unique positive solution \( y^i \) to equation \( y^i = d^i(\hat{y}^i, y^{-i}) \) is established in §B.3.1. Moreover, the reasoning implies that \( y^i \leq d^i(y^i, y^{-i}) \) for any \( y^i \leq \hat{y}^i \). Thus, if \( y^i \leq d^i(y^i, y^{-i}) \), retailer \( i \) sells only in the first period and, by (1), its profit function is \( v^i = (p_1 - c)y^i \), which is increasing in \( y^i \) for any \( y^i \in [0, \hat{y}^i] \). Therefore, inventory \( y^i \) of a profit-maximizing retailer is never less than the first-period demand, i.e., \( y^i \geq \hat{y}^i \) holds.

Part (II). Claim (a) is straightforward when \( y^i \geq \hat{y}^i \) holds and when retailer \( i \) sets the inventory above the symmetric level \( \bar{Y}/n \). In that case, the first-period demand of other retailers decreases compared to \( \bar{Y}/n \) (constant for \( \gamma = 0 \)), which cannot lead to stockouts.

Stockouts may potentially arise only when retailer \( i \) sets the inventory below \( \bar{Y}/n \), increasing the first-period market share of other retailers above the symmetric level. In this case, we show that the first-period demand \( d^i \) of any retailer \( j \neq i \) is not greater than inventory \( y^i = \bar{Y}/n \). Suppose that \( y^i = \bar{y}^i \), which is the minimum possible inventory of a retailer rationally responding to a symmetric profile, and that \( \bar{y}^i \leq \bar{y}^i \). Then \( d^i = \frac{(1-v_{\min})(\bar{Y}/n)^{\gamma}}{(n-1)(\bar{Y}/n)^{\gamma} + (\bar{y}^i)^{\gamma}} \), and the no-stockout condition \( d^i \leq \bar{Y}/n \) can be written as

\[
(1-v_{\min})\left(\frac{\bar{Y}/n}{(n-1)(\bar{Y}/n)^{\gamma} + (\bar{y}^i)^{\gamma}}\right)^{\gamma-1} \leq (n-1)\left(\frac{\bar{Y}/n}{(n-1)(\bar{Y}/n)^{\gamma} + (\bar{y}^i)^{\gamma}}\right)^{\gamma}.
\]

Since \( \bar{y}^i = d^i = \frac{(1-v_{\min})(\bar{y}^i)^{\gamma}}{(n-1)(\bar{Y}/n)^{\gamma} + (\bar{y}^i)^{\gamma}} \), the RHS of the last inequality equals \((1-v_{\min})\left(\frac{\bar{Y}/n}{(n-1)(\bar{Y}/n)^{\gamma} + (\bar{y}^i)^{\gamma}}\right)^{\gamma-1} \). Then \( d^i \leq \bar{Y}/n \) trivially holds for \( \gamma = 1 \) and, for \( \gamma \in [0,1) \), is equivalent to \( \bar{y}^i \leq \bar{Y}/n \) (since \( \gamma < 1 \)), which holds by the assumption.

Part (II) (b) follows from part (II) (a).

Part (II) (c). The second-period total inventory is \( Y - Q = Y - (1-v_{\min}) \). Suppose this number is positive. The number of consumers remaining in the market is \( v_{\min} \), and the upper bound of their second-period valuations is \( \beta v_{\min} \). Therefore, as long as \( p_2 \geq s \), the market clearing condition for the second period takes the form \( v_{\min} - \beta v_{\min} = Y - 1 + v_{\min} \), or, equivalently, \( p_2 = \beta(1-Y) \).

If \( \beta(1-Y) < s \), bargain-hunters absorb any excess supply at price \( s \). Combining these two cases, we get the second-period price in the form \( p_2 = s \lor [\beta(1-Y)] \), which is continuous in \( y^i \), \( i \in I \).

A.5. **Proof of Lemma 4 (\( p_2 < \beta p_1 \)).** From Lemma 1, we have \( v_{\min} = p_1 \) if and only if \( \rho = \beta p_1 \leq p_1 \), which can be equivalently rewritten as \( \alpha \rho \beta p_1 \leq \rho \beta p_2 \). Within feasible parameter values, the later holds if and only if either \( \alpha = 0 \), \( \rho = 0 \), or \( \beta p_1 = \beta p_2 \). By Lemma (6), \( Y \geq 1 - v_{\min} \). Thus, either of \( \rho = 0 \), \( \alpha = 0 \) or \( \beta p_1 \leq \beta p_2 \) implies that \( Y \geq 1 - p_1 \). Moreover, \( Y = 1 - p_1 \) means there are no sales in the second period, while \( Y > 1 - p_1 \) means that these sales occur at price \( p_2 < \beta p_1 \).

Part (1): We conclude that \( \beta p_2 \geq \beta p_1 \) would never be rational and, in any rational expectations equilibrium, we must have \( p_2 < \beta p_1 \).

Part (2): By the above reasoning, \( \alpha = 0 \) implies \( v_{\min} = p_1 \) and \( Y \geq 1 - p_1 \). However, \( Y > 1 - p_1 \) in combination with \( v_{\min} = p_1 \) means that there are sales in the second period and \( \alpha = 0 \) is not rational.

If \( \alpha = 1 \), by part (1) and condition (3), we have \( \beta(1-Y) \leq s \lor [\beta(1-Y)] = p_2 < \beta p_1 \). Thus, \( Y > 1 - p_1 \) in any rational expectations equilibrium with \( \alpha = 1 \).

Part (3): Since in any rational expectations equilibrium, \( \beta p_2 = p_2 \) and \( \alpha = 1 \) if there are sales in the second period, Lemma 1 implies that, if there are sales in both periods, \( v_{\min} < 1 \), which, using (3), is equivalent to \( p_1 - \rho \beta(1-Y) < 1 - \rho \beta \rho \beta Y < 1 - p_1 \). If there are sales only in the second period, \( p_1 - \rho \beta(1-Y) \geq 1 - \rho \beta \rho \beta Y \geq 1 - p_1 \); \( p_2 \geq c \) since, in this case, \( r^i = (p_2 - c)y^i \), and retailers are profit-maximizing.

Part (4): Since \( \beta p_2 \geq \beta p_1 \) would never be rational, \( v_{\min} = p_1 \) can occur in a rational expectations equilibrium if and only if \( \alpha = 0 \) or \( \rho = 0 \).
A.6. Proof of Theorem 1 (RESE with \( p_2^* > s \)). The theorem exhaustively considers all possible market outcomes without salvaging: no sales in the first period (RESE1), no sales in the second period (RESE2), and sales in both periods (RESE3). Logically, these three outcomes are mutually exclusive but it is not obvious a priori that they cannot exist under the same model inputs. In the course of the proof we establish that these outcomes also do not overlap in the sense of their necessary and sufficient conditions on model parameters. Parts (1) and (2) of the RESE definition (§3.2) rely on the notion of a symmetric equilibrium for given consumer expectations. The structure of such an equilibrium is one of the major sources of necessary and sufficient conditions. Another source is the rationality of consumer expectations. We first classify the outcomes by the presence of second-period sales.

No second-period sales: RESE2. The absence of second-period sales along with retailer rationality, by Lemma 6, means that the best response in a symmetric equilibrium occurs with \( Y = 1 - v^\min \).

Consumer rationality in this case demands that \( \bar{\alpha} = 0 \) and \( v^\min = p_1 \) implying that the candidate RESE is described by \( v^* = p_1, Y^* = 1 - v^* \), and, therefore, \( \alpha^* = 0 \) and \( r^* = \frac{1}{n} (p_1 - c)(1 - p_1) \).

This implies that \( \frac{n - 1}{n} Y^* = \frac{n - 1}{n} (1 - p_1) < 1 - p_1 < 1 - \frac{s}{\beta} \) and condition of part (1) of Lemma 7 is satisfied.

Since, by part (1.3) of Lemma 7, \( r^i \) is pseudoconcave on the interval \( (1 - v^\min - Y^{-i})^+ \leq y^i < 1 - \frac{s}{\beta} - Y^{-i} \), the candidate RESE exists if and only if

(i) there is a local maximum of \( r^i \) at \( y^i = 1 - v^* - \frac{n - 1}{n} Y^* = \frac{Y^*}{n} \) and

(ii) the profit \( r^i \) at this maximum is greater than at a potential local maximum on the interval \( y^i > 1 - \frac{s}{\beta} - \frac{n - 1}{n} Y^* \).

Condition (i) is equivalent to \( \frac{\partial r^i}{\partial y^i} \bigg|_{y^i=1-v^*} \leq 0 \). Since \( y^i = \frac{1}{n} (1 - p_1) \), the last inequality, using (11), becomes \( \beta v^* - c + p_1 - \beta v^* + \frac{1}{n} (1 - p_1) \left[ -(p_1 - \beta v^*) \frac{1}{1-v^*} \right] \leq 0 \), which, after the substitution for \( v^* = p_1 \) and multiplication by \( n \), takes the form \( np_1 - p_1 (1 - \beta) \leq nc \) or \( p_1 \leq \frac{nc}{\beta + n - 1} = P_2 \). We showed that this condition is necessary.

Condition (ii) is satisfied if \( r^i \) is nonincreasing for \( y^i > 1 - \frac{s}{\beta} - \frac{n - 1}{n} Y^* \). Since \( r^i \) is concave on this interval by part (1.4) of Lemma 7, it is nonincreasing if \( \frac{\partial r^i}{\partial y^i} \bigg|_{y^i=1-v^*} \leq 0 \). The latter, using (15), can be written as

\[
-c + s + \frac{n - 1}{n} (1 - p_1) \left( 1 - \frac{s}{\beta} \right)^2 (1 - p_1)(1 - p_1) \leq 0 \quad \text{or} \quad \frac{1}{n} \left( p_1 - s \right)^2 \left( 1 - \frac{s}{\beta} \right)^2 \leq 1. \tag{17}
\]

Since \( p_1 > \frac{s}{\beta} \), we have \( \frac{(1 - p_1)^2}{(1 - s/\beta)} \leq 1 \), and (17) is implied by \( (n - 1)(p_1 - s) \leq n(c - s) \). The latter holds because, by (already proved as necessary) condition \( p_1 \leq P_2 \), \( n(c - s) \geq (n - 1 + \beta) p_1 - ns = (n - 1)(p_1 - s) + \beta p_1 - s > (n - 1)(p_1 - s) \). Therefore, condition \( p_1 \leq P_2 \) is necessary and sufficient for the existence of RESE2.

There are second-period sales: RESE1 or 3. When sales in the second period do occur, a symmetric equilibrium \( Y = \hat{Y} \) is provided by \( \hat{Y} > 1 - v^\min \), which is an internal maximum of the profit function for each retailer. For \( p_2 > s \), the first-order optimality condition \( \frac{\partial r^i}{\partial y^i} = 0 \) is provided by setting (12) (see §A.2.2) to zero with substitutions \( y^i = \frac{Y}{n} \) and \( Y^{-i} = \frac{n - 1}{n} Y \):

\[
\beta \left( 1 - \frac{n - 1}{n} Y \right) - c + \beta(1 - v^\min) - 2\beta \frac{Y}{n} + (p_1 - \beta)(1 - v^\min) \frac{n - 1}{n} Y^2 = 0
\]

or

\[
- \beta \frac{n + 1}{n} Y - c + \beta(2 - v^\min) + (p_1 - \beta)(1 - v^\min) \frac{n - 1}{n} \frac{1}{Y} = 0.
\]
Multiplication of the last equation by $-\frac{n}{\beta(n+1)}Y$ yields

$$Y^2 - Y \frac{n}{n+1} \left( 2 - v_{\min} - \frac{c}{\beta} \right) - \frac{n-1}{n+1} \left( \frac{p_1}{\beta} - 1 \right) (1 - v_{\min}) = 0. \quad (18)$$

Equation (18) along with the relation between $v_{\min}$ and $Y$ from Lemma 1 and inequality $Y > 1 - p_1$ (from part (2) of Lemma 4) provide the necessary conditions for any equilibria with sales in the second period and $p_2 = \beta(1 - Y) > s$.

It is convenient to analyze RESE existence in terms of $v_{\min}$ as a function of $Y$. For rational expectations $\bar{\alpha} = 1$ and $\bar{p}_2 = p_2 = \beta(1 - Y)$, denote the mapping from $Y$ to $v_{\min}$ resulting from Lemma 1 as function

$$v_{\min}^1(Y) \triangleq p_1 \vee \left( \frac{p_1 - \rho \beta(1 - Y)}{1 - \rho \beta} \wedge 1 \right). \quad (19)$$

When $\rho > 0$, this function is increasing and piecewise linear with two breakpoints. It is straightforward to check that the first break-point occurs exactly at $Y = 1 - p_1$ while the second at $Y = \frac{1 - p_1}{\rho \beta}$.

When $\rho = 0$, $v_{\min}^1 \equiv p_1$.

Equation (18) yields another mapping from $Y$ to $v_{\min}$:

$$v_{\min}^2(Y) \triangleq 1 - \frac{Y^2 - Y \frac{n}{n+1} \left( 1 - \frac{c}{\beta} \right)}{Y \frac{n}{n+1} + \frac{n-1}{n+1} \left( \frac{p_1}{\beta} - 1 \right)}. \quad (20)$$

When $p_1 \neq \beta$ and $n > 1$, this function is a hyperbola with a vertical asymptote $Y = \frac{n}{n+1} \left( 1 - \frac{p_1}{\beta} \right)$ and an asymptote with a negative slope $-\frac{n+1}{n}$. When $Y = 0$ or $Y = \frac{n}{n+1} \left( 1 - \frac{\bar{c}}{\bar{\beta}} \right)$, $v_{\min}^2(Y) = 1$.

Implicit differentiation of (18) yields

$$2Y - \frac{n}{n+1} \left( 2 - v_{\min} - \frac{c}{\beta} \right) + Y \frac{n}{n+1} \frac{\partial v_{\min}^2}{\partial Y} + \frac{n-1}{n+1} \left( \frac{p_1}{\beta} - 1 \right) \frac{\partial v_{\min}^2}{\partial Y} = 0$$

resulting in $(n - 1)(p_1 - \beta) \frac{\partial v_{\min}^2}{\partial Y} \Big|_{Y=0} = n(\beta - c)$.

When $p_1 > \beta$ and $n > 1$, the vertical asymptote is located to the left of $Y = 0$ implying that points $(0, 1)$ and $\left( \frac{n}{n+1} \left( 1 - \frac{c}{\beta} \right), 1 \right)$ in the $(Y, v_{\min})$-plane belong to the same branch of the hyperbola. In this case, $\frac{\partial v_{\min}^2}{\partial Y} \big|_{Y=0} > 0$ and it must be true that $\frac{\partial v_{\min}^2}{\partial Y} < 0$ for all $Y \geq \frac{n}{n+1} \left( 1 - \frac{c}{\beta} \right)$. Relevant equilibrium candidates can only be on the downward-sloping segment of $v_{\min}^2(Y)$ to the right of $Y = \frac{n}{n+1} \left( 1 - \frac{c}{\beta} \right)$ and in the range $p_1 \leq v_{\min} \leq 1$. This case is depicted in Figure 15 (a), where a solid curve is $v_{\min}^2(Y)$, dotted lines represent its asymptotes, and the dashed lines indicate the lower and upper bounds on the relevant range of $v_{\min}$.

When $p_1 < \beta$ and $n > 1$, the vertical asymptote is located to the right of $Y = 0$ implying that points $(0, 1)$ and $\left( \frac{n}{n+1} \left( 1 - \frac{c}{\beta} \right), 1 \right)$ belong to different branches of the hyperbola. We have $\frac{\partial v_{\min}^2}{\partial Y} < 0$ for all $Y$, and the entire left branch is irrelevant since the vertical asymptote is to the left of $Y = 1 - p_1$. Indeed, $\frac{n}{n+1} \left( 1 - \frac{p_1}{\beta} \right) < 1 - p_1$ is equivalent to $np_1 - (n-1)\frac{p_1}{\beta} < 1$ which always holds for $p_1 < \beta$. All possible equilibrium candidates are again on the downward-sloping segment of $v_{\min}^2(Y)$ to the right of $Y = \frac{n}{n+1} \left( 1 - \frac{c}{\beta} \right)$ and in the range $p_1 \leq v_{\min} \leq 1$. This case is illustrated in Figure 15 (b).

When $p_1 = \beta$ or $n = 1$, the relevant part of $v_{\min}^2(Y)$ is decreasing linear: $v_{\min}^2(Y) = 2 - \frac{c}{\beta} - \frac{n+1}{n}Y$, which also satisfies $\frac{n}{n+1} \left( 1 - \frac{c}{\beta} \right) = 1$. Thus, regardless of $n$ and the relation between $p_1$ and $\beta$, the geometric structure of potential equilibrium candidates is essentially the same.
RESE1: There are no sales in the first period at a RESE if and only if $v^* = 1$. The geometric structure described above implies that such an equilibrium can be realized only if $v_1^{\min}(Y)$ intersects with $v_2^{\min}(Y)$ at a point corresponding to $Y^* = \frac{n}{n+1} \left(1 - \frac{c}{\beta}\right)$, i.e., $v_1^{\min}(Y^*) = 1$ or $p_1 - \rho \beta \left[1 - \frac{n}{n+1} \left(1 - \frac{c}{\beta}\right)\right] \geq 1 - \rho \beta$, which is equivalent to $p_1 \geq P_1 = 1 - \frac{n}{n+1} \rho (\beta - c)$. This necessary condition is also sufficient for RESE1. Indeed, given that $v_1^{\min}(Y^*) = 1$, the equilibrium values are in the form of RESE1, $p_2^* = \beta \left[1 - \frac{n}{n+1} \left(1 - \frac{c}{\beta}\right)\right] = \frac{nc + \beta}{n+1} > c > s$ and $y^i = \frac{Y^*}{n}$ indeed delivers the best response of retailer $i$ since $Y^* = \frac{n}{n+1} \left(1 - \frac{c}{\beta}\right) < 1 - \frac{c}{\beta} < 1 - \frac{s}{\beta}$ and $\frac{\partial v^i}{\partial y^i}\bigg|_{y^i = 1 - \frac{s}{\beta} - \frac{n-1}{n} Y^* + 0} = -c + s < 0$ implying, by part (1.5) of Lemma 7, that $r^i$ is pseudoconcave.

The description of RESE1 is completed by substituting $p_2^*$, $Y^*$ and $v^*$ into (10):

$$r^* = \frac{Y^*}{n} \left[\frac{\beta + nc}{n+1} - c\right] = \frac{1}{n+1} \left(1 - \frac{c}{\beta}\right) \left[\frac{\beta + nc}{n+1} - c\right] = \frac{(\beta - c)}{(n+1)\beta} \left[\frac{\beta + nc - nc - c}{n+1}\right] = \frac{(\beta - c)^2}{(n+1)^2\beta}.$$ 

The $p_1$-ranges in RESE1 and 2 do not overlap because the minimal lower bound for $p_1$ in RESE1, which corresponds to $n \to \infty$, exceeds the maximal upper bound in RESE2 (at $n = 1$): $1 - \rho (\beta - c) > \frac{s}{\beta} \iff \beta (1 - \rho \beta) > c (1 - \rho \beta)$.

RESE3: This case is characterized by $Y^* > 1 - v^*$ (there are sales in the second period) and $p_1 \leq v^* < 1$ (there are sales in the first period) with $v^* = p_1$ only if $\rho = 0$. Translating this into the geometric structure described above, necessary conditions for RESE3 are $v_1^{\min}(Y) = \frac{n}{n+1} \left(1 - \frac{s}{\beta}\right) < 1$ and $v_2^{\min}(1 - p_1) > p_1$. The first condition is equivalent to the negation of $p_1 \geq P_1$, i.e., the strict upper limit of $p_1$-range for RESE3. The second condition ensures that $v_2^{\min}(Y)$ intersects $v_1^{\min}(Y)$ for $Y > 1 - p_1$ and is equivalent to

$$1 - \frac{(1 - p_1)^2 - (1 - p_1) \frac{n}{n+1} \left(1 - \frac{s}{\beta}\right)}{(1 - p_1) \frac{n}{n+1} + \frac{n-1}{n+1} \left(\frac{p_1}{\beta} - 1\right)} > p_1,$$
Collecting like terms inside $[\cdot]$ yields $(n - 1 + \beta)p_1 > nc$ which is the negation of the necessary and sufficient condition $p_1 \leq P_2$ of RESE2, i.e., the strict lower limit of $p_1$-range for RESE3.

Given that necessary condition $P_3 < p_1 < P_1$ holds and there are sales in both periods, the candidate point for the equilibrium, by Lemma 1, satisfies

$$v^* = \frac{p_1 - \rho\beta(1 - Y^*)}{1 - \rho\beta}$$

and $v^* \in [p_1, 1)$. Substitution for $v^{\text{min}} = v^*$ into (18) results in the following equation for $Y^*$:

$$Y^{2} - Y \frac{n}{n+1} \left(\frac{2 - (p_1 - \rho\beta(1 - Y) - \frac{c}{\beta})}{1 - \rho\beta} - \frac{n - 1}{n+1} \left(\frac{p_1}{\beta} - 1\right) \frac{n - 1}{n+1} \left(\frac{p_1}{\beta} - 1\right) \left(1 - \frac{p_1 - \rho\beta(1 - Y)}{1 - \rho\beta}\right)\right) = 0,$$

which, after collecting the terms with $Y$, becomes

$$Y^{2} \left(1 + \frac{n}{n+1} \frac{\rho\beta}{1 - \rho\beta}\right) - Y \left[\frac{n}{n+1} \left(\frac{2 - (p_1 - \rho\beta(1 - Y) - \frac{c}{\beta})}{1 - \rho\beta} - \frac{n - 1}{n+1} \left(\frac{p_1}{\beta} - 1\right) \frac{n - 1}{n+1} \left(\frac{p_1}{\beta} - 1\right) \left(1 - \frac{p_1 - \rho\beta(1 - Y)}{1 - \rho\beta}\right)\right] = 0.$$

The coefficient in front of $Y^2$ is

$$1 + \frac{n}{n+1} \frac{\rho\beta}{1 - \rho\beta} = \frac{n + 1 - \rho\beta}{(n+1)(1 - \rho\beta)},$$

and the coefficient in front of $Y$ is

$$-\frac{1}{(n+1)(1 - \rho\beta)} \left[n \left(2 - 2\rho\beta - p_1 + \rho\beta - \frac{c}{\beta}(1 - \rho\beta)\right) - (n - 1) \left(\frac{p_1}{\beta} - 1\right) \rho\beta\right],$$

where the first term in the bracket $[\ldots]$ is

$$n \left(2 - \rho\beta - p_1 - \frac{c}{\beta} (1 - \rho\beta)\right) = n(1 - \rho\beta) \left(1 - \frac{c}{\beta}\right) + n(1 - p_1).$$

Then multiplication of (22) by $\frac{\beta(n+1)(1 - \rho\beta)}{\beta(n+1 - \rho\beta)}$ results in

$$y^2 \frac{\beta(n+1)(1 - \rho\beta)}{\beta(n+1 - \rho\beta)} \left(\frac{2 - \rho\beta(1 - Y) + (p_1 - \rho\beta)(n - 1)}{n + 1 - \rho\beta}\right) \frac{n - 1}{n+1} \left(\frac{p_1}{\beta} - 1\right) \left(1 - \frac{p_1 - \rho\beta(1 - Y)}{1 - \rho\beta}\right) = 0.$$

By geometric structure under condition $P_2 < p_1 < P_1$, the larger root of this equation does belong to the region $Y > 1 - p_1$ and the smaller root is irrelevant.

The conditions for RESE3 will become necessary and sufficient if (23), (21), and $P_2 < p_1 < P_1$ are complemented with the conditions guaranteeing that the larger root $Y^*$ of (23) is such that $Y^* < 1 - \frac{s}{\beta}$ (implying $p^*_2 > s$ and included as a condition of the theorem) and either

(a) the profit $r^i$ of retailer $i$ deviating from this RESE so that $p_2 = s$ (the total inventory is greater than $1 - \frac{s}{\beta}$) has no maximum for $Y > 1 - \frac{s}{\beta}$, or

(b) if $r^i = \max r^i$ exists for $Y > 1 - \frac{s}{\beta}$, then the inequality $r^i < r^*$ holds.

Since, by part (1.4) of Lemma 7, $r^i$ is concave for $y^i \geq 1 - \frac{s}{\beta} - \frac{n - 1}{n} Y^*$ (or, equivalently, $Y \geq 1 - \frac{s}{\beta}$), $r^i$ is nonincreasing for $y^i \geq 1 - \frac{s}{\beta} - \frac{n - 1}{n} Y^*$ if and only if $\partial r^i / \partial y^i \bigg|_{y^i=1-\frac{s}{\beta}-\frac{n-1}{n}Y^*+0} \leq 0$. Thus, the latter
This expression can be also written as follows: \( \tilde{r} \) is equivalent to (a). Using (15) with \( v^\text{min} = v^* \), \( Y^{-i} = \frac{n-1}{n} Y^* \) and \( Y = 1 - \frac{s}{\beta} \), this condition can be written as \(-c + s + \frac{n-1}{n} \frac{Y^*}{(1-s/\beta)} (p_1 - s) (1 - v^*) \leq 0\), yielding condition (a).

If \( \frac{\partial^i Y}{\partial y^i} \bigg|_{y^i = 1 - \frac{s}{n} - Y^{-i} + 0} > 0 \), then, since \( \frac{\partial^i Y}{\partial y^i} \) becomes negative for sufficiently large \( Y \) by (15), \( \tilde{r}^i = \max r^i \) exists for \( Y > 1 - \frac{s}{\beta} \). Therefore, the RESE exists in this case if \( r^* \geq \tilde{r}^i \) (condition (b)).

In order to provide the expression for \( \tilde{r}^i \), denote the maximized deviator's inventory decision by \( \tilde{y}^i \triangleq \arg \max r^i > \frac{1}{n} Y^* \). As a result of this deviation, the total inventory becomes \( \tilde{Y} = \tilde{y}^i + \frac{n-1}{n} Y^* \). Then, using (15) with \( v^\text{min} = v^* \), we obtain the following equation in \( \tilde{Y} \): \( \frac{\partial^i Y}{\partial y^i} \bigg|_{y^i = \tilde{y}^i} = 0 = -(c - s) + \frac{n-1}{n} \frac{Y^*}{Y^*} (p_1 - s) (1 - v^*) \), which yields \( \tilde{Y} = \sqrt{\frac{n-1}{n} Y^* (p_1-s)(1-v^*)} \). Substitution of \( Y = \tilde{Y} \) and \( y^i = \tilde{Y} - \frac{n-1}{n} Y^* \) into the equation for profit (14), results in

\[
\tilde{r}^i = \left\{ \sqrt{\frac{n-1}{n} Y^* (p_1-s)(1-v^*)} - \frac{n-1}{n} Y^* \right\} \times \left\{ -(c - s) + \frac{(p_1-s)(1-v^*)}{\sqrt{\frac{n-1}{n} Y^* (p_1-s)(1-v^*)}} \right\},
\]

which, after factoring out \( \frac{n-1}{n} Y^* \) from the first curly bracket and \( c-s \) from the second one, becomes

\[
\tilde{r}^i = \left( \frac{n-1}{n} Y^* (c-s) \right) \left\{ \sqrt{\frac{n}{(n-1)} \frac{(p_1-s)(1-v^*)}{(c-s)Y^*}} - 1 \right\}^2.
\]

This expression can be also written as follows: \( \tilde{r}^i = \left\{ (p_1-s)(1-v^*) - \sqrt{\frac{n-1}{n} Y^* (c-s)} \right\}^2 \), which coincides with the expression for \( \tilde{r}^i \) in the theorem statement.

Expression for \( r^* \) follows immediately from (1) and Lemma 1.

We complete the proof of the main part of the theorem by a simple observation that equilibrium characteristics are continuous on the boundaries between RESE1 and 3 as well as RESE2 and 3. Figure 16, in its subplot (a), depicts a typical configuration of \( v^\text{min}_1(Y) \) and \( v^\text{min}_2(Y) \) when RESE3 exists, while subplots (b) and (c) depict this configuration at the points of change to RESE1 and 2, respectively.

RESE3 continuously changes into RESE1 as the intersection point of \( v^\text{min}_1(Y) \) and \( v^\text{min}_2(Y) \) representing RESE3 moves toward the point \( \left( \frac{n}{n+1} \left(1 - \frac{s}{\beta} \right), 1 \right) \) on \( v^\text{min}_2(Y) \) representing RESE1. The latter point is to the left of all possible candidates for RESE3 located on \( v^\text{min}_2(Y) \) implying that,
in RESE3, \( Y^* \geq \frac{n}{n+1} \left[ 1 - \frac{c}{\beta} \right] \). Similarly, RESE3 continuously changes into RESE2 as the intersection point of \( v_1^{\min}(Y) \) and \( v_2^{\min}(Y) \) moves toward \( v_1^{\min}(Y) \)'s break-point \((1 - p_1, p_1)\) (representing RESE2). The continuity of \( r^* \) follows from the continuity of the expression for \( r^i \), given by (8), in all the parameters and continuity of \( v^* \) and \( Y^* \) (using \( y^i = \frac{1}{n}Y^* \)).

It remains to examine the convergence under deviations from rational expectations. The geometric structure of candidates for RESE3 and 1 implies that the areas of inputs where these RESE exist do not intersect. Suppose that (i) \( \bar{\alpha} = \alpha^* = 1 \), i.e., one and only one of RESE3 or 1 can be realized for given inputs; and (ii) consumer expectations of the second-period price deviate from rational ones with \( p_2^0 < p_2^* \), and the game is repeated with the same inputs. As shown above, \( \frac{\partial v_2^{\min}}{\partial Y} < 0 \), implying \( \frac{\partial BR}{\partial p_2} < 0 \), where \( BR = BR[v_2^{\min}(\hat{p}_2)] \) is a symmetric best response, given \( \hat{p}_2 \). By Lemma 1, \( \frac{\partial v_2^{\min}}{\partial p_2} \leq 0 \), which leads to \( \frac{\partial BR}{\partial p_2} \geq 0 \). Then \( \hat{p}_2 < p_2^* \Rightarrow BR(\hat{p}_2) \leq BR(p_2^*) = Y^* < 1 - \frac{c}{\beta} \), implying that the realized price \( p_2^* = \beta[1 - BR(\hat{p}_2)] \) is bigger than \( p_2^* > s \). Moreover, for any moment of time \( t \in [0, \infty) \), the realized \( (p_2^t) \) and expected \( (\hat{p}_2^t) \) second-period prices are such that \( p_2^t \geq p_2^* > \hat{p}_2^t \) if consumer expectations follow a linear adjustment process \( \hat{p}_2^{t+1} = \mu \hat{p}_2^t + (1 - \mu) \hat{p}_2^t \) with a sufficiently small \( \mu \). This process, (3), can be written as \( \hat{p}_2^{t+1} = \beta(1 - Y^t) + (1 - \mu) \hat{p}_2^t \), where \( Y^t = BR(\hat{p}_2^t) \). Then \( |p_2^t - \hat{p}_2^{t+1}| = |p_2^t - \beta[1 - BR(\hat{p}_2^t)] + p_2^t - p_2^*| = \beta(1 - \mu)(\hat{p}_2^t - p_2^*) \). By the mean value theorem, there exists \( \hat{p}_2 \in (p_2^*, \hat{p}_2^t) \) such that \( Y^t - BR(\hat{p}_2^t) = \frac{\partial BR(p_2^t)}{\partial p_2} \bigg|_{\hat{p}_2} (\hat{p}_2^t - \hat{p}_2^t) \). Then, if \( \mu \frac{\partial BR(p_2^t)}{\partial p_2} \leq 2(1 - \mu) \mu \mu \frac{\partial BR(p_2^t)}{\partial p_2} \leq 2(1 - \mu) \leq 2/(2 + \beta \frac{\partial BR(p_2^t)}{\partial p_2}) < 1 \), we have \( |p_2^t - \hat{p}_2^{t+1}| \leq (1 - \mu)|p_2^* - \hat{p}_2^t| = (1 - \mu)|p_2^* - \hat{p}_2^t| \), which goes to zero with \( t \to \infty \) for any \( \hat{p}_2 < p_2^* \). Since \( \frac{\partial v_2^{\min}}{\partial p_2} \) is restricted for any \( \rho \in [0, 1) \) and, by (20), \( \frac{\partial BR}{\partial p_2} \) is restricted in the relevant region for \( Y^* \), there exists a sufficiently small \( \mu \) such that \( \mu < 2/(2 + \beta \frac{\partial BR(p_2^t)}{\partial p_2}) \) leading to the convergence of the adjustment process to \( p_2^* \).

The adjustment process can be specified using inequalities \( \frac{\partial v_2^{\min}}{\partial p_2} \leq 0 \) and \( p_2^0 < p_2^* \), which imply \( v_2^{\min}(\hat{p}_2^t) \geq v^* = v_2^{\min}(p_2^t) \). This property leads to three cases. (a) \( v^* < 1 \) and \( v_2^{\min}(\hat{p}_2^t) < 1 \), which corresponds to the adjustment process above; (b) \( v^* = 1 \) (RESE1 is realized at \( p_2 = p_2^t \)). In this case, all \( p_2^t \) are such that \( v_2^{\min}(\hat{p}_2^t) = 1 \), i.e., retailers’ decisions do not depend on \( p_2^t \) and the adjustment becomes \( p_2^{t+1} = p_2^t + (1 - \mu)p_2^t \). Then \( |p_2^{t+1} - p_2^t| \leq |1 - \mu|^t |p_2^t - p_2^t| \), which converges to \( p_2^t \) for any \( \mu \in (0, 2) \) and \( p_2^0 < p_2^* \). (c) \( v^* < 1 \) and \( v_2^{\min}(\hat{p}_2^t) = 1 \). In this case, the initial adjustment steps are \( p_2^{t+1} = \beta(1 - BR(v_2^{\min} - 1)) + (1 - \mu)p_2^t \). Since \( \frac{\partial BR}{\partial v_2^{\min}} < 0 \), \( \hat{p}_2^t \) in this process increases faster than for \( BR[v_2^{\min} < 1] \). Then, by continuity and monotonicity of \( v_2^{\min} \) in \( p_2^t \) and monotonicity of \( \hat{p}_2^t \) in \( t \), there exists such \( \hat{t} \) that the adjustment process switches to case (a) and follows it for any \( t \geq \hat{t} \).

A.7. Proof of Proposition 1 (profit of a deviator from MSRP). Assume (too optimistically) that retailer \( i \) obtain the entire first-period demand if its first-period price is \( p_1^i < p_1 \). This assumption overestimates retailer \( i \)'s profit because in reality consumers buy at different prices due to their search costs. Moreover, the assumption treats consumers as myopic, which, by Lemma 1, maximizes the first-period demand and, as a result, the profit of a retailer who sells only in the first period. Using the general expression for profit (8) with an additional cost \( K \geq 0 \) and unit cost \( c^H > c \), the profit upper bound, for \( p_2 > s \), is \( UB^i = -K - c^H y^i + p_1^i (1 - p_1^i) + \beta (1 - Y) [ y^i - (1 - p_1^i) ] \). Consider the problem of maximizing \( UB^i(y^i, p_1^i) \) s.t. \( 1 - p_1^i \leq y^i \) (by retailer rationality) and \( p_1^i \leq p_1 \) (we consider a non-strict inequality, which also overestimates \( r^i \)). The Lagrangian of this problem is \( L^i = r^i - \lambda (1 - p_1^i - y^i) - \mu (p_1^i - p_1) \) (which is concave since the Hessian is negative definite and
the feasible region is convex) and the first-order conditions are
\[
\frac{\partial L_i}{\partial p'_i} = 1 - 2p'_i + \beta(1 - Y) + \lambda - \mu = 0, \tag{24}
\]
\[
\frac{\partial L_i}{\partial y^i} = -e^H + \beta(2 - Y - p'_i - y^i) + \lambda = 0, \tag{25}
\]

\[
\lambda, \mu \geq 0, \lambda(1 - p'_i - y^i) = 0, \mu(p'_i - p_i) = 0, \tag{26}
\]

where \( Y = y^i + Y^{-i} \). Profit upper bound \( UB^i \) can be written as \( UB^i = -K + (1 - p'_i)[p'_i - \beta(1 - Y)] + y^i \{ \beta(1 - Y) - c^H \}, \) where, by (24), \( [\cdot] = 1 - p'_i + \lambda - \mu \) and, by (25), \( \{\cdot\} = \beta(y^i + p'_i - 1) - \lambda \).

Then \( UB^i = -K + (1 - p'_i)(1 - p'_i + \lambda - \mu) + y^i[\beta(y^i + p'_i - 1) - \lambda] \).

When \( \lambda > 0 \), retailer \( i \) has sales only in the first period \( (y^i = 1 - p'_i) \). Consider two cases: \( p'_i < p_1 \) (\( \mu = 0 \)) and \( p'_i = p_1 \) (in the latter case system (24), (25) yields the first-period price greater than \( p_1 \) and \( \mu > 0 \)).

When \( \mu = 0 \), the subtraction of doubled (24) from (25) leads to \((4 - \beta)p'_i = 2 + \lambda + e^H - \beta Y^{-i} \) or \( p'_i = (2 + \lambda + e^H - 3Y^{-i})/(4 - \beta) \), which, substituted into (24), yields \( y^i = \frac{1}{\beta}[1 + \beta(1 - Y^{-i}) + \lambda - 2(2 + \lambda + e^H - \beta Y^{-i})] \).

Then condition \( 1 - p'_i - y^i \leq 0 \) is equivalent to \( \beta(4 - \beta)(1 - p'_i - y^i) = 4\beta - \beta^2 - [2\beta + \beta\lambda + \beta e^H - \beta^2 Y^{-i}] - [3\lambda + 3\lambda - 2\beta e^H - 2\beta Y^{-i} - \beta^2(1 - Y^{-i})] \).

Then \( UB^i = -K - (1 - p'_i)(1 - p'_i + \lambda - \mu) + y^i[\beta(y^i + p'_i - 1) - \lambda] \).

When \( \mu > 0 \), we have \( p'_i = p_1 \) and \( y^i = 1 - p_1 \). Then \( UB^i = -K + (1 - c^H)(1 - p_1) \) and, by (25), \( \lambda = c^H + \beta(Y^{-i} - p_1) \), which, substituted into (24), yields \( \mu = 1 - 2p_1 + e^H \).

Inequalities \( \mu > 0 \) and \( \lambda > 0 \) are equivalent to \( c^H > 2p_1 - 1 \) and \( e^H > \beta(p_1 - Y^{-i}) \) respectively.

When, for \( \mu = 0 \), the \( e^H \)-range \( (\beta(1 - 2Y^{-i})/(2 - \beta), 2p_1 - 1) \) is not empty, the \( c^H \)-range for \( \mu > 0 \) is \( c^H > 2p_1 - 1 \). Indeed, \( 2p_1 - 1 > \beta(1 - 2Y^{-i})/(2 - \beta) \) if \( Y^{-i} > \frac{1}{2}[1 - (2 - \beta)(2p_1 - 1)/\beta] = [1 - (2 - \beta)p_1]/\beta \) leading to \( \beta(p_1 - Y^{-i}) < \beta(p_1 - 1 - 2(p_1 - 1)/\beta) = 2p_1 - 1 \).

A.8. Proof of Corollary 1 (no deviation from MSRP under RESE1). By part RESE1 of Theorem 1, \( Y^{-i} = \frac{n-1}{n+1}(1 - c/\beta) \) and \( r^* = \frac{(\beta - c)^2}{(n-1)/\beta} \). The deviation from MSRP is unprofitable iff \( r^* \geq r^4 \). By Proposition 1, this inequality holds if \( \frac{(\beta - c)^2}{(n-1)/\beta} \geq -K + \frac{1}{4}(1 - e^H)^2 \) if \( \beta(1 - 2Y^{-i})/(2 - \beta) < c^H \leq 2p_1 - 1 \) and \( \frac{(\beta - c)^2}{(n-1)/\beta} \geq -K + (1 - c^H)(1 - p_1) \) if \( c^H > \max\{2p_1 - 1, \beta(p_1 - Y^{-i})\} \) yielding the result.

A.9. Proof of Corollary 2 (irrational \( \hat{p}_2 \), RESE3). The derivative \( \frac{\partial Y}{\partial \hat{p}_2} \) can be written as \( \frac{\partial Y}{\partial \hat{p}_2} = \frac{\partial Y}{\partial p_{min}} \frac{p_{min}}{\partial \hat{p}_2} \), which, by the implicit function theorem, is \( \frac{\partial Y}{\partial \hat{p}_2} = \frac{\partial Y}{\partial p_{min}} / \frac{\partial \hat{p}_2}{\partial Y} \). By Lemma 1, \( \frac{\partial Y}{\partial \hat{p}_2} = -\frac{\partial \hat{p}_2}{\partial \hat{p}_3} \), which is \( -\frac{\rho}{1-\rho^3} \) under RESE3. By (20), \( \frac{\partial p_{min}}{\partial \hat{p}_2} = \frac{\partial \hat{p}_2}{\partial \hat{p}_3} \)

\[
\frac{\partial \hat{p}_2}{\partial \hat{p}_3} = \frac{[2Y - \frac{n}{n+1}(1 - c/\beta)][(n-1)/n+1][p_1(1-\beta)/\beta - 1] - [2Y^2 - \hat{Y} \frac{n}{n+1}(1 - c/\beta)] \frac{n}{n+1}}{[\hat{Y} \frac{n}{n+1} + \frac{n-1}{n+1}(p_1(1-\beta)/\beta - 1)]^2} = \frac{\hat{Y} \frac{n}{n+1} + 2\hat{Y} \frac{n-1}{n+1}(p_1(1-\beta)/\beta - 1) - \frac{n}{n+1}(1 - c/\beta)(p_1(1-\beta)/\beta - 1)}{[\hat{Y} \frac{n}{n+1} + \frac{n-1}{n+1}(p_1(1-\beta)/\beta - 1)]^2}. \tag{27}
\]

Since both \( \frac{\partial p_{min}}{\partial \hat{p}_2} \) and \( \frac{\partial \hat{p}_2}{\partial \hat{p}_3} \) are negative (the latter – by the geometric argument in the proof of Theorem 1 illustrated in Figure 15), the upper bound on \( \frac{\partial Y}{\partial \hat{p}_2} \) corresponds to the upper bound on
\[ \frac{\partial v_{2 \min}}{\partial Y} \], which for \( p_1 \geq \beta \), (see Figure 15 (a)) is provided by \( \frac{\partial v_{2 \min}}{\partial Y} \leq \frac{\partial v_{2 \min}}{\partial Y} \bigg|_{Y = \frac{n}{n+1}(1-c/\beta)} = \]

\[ -\frac{n}{n+1}(1-c/\beta)\left(\frac{n}{n+1}\right)^2(1-c/\beta) + \frac{n-1}{n+1}(p_1/\beta - 1)^2 \]

where denominator is positive, increases in \( n \) with the limit \( (p_1 - c)/\beta \) when \( n \to \infty \). Therefore, \( \frac{\partial Y}{\partial p_2} \leq \frac{\rho + p_1 - c}{1 - \rho \beta} \) when \( p_1 \geq \beta \).

When \( p_1 < \beta \), we have (see Figure 15 (b)) \( \frac{\partial v_{2 \min}}{\partial Y} \leq \frac{\partial v_{2 \min}}{\partial Y} \bigg|_{Y = 1 - p_1} = \]

\[ (1-p_1)^2 \frac{n}{n+1} + 2 \frac{n-1}{n+1}(p_1/\beta - 1)(1-p_1) - \frac{n(n-1)}{(n+1)^2}(1-c/\beta)(p_1/\beta - 1) \]

The last term in the numerator can be written as \( -\frac{n(n-1)}{(n+1)^2}(1-p_1/\beta + (p_1 - c)/\beta)(p_1 - 1) = \frac{n(n-1)}{(n+1)^2}(1-p_1/\beta)^2 + \frac{n(n-1)}{(n+1)^2}(1-p_1/\beta)(p_1 - c)/\beta \). Then the numerator is \( \frac{n}{n+1} \left( (1-p_1)^2 + 2 \frac{n-1}{n}(p_1/\beta - 1)(1-p_1) + \frac{n-1}{n} \right) \)

By adding and subtracting \( \frac{n-1}{n}(p_1/\beta - 1) \), the expression in \{\} becomes \{\} = \left[ 1 - p_1 + \frac{n-1}{n}(p_1/\beta - 1) \right] + \frac{n-1}{n}(p_1/\beta - 1) \]

\[ \frac{\partial v_{2 \min}}{\partial Y} \bigg|_{Y = 1 - p_1} = -\frac{n+1}{n} \left[ 1 + \frac{n-1}{n(n+1)}(p_1/\beta - 1)^2 + \frac{n-1}{n+1}(1-p_1/\beta)(p_1 - c)/\beta \right] \]

\[ \leq -\frac{n+1}{n} < -1, \]

i.e., \( \frac{\partial Y}{\partial p_2} \leq \frac{\rho + p_1 - c}{1 - \rho \beta} \) when \( p_1 < \beta \).

### A.10. Proof of Corollary 3 (RESE3, monopoly). For \( n = 1 \), sufficient condition (a) always holds and (18) is \( Y - 1 + \frac{1}{2}(v_{\min} + \frac{c}{\beta}) \mid Y = 0 \), yielding \( Y^* = 1 - \frac{1}{2}(v^* + \frac{c}{\beta}) \). The equation for \( v^* \) is \( v^* = \frac{p_1 - \frac{1}{2} \rho \beta (v^* + \frac{c}{\beta})}{1 - \rho \beta} \) which is equivalent to \( v^*(2 - \rho \beta) = 2p_1 - c \rho c \), resulting in the equilibrium \( v^* \). Substitution of \( Y^* \) into (3) leads to the expression for \( p_2^* \).

### A.11. Proof of Corollary 4 (RESE3, \( \rho = 0 \), second-period sales at loss). For \( \beta = 1 \) and \( \rho = 0 \), \( p_1 \)-range in RESE3 is \( c < p_1 < 1 \). Thus, RESE1 and 2 cannot be realized. By the proof of Theorem 1, \( Y^{*,3} \leq 1 - \frac{c}{\beta} \) is equivalent to \( v^* = v_{2 \min}^* \left( 1 - \frac{c}{\beta} \right) \) since \( v_{2 \min}^*(Y) \), given by (20), is decreasing in the relevant range of \( Y \). Using \( Y = 1 - c \) and \( \beta = 1 \) in (20), we get

\[ v_{2 \min}^*(1-c) = 1 - \frac{(1-c)^2 - (1-c)^2 \frac{n}{n+1}}{(1-c) \frac{n}{n+1} + \frac{n-1}{n+1}(p_1 - 1)} = 1 - \frac{(1-c)^2}{n(p_1 - c) + 1 - p_1}. \]

Thus, under conditions of the corollary, \( p_2^* \geq c \) if and only if \( v^* = p_1 \geq 1 - \frac{(1-c)^2}{n(p_1 - c) + 1 - p_1} \). Rearranging this inequality we obtain \( \frac{(1-c)^2}{n(p_1 - c) + 1 - p_1} \geq 1 - p_1 \), and solving for \( n \) we get

\[ n \leq \frac{1}{p_1 - c} \left( \frac{(1-c)^2}{1 - p_1} - (1-p_1) \right) = \frac{2 - c - p_1}{1 - p_1} = 2 + \frac{p_1 - c}{1 - p_1}. \]

### A.12. Proof of Corollary 5 (RESE3, perfect competition). If \( P_2 < p_1 < P_1 \) for \( n \to \infty \), the limits \( v_{\infty}^* \) and \( Y_{\infty}^* \) of, respectively, \( v^* = \frac{p_1 - \rho \beta (1-Y^*)}{1 - \rho \beta} \) and \( Y^* \) defined by (23) exist. This follows from the geometric structure of curves \( v_{1 \min}^*(Y) \) and \( v_{2 \min}^*(Y) \) in the limiting case (see equations (19) and (20) and their analysis in the proof of Theorem 1).
The violation of condition (a) in the limit of $n \to \infty$ means that, for some $\epsilon > 0$,
\[
\frac{(p_1 - s)(1 - v_\infty^*)Y_\infty^*}{(c - s)\left(1 - \frac{s}{\beta}\right)^2} = 1 + 2\epsilon.
\]
There exists $N$ such that condition (a) is violated for any $n > N$ by at least $\epsilon$:
\[
\frac{n - 1}{n} \frac{(p_1 - s)(1 - v^*)Y^*}{(c - s)\left(1 - \frac{s}{\beta}\right)^2} \geq 1 + \epsilon. \tag{28}
\]

There are two cases: $Y_\infty^* > 1 - \frac{s}{\beta}$ and $Y_\infty^* \leq 1 - \frac{s}{\beta}$. If $Y_\infty^*>1 - \frac{s}{\beta}$, there exists $N'$ such that $Y^* > 1 - \frac{s}{\beta}$ for any $n > N'$ implying, by condition (b) of Theorem 1, that RESE3 does not exist for these $n$, and the claim of the corollary is established. If $Y_\infty^* \leq 1 - \frac{s}{\beta}$, there exist sufficiently small $\epsilon' > 0$ and $N'$ such that
\[
\frac{1 - s/\beta}{Y^*} \geq \frac{1 + \epsilon'}{\sqrt{1 + \epsilon}} \tag{29}
\]
for all $n > N'$. Inequality (28) is equivalent to $\frac{\partial r^i}{\partial y^i}\bigg|_{y^i = 1 - \frac{s}{\beta} - Y^{-i} + 0} > 0$ (see the analysis of RESE3 in the proof of Theorem 1) and implies that there exists $\tilde{r}^i$, which is a unique maximum of $r^i$ for $y^i > 1 - \frac{s}{\beta} - Y^{-i}$. Using the proof of condition (b) in Theorem 1, we have
\[
\tilde{r}^i = \frac{n - 1}{n} Y^* (c - s) \left\{ \sqrt{\left\{ \frac{n - 1}{n} \frac{(p_1 - s)(1 - v^*)}{(c - s)Y^*} - 1 \right\}^2} \right\}.
\]

Bounds (28) and (29) imply
\[
\frac{n}{n - 1} \frac{(p_1 - s)(1 - v^*)}{(c - s)Y^*} \geq \left( \frac{n - 1}{n} \frac{1 - s/\beta}{Y^*} \right)^2 (1 + \epsilon) \geq (1 + \epsilon')^2.
\]

Then, using $\frac{n - 1}{n} \geq \frac{1}{2}$ and $Y^* \geq 1 - p_1$, $\tilde{r}^i$ is bounded from below as follows:
\[
\tilde{r}^i \geq \frac{n - 1}{n} Y^* (c - s) \left\{ \sqrt{(1 + \epsilon')^2 - 1} \right\}^2 \geq \frac{1}{2} (1 - p_1)(c - s)(\epsilon')^2
\]
for all $n > \max\{N, N'\}$. That is, $\tilde{r}^i$ is separated from zero by a positive constant for all sufficiently large $n$. On the other hand, the following lemma immediately implies that $\lim_{n \to \infty} r^* = 0$.

**Lemma 8.** The equilibrium profit in RESE 3 can be expressed as
\[
r^* = \frac{\beta}{n(1 - \rho \beta)} \left\{ - (Y^*)^2 + Y^* \left[ 2 - p_1 - \frac{c}{\beta} - \rho (p_1 - c) \right] + \left( \frac{p_1}{\beta} - 1 \right)(1 - p_1) \right\}. \tag{30}
\]

Indeed, $Y^*$ is bounded, implying that the expression inside $\cdot\cdot\cdot$ is also bounded, while the coefficient in front of $\cdot\cdot\cdot$ tends to zero as $n \to \infty$. Then there exists $N'' \geq \max\{N, N'\}$ such that $\tilde{r}^i > r^*$ for all $n > N''$ and RESE3 does not exist.

**A.13. Proof of Theorem 2 (RESE with $p_2^* = s$).** We start by identifying candidate solutions for a symmetric equilibrium with given expectations. When $p_2 = s$, the equilibrium is possible only with sales in both periods, and rationality requires that $v^\min < 1$ and $\alpha = 1$.

By parts (1.4) and (2) of Lemma 7, the profit function is strictly concave when $y^i \geq \left(1 - \frac{s}{\beta} - Y^{-i}\right)^{\dagger}$. Thus, a symmetric equilibrium inventory decision is found by solving the first-order optimality condition for the best response. Since the optimum of the profit function cannot occur at
\( y^i = 1 - \frac{s}{\beta} - Y^{-i} \) by part (1.1) of Lemma 7, the candidate is found by setting the derivative of the profit (15) \( \partial r^i \) to zero while, by symmetry, \( Y^{-i} = \frac{n-1}{n}Y \):

\[
0 = \frac{\partial r^i}{\partial y^i} = -(c - s) + \frac{Y^{-i}}{2} (p_1 - s) (1 - v^{\min}) = -(c - s) + \frac{n-1}{nY} (p_1 - s) (1 - v^{\min}).
\]

The resulting unique solution is \( \hat{Y} = \frac{n-1}{n} \frac{(p_1 - s)(1 - v^{\min})}{c - s} \). Combining it with rationality of expectations, we find, by Lemma 1, that \( v^{\min} = v^* = \frac{p_1 - \rho s}{1 - \rho^2} \) and \( Y^* = \frac{n-1}{n} \frac{(p_1 - s)(1 - v^*)}{c - s} \). This is the only candidate point for RESE with \( p_2^* = s \). The equilibrium profit of a retailer at this point is

\[
r^* = \frac{1}{n} \left\{ -cY^* + p_1 (1 - v^*) + s (Y^* - (1 - v^*)) \right\} = \frac{1}{n} \left\{ -(c - s)Y^* + (p_1 - s)(1 - v^*) \right\},
\]

which yields the expression for \( r^* \) in the theorem.

We now analyze when the candidate point is indeed a RESE with \( p_2^* = s \), and start by checking that it is contained within the valid ranges \( p_1 \leq v^* < 1 \) and \( Y^* \geq 1 - \frac{s}{\beta} \), which provide necessary conditions for RESE existence. The second condition is the domain restriction of \( \S A.2.3 \). It is equivalent to \( p_2^* = s \) and follows from either of the mutually exclusive cases (a), (b), and (c) in the statement of the theorem. Since the equilibrium cannot result in \( Y^* = 1 - \frac{s}{\beta} \), by part (1.1) of Lemma 7, the second condition is strengthened to \( Y^* > 1 - \frac{s}{\beta} \), under which cases (a), (b) and (c) become exhaustive. Since \( 1 - \frac{s}{\beta} > 0 \) and \( Y^* \) is proportional to \( 1 - v^* \), the resulting strict positivity of \( Y^* \) implies that \( v^* < 1 \). Similarly to RESE3, \( v^* = p_1 \) if \( \rho = 0 \), and it can be shown that \( v^* > p_1 \) if \( \rho > 0 \). Indeed, inequality \( v^* > p_1 \) is equivalent to \( \frac{p_1 - \rho s}{1 - \rho^2} > p_1 \Leftrightarrow p_1 - \rho s > 0 \Leftrightarrow -p_1 \rho \beta \Leftrightarrow p_1 > \frac{s}{\beta} \), which always holds in this problem.

It remains to establish that the exact conditions ensuring that \( Y^* \) provides the global optimum of the profit function are indeed provided by the mutually exclusive and exhaustive (under condition \( Y^* > 1 - \frac{s}{\beta} \)) cases (a), (b), and (c).

Condition (a), i.e., \( \frac{n-1}{n} Y^* \geq 1 - \frac{s}{\beta} \), means, by (3), that \( p_2 = s \) independently of the inventory decisions of individual retailers. By part (2) of Lemma 7, the profit function is globally strictly concave in this case and \( Y^* \) is indeed its unique global maximum.

In case (b) of the theorem, condition (a) does not hold, which means that \( p_2 = s \) may or may not hold depending on the decisions of individual retailers. Nevertheless, the maximum of the profit function is unique and occurs when \( p_2 = s \) as long as the profit function is strictly increasing in the interval corresponding to \( p_2 > s \). This is ensured by the condition \( \frac{\partial r^i}{\partial y^i} \big|_{y^i = 1 - \frac{s}{\beta} - Y^{-i} = 0} \geq 0 \) which, by part (1.5) of Lemma 7, implies pseudoconcavity of the profit function. Using (12), the last condition takes the following form:

\[
\frac{\partial r^i}{\partial y^i} \big|_{y^i = 1 - \frac{s}{\beta} - Y^{-i} = 0} = \beta (1 - Y^{-i}) - c + \beta (1 - v^{\min}) - 2\beta \left( 1 - \frac{s}{\beta} - Y^{-i} \right) + \frac{(p_1 - \beta)(1 - v^{\min})Y^{-i}}{(1 - \frac{s}{\beta})^2} \geq 0,
\]

which, after collecting the terms and substituting \( Y^{-i} = \frac{n-1}{n} Y^* \) and \( v^{\min} = v^* \), can be rewritten as

\[
\left( \beta + \frac{(p_1 - \beta)(1 - v^*)}{(1 - \frac{s}{\beta})^2} \right) \frac{n-1}{n} Y^* \geq c + \beta v^* - 2s,
\]

yielding condition (b).

In case (c) of the theorem, condition (b) does not hold, i.e. \( \frac{\partial r^i}{\partial y^i} \big|_{y^i = 1 - \frac{s}{\beta} - Y^{-i} = 0} < 0 \). Then, there exists a local maximum of \( r^i \) without or with the sales in the second period and \( p_2 > s \). In other words, there exists such an inventory decision \( \tilde{y}^i \) of a deviating retailer that

\[
\tilde{y}^i = \text{arg max} \left\{ r^i(y^i) \mid y^i \in \left[ \max \left\{ 0, 1 - v^* - \frac{n-1}{n} Y^* \right\}, 1 - \frac{s}{\beta} - \frac{n-1}{n} Y^* \right] \right\}
\]
or, denoting \( \tilde{Y} \triangleq \tilde{y}^i + \frac{n-1}{n}Y^* \), \( \tilde{Y} \in \left[ \max \left\{ 1 - v^*, \frac{n-1}{n}Y^* \right\}, 1 - \frac{s}{\beta} \right) \). Then the equilibrium with \( p_2^s = s \) exists only if

\[
\tilde{r}^i \triangleq r^i(\tilde{y}^i) \leq r^* .
\]

Consider this condition at the left boundary of the range for \( y^i \). If \( \tilde{y}^i = 0 \), then \( \tilde{r}^i = 0 \) and (31) holds trivially. If \( \tilde{y}^i = 1 - v^* - \frac{n-1}{n}Y^* = (1 - v^*) \left[ 1 - \left( \frac{n-1}{n} \right)^2 \frac{p_1 - s}{c-s} \right] \), then, by §A.2.1, there are no sales in the second period and \( \tilde{r}^i = (1 - v^*) \left[ 1 - \left( \frac{n-1}{n} \right)^2 \frac{p_1 - s}{c-s} (p_1 - c) \right] \). After substitutions for \( \tilde{r}^i \) and \( r^* \), and multiplication of both sides by \( \frac{n^2}{(1-v^i)(p_1-c)} \), condition (31) becomes \( n^2 - (n-1)^2 \frac{p_1 - s}{c-s} \leq \frac{p_1 - s}{p_1 - c} \), which always holds. Indeed, let \( g(n) \triangleq n^2 - (n-1)^2 \frac{p_1 - s}{c-s} \). Then \( g'(n) = 2n - 2(n-1) \frac{p_1 - s}{c-s} = 2 \left[ -n \frac{p_1 - s}{c-s} + \frac{p_1 - s}{c-s} \right] \) and \( g''(n) = -2 \frac{p_1 - s}{c-s} < 0 \). Therefore, the unique maximum of \( g \), defined by the condition \( g'(n) = 0 \), is \( n_{\text{max}} = \frac{p_1 - s}{p_1 - c} \) and

\[
g_{\text{max}} = g(n_{\text{max}}) = \left( \frac{p_1 - s}{p_1 - c} \right)^2 - \left( \frac{p_1 - s}{p_1 - c} \right) \frac{p_1 - s}{c-s} = \frac{p_1 - s}{p_1 - c} \left( \frac{p_1 - s}{p_1 - c} - \frac{c-s}{p_1 - c} \right) = \frac{p_1 - s}{p_1 - c}.
\]

Finally, the RESE with \( p_2^s = s \) may also exist if there exists an internal local maximum \( \tilde{r}^i(\tilde{y}^i) \leq r^* \) with \( \tilde{y}^i = \tilde{Y} - \frac{n-1}{n}Y^* \) such that \( \max \left\{ 1 - v^*, \frac{n-1}{n}Y^* \right\} < \tilde{Y} < 1 - \frac{s}{\beta} \) and \( \frac{\partial r^i}{\partial y^i} \bigg|_{y^i=\tilde{y}^i} = 0 \). In this case, formula (10) from §A.2.2 yields the expression for \( \tilde{r}^i \) in condition (c):

\[
\tilde{r}^i = \left[ \beta \left( 1 - \tilde{Y} \right) - c + \beta (1 - v^*) + \frac{(p_1 - \beta)(1 - v^*)}{\tilde{Y}} \right] ,
\]

where \( \tilde{Y} \) is a zero of the profit function derivative (12), which, in this case, is

\[
0 = \frac{\partial r^i}{\partial y^i} = \beta \left( 1 - \frac{n-1}{n}Y^* \right) - c + \beta (1 - v^*) - 2\beta \left( \frac{n-1}{n}Y^* \right) + (p_1 - \beta)(1 - v^*) \frac{n-1}{n}Y^* \frac{1}{\tilde{Y}} .
\]

After multiplication by \( -Y^2/\beta \) this equation becomes

\[
2Y^3 + a_2 Y^2 + a_0 = 0 ,
\]

which is equation (4) if one substitutes the coefficients

\[
a_2 \triangleq \frac{c}{\beta} - (1 - v^*) - \left( 1 + \frac{n-1}{n}Y^* \right) = -(1 - v^*) - \left( 1 - \frac{c}{\beta} \right) - \frac{n-1}{n}Y^* < 0 ,
\]

\[
a_0 \triangleq (1 - \frac{p_1}{\beta})(1 - v^*) \frac{n-1}{n}Y^* .
\]

Since, by part (1.3) of Lemma 7, the profit function of the deviating retailer is pseudoconcave on the interval \( (1 - v^* - \frac{n-1}{n}Y^*)^+ \leq y^i \leq 1 - \frac{s}{\beta} - \frac{n-1}{n}Y^* \), equation (32) may have at most one root on this interval.

Any cubic equation with real coefficients has at least one and up to three real roots. If neither of the roots is relevant, it means that there is no internal maximum and the boundary maximum cannot exceed \( r^* \) as shown above. If there is a relevant root, a direct comparison between \( r^* \) and \( \tilde{r}^i \) determines the existence of RESE.

Suppose that consumer expectations of the second-period price deviate from rational ones with the initial deviation \( p_2^0 > s \) and the game is repeated with the same inputs. As shown above, a symmetric best response is \( \text{BR}(p_2) = \frac{n}{n} \frac{(p_1-s)(1-v^{\text{min}}(p_2))}{c-s} \), which is increasing in \( p_2 \). For any inputs where RESE exists, \( Y^* = \text{BR}(s) > 1 - \frac{s}{\beta} \) and, since \( p_2^0 > s \), \( \text{BR}(p_2^0) > 1 - \frac{s}{\beta} \). Then, for any \( t \geq 0 \), the realized second-period price \( p_2^t \) equals \( s \) if consumer expectations follow a linear adjustment
process \( \hat{p}_{2}^{t+1} = \mu s + (1 - \mu) \hat{p}_2 \) with \( \mu < 1 \). Under this process, \( |s - \hat{p}_{2}^{t+1}| = |(1 - \mu)(s - \hat{p}_2)| = |(1 - \mu)(s - \hat{p}_2^0)| \), which goes to zero with \( t \to \infty \) for any \( \mu \in (0, 1) \) and \( \hat{p}_2^0 \in (s, \infty) \).

A.14. **Proof of Corollary 6 (irrational \( \hat{p}_2 \), RESE4).** The result follows from \( \frac{\partial Y}{\partial \mu} = \frac{\partial Y}{\partial \hat{p}_2} \frac{\partial \hat{p}_2}{\partial \mu} \), where, by Theorem 2, \( \frac{\partial Y}{\partial \mu} = -\frac{n-1}{n} \frac{p_1 - s}{c - s} \) and, by Lemma 1, \( \frac{\partial \hat{p}_2}{\partial \mu} = -\rho \frac{1}{1 - \rho \beta} \).

A.15. **Proof of Proposition 2 (Uniqueness of RESE).** Part (a). We start by discussing model inputs satisfying conditions of RESE1 and 2. By Theorem 1, these conditions rule out RESE3 and guarantee that, for the corresponding structure, one and only one equilibrium exists. Thus, it remains to rule out RESE4.

RESE4 cannot exist under the same conditions as RESE1 because \( p_1 \)-lower bound \( P_1 \) in RESE1 exceeds the upper bound \( P_2 \) in RESE2 since the latter is incompatible with necessary condition \( Y^* > 1 - \frac{s}{\beta} \) for RESE4. Indeed, consider \( Y^* = \frac{n-1}{n} \frac{p_1 - s}{c - s} (1 - v^* \) for RESE4. Condition \( p_1 \leq P_2 \) implies \( n(c - s) \geq (n - 1 + \beta) p_1 - ns = (n - 1)(p_1 - s) + \beta p_1 - s > (n - 1)(p_1 - s) \). Since \( 1 - v^* \leq 1 - p_1 < 1 - \frac{s}{\beta} \), we get \( Y^* < 1 - \frac{s}{\beta} \).

Part (b). It remains to show that, when conditions of RESE1 or 2 do not hold, condition (b.2) guarantees the existence of RESE3 and non-existence of RESE4. Indeed, (b.2) implies that RESE4 total equilibrium supply violates a necessary condition \( Y^* > 1 - \frac{s}{\beta} \) for RESE4 because \( v^* \geq p_1 \) and we have \( Y^* = \frac{n-1}{n} \frac{p_1 - s}{c - s} (1 - v^*) \leq 1 - \frac{s}{\beta} \).

Finally, for the existence of RESE3, we show that (b.2) implies \( Y^* < 1 - \frac{s}{\beta} \) and condition (a) of Theorem 1. Indeed, as long as \( Y^* < 1 - \frac{s}{\beta} \) and since \( v^* \geq p_1 \), the LHS of (a) is smaller than the LHS of (b.2). We show that \( Y^* < 1 - \frac{s}{\beta} \) by demonstrating that \( 1 - \frac{s}{\beta} \) exceeds the larger root of (23) under condition (b.2). Recall, from the proof of Theorem 1, that (23) is obtained as a characterization of the intersection point \( (Y^*, v^*) \) of functions \( v_1^{\text{min}}(Y) \) and \( v_2^{\text{min}}(Y) \) in the range of \( Y \geq \frac{n}{n+1} (1 - \frac{s}{\beta}) \) where \( v_2^{\text{min}}(Y) \) is decreasing (see Figure 16(a)). Since the smallest possible value of \( v_1^{\text{min}}(Y) \) is \( p_1 \), \( Y^* < 1 - \frac{s}{\beta} \) holds as long as \( v_2^{\text{min}}(1 - \frac{s}{\beta}) < p_1 \), i.e.,

\[
1 - \left(1 - \frac{s}{\beta}\right)^2 - \left(1 - \frac{s}{\beta}\right) \frac{n}{n+1} \left(1 - \frac{c}{\beta}\right) = 1 - \frac{n}{n+1} \left(1 - \frac{s}{\beta}\right)^2 + \left(1 - \frac{s}{\beta}\right) \frac{n}{n+1} \frac{c-s}{\beta} < p_1, \quad \text{or}
\]

\[
(n-1)(1-p_1) \frac{p_1 - s}{\beta} + (1-p_1) \left(1 - \frac{s}{\beta}\right) - \left(1 - \frac{s}{\beta}\right)^2 < n \left(1 - \frac{s}{\beta}\right) \frac{c-s}{\beta}.
\]

The latter is implied by (b.2) since \( 1 - p_1 < 1 - \frac{s}{\beta} \).

A.16. **Proof of Proposition 3 (Switches between RESE).** The \( p_1 \)-bounds in the claim of the proposition satisfy the following chain of inequalities for all valid model inputs: \( \frac{s}{\beta} \leq 1 - \beta + c < 1 - \rho(\beta - c) \leq 1 - \frac{n}{n+1} \rho(\beta - c) \). The value \( 1 - \beta + c \) provides the exact lower bound on \( p_1 \)-values corresponding to RESE1 over all \( n \geq 1 \) and \( \rho \in [0, 1] \), while \( \frac{s}{\beta} \) provides the exact upper bound on \( p_1 \) corresponding to RESE2. Thus, \( p_1 \) corresponding to RESE1 for some model inputs cannot result in RESE2 under any other inputs and vice versa. Consider each of the possible \( p_1 \)-ranges.

Part (1): By Theorem 1, if \( p_1 \leq \frac{s}{\beta} \) and \( n = 1 \), the RESE is realized in the form 2 and not form 3. The necessary and sufficient condition for RESE2 can be rewritten as \( p_1 - n(1 - \beta) \leq nc \) or, equivalently, \( n \leq n_2 = \frac{p_1(1-\beta)}{p_1-c} \). For \( n > n_2 \), RESE2 cannot exist and \( p_1 \) falls into the range of RESE3 (and, as argued above, cannot fall into the range of RESE1). That is, as the level
of competition increases, the equilibrium with no sales in the second period (RESE2) becomes impossible and is replaced by the equilibrium with sales in both periods (RESE3).

Part (4): When $p_1 > \frac{c}{\beta}$, we have $n_2 < 1$, i.e., even a monopolist cannot realize RESE2. If, in addition to this condition, $p_1 < 1 - \beta + c$, only RESE3 is possible.

Part (2): Since the RESE3 upper bound on $p_1$ is decreasing in $n$, RESE3 may exist only if $p_1 < 1 - \frac{1}{2} \rho (\beta - c)$ ($P_1$ for $n = 1$). RESE3 $p_1$-bounds imply the following bounds on $n$:

$$n_2 < n < \left\{ \begin{array}{ll} n_1 = \frac{1 - p_1}{p_1 - 1 + p_1 (\beta - c)} & \text{if } p_1 > 1 - \rho (\beta - c), \\ \infty & \text{otherwise}. \end{array} \right.$$ 

That is, as $n$ increases, RESE3 becomes impossible if $p_1 > 1 - \rho (\beta - c)$ and is replaced by RESE1.

Part (3) of this proposition can be shown in the same way, using the boundary on $p_1$ between RESE1 and 3 as a function of $\rho$.

A.17. Proof of Proposition 4 (Monotonicity of $v^*$, $v^*$, and $n^*$). Monotonicity of $v^*$ and $Y^*$. By Theorem 1, $v^*$ is constant in $n$ and $\rho$ for RESE1 and 2; $Y^*$ is increasing in $n$ and constant in $\rho$ for RESE1 and constant in $n$ and $\rho$ for RESE2. By continuity of $v^*$ and $Y^*$, it remains to show the correspondent monotonicity of these values for RESE3.

Monotonicity of $v^*$ and $Y^*$ in $\rho$. Recall that, for RESE3, $Y^*$ and $v^*$ satisfy (18) for $Y^*$. The derivative of this equation in $\rho$ is

$$2Y^* \frac{\partial Y^*}{\partial \rho} - \frac{\partial Y^*}{\partial \rho} \frac{n}{n + 1} \left( 2 - v^* - \frac{c}{\beta} \right) + \frac{n}{n + 1} Y^* \frac{\partial v^*}{\partial \rho} + \frac{n}{n + 1} \left( \frac{p_1}{\beta} - 1 \right) \frac{\partial v^*}{\partial \rho} = 0,$$

which can be written as

$$\frac{\partial Y^*}{\partial \rho} \left[ 2Y^* - \frac{n}{n + 1} \left( 2 - v^* - \frac{c}{\beta} \right) \right] = - \frac{\partial v^*}{\partial \rho} \frac{1}{n + 1} \left[ nY^* + (n - 1) \left( \frac{p_1}{\beta} - 1 \right) \right]. \quad (33)$$

Since $Y^* > 1 - p_1$ (by Lemma 4), and $\frac{p_1}{\beta} \geq p_1$, the lower bound for the square bracket in the RHS is $n(1 - p_1) + (n - 1)(p_1 - 1) = 1 - p_1 > 0$. The square bracket in the LHS of (33) is also positive:

$$2Y^* - \frac{n}{n + 1} \left( 2 - v^* - \frac{c}{\beta} \right) > 0 \quad (34)$$

because $Y^* > 1 - p_1 \geq 1 - v^* \geq \frac{n}{n + 1} (1 - v^*)$ and $\frac{n}{n + 1} \left( 1 - \frac{c}{\beta} \right)$ is a lower bound for $Y^*$ in RESE3 (by Theorem 1).

For RESE3, $Y^*$ and $v^*$ satisfy (21), which can be written as $(1 - \rho \beta) v^* - \rho \beta Y^* = p_1 - \rho \beta$ with the following derivative in $\rho$ : $(1 - \rho \beta) \frac{\partial v^*}{\partial \rho} - \rho \beta \frac{\partial Y^*}{\partial \rho} = \beta (v^* + Y^* - 1)$, where the RHS is positive since $v^* \geq p_1$ and $Y^* > 1 - p_1$. The combination of the last equation with (33) results in the linear system in $\frac{\partial v^*}{\partial \rho}$ and $\frac{\partial Y^*}{\partial \rho}$ with positive $a_i$ and $b_i$:

$$a_2 \frac{\partial v^*}{\partial \rho} = - a_1 \frac{\partial Y^*}{\partial \rho},$$
$$b_2 \frac{\partial v^*}{\partial \rho} = b_1 \frac{\partial Y^*}{\partial \rho} + b_0.$$

The first equation describes a straight line with zero intercept and negative slope. The second straight line goes through the points $\left( \frac{\partial v^*}{\partial \rho}, \frac{\partial v^*}{\partial \rho} \right) = (0, b_0)$ and $\left( \frac{\partial Y^*}{\partial \rho}, \frac{\partial v^*}{\partial \rho} \right) = \left( - \frac{b_0}{b_1}, 0 \right)$ with a positive slope. A unique intersection of these lines belongs to the area where $\frac{\partial v^*}{\partial \rho} > 0$ and $\frac{\partial Y^*}{\partial \rho} < 0$.

Monotonicity of $v^*$ and $Y^*$ in $n$. Denote $z \triangleq \frac{n}{n + 1}$, which implies $\frac{n - 1}{n + 1} = 2z - 1$. Since $z$ increases in $n$, monotonicity of $v^*$ and $Y^*$ in $z$ is equivalent to monotonicity in $n$. Equation (18) for $Y^*$ can
be written as \((Y^*)^2 - Y^*z\left(2 - v^* - \frac{c}{\beta}\right) - (2z - 1)\left(\frac{p_1}{\beta} - 1\right)(1 - v^*) = 0\) with the derivative in \(z\)

\[
2Y^* \frac{\partial Y^*}{\partial z} - \frac{\partial Y^*}{\partial z} z\left(2 - v^* - \frac{c}{\beta}\right) - Y^*\left(2 - v^* - \frac{c}{\beta}\right) + Y^* z \frac{\partial v^*}{\partial z} - 2\left(\frac{p_1}{\beta} - 1\right)(1 - v^*) + (2z - 1)\left(\frac{p_1}{\beta} - 1\right) \frac{\partial v^*}{\partial z} = 0.
\]

After collecting the terms with \(\frac{\partial v^*}{\partial z}\) and \(\frac{\partial Y^*}{\partial z}\), this equation becomes

\[
\frac{\partial Y^*}{\partial z} \left[2Y^* - z\left(2 - v^* - \frac{c}{\beta}\right)\right] + \frac{\partial v^*}{\partial z} \left[Y^* z + (2z - 1)\left(\frac{p_1}{\beta} - 1\right)\right] = Y^*\left(2 - v^* - \frac{c}{\beta}\right) + 2\left(\frac{p_1}{\beta} - 1\right)(1 - v^*). \tag{35}
\]

The first square bracket in the LHS is positive by (34). The second square bracket in the LHS is also positive since it is positive for \(p_1 \geq \beta\), and, for \(p_1 < \beta\), it is bounded from below as follows:

\[
Y^* \frac{n}{n + 1} + \frac{n - 1}{n + 1} \left(\frac{p_1}{\beta} - 1\right) > \frac{n\left[Y^* - (1 - \frac{p_1}{\beta})\right]}{n + 1} > \frac{n\left[Y^* - (1 - p_1)\right]}{n + 1} > 0.
\]

The RHS of (35) is positive since it is linear in \(v^*\), positive at \(v^* = 1\), and positive at \(v^* = p_1\):

\[
Y^* \left(2 - p_1 - \frac{c}{\beta}\right) + 2\left(\frac{p_1}{\beta} - 1\right)(1 - p_1) \\
\geq (1 - p_1) \left[2 - p_1 - \frac{c}{\beta} + 2\frac{p_1}{\beta} - 2\right] = (1 - p_1) \left[\frac{p_1}{\beta} - p_1 + \frac{p_1 - c}{\beta}\right] > 0.
\]

The derivative of (21) in \(z\) is

\[
\frac{\partial v^*}{\partial z} = \frac{\rho \beta}{1 - \rho \beta} \frac{\partial Y^*}{\partial z}. \tag{36}
\]

If \(\rho > 0\), then \(\frac{\partial v^*}{\partial z}\) and \(\frac{\partial Y^*}{\partial z}\) satisfy the following system with positive \(a_i\) and \(b_i\):

\[
a_1 \frac{\partial Y^*}{\partial z} = a_2 \frac{\partial v^*}{\partial z},
\]

\[
b_1 \frac{\partial Y^*}{\partial z} + b_2 \frac{\partial v^*}{\partial z} = b_0.
\]

The first equation describes a straight line with zero intercept and positive slope. The second straight line goes through the points on the axes \(\left(\frac{\partial Y^*}{\partial z}, \frac{\partial v^*}{\partial z}\right) = (\frac{b_0}{b_1}, 0)\) and \(\left(\frac{\partial Y^*}{\partial z}, \frac{\partial v^*}{\partial z}\right) = (0, \frac{b_0}{b_2})\) with a negative slope. A unique intersection of these lines belongs to the area where \(\frac{\partial v^*}{\partial z} > 0\) and \(\frac{\partial Y^*}{\partial z} > 0\).

If \(\rho = 0\), (36) becomes \(\frac{\partial v^*}{\partial z} = 0\) yielding the solution \(\left(\frac{\partial y^*}{\partial z}, \frac{\partial v^*}{\partial z}\right) = (\frac{b_0}{b_1}, 0)\).

Monotonicity of \(nr^*\) in \(n\). By Theorem 1, \(nr^*\) is constant in \(n\) for RESE2 and monotonically decreasing for RESE1. By global continuity of \(nr^*\), it remains to show the correspondent monotonicity of \(nr^*\) for RESE3.

By the alternative expression (30) for RESE3 profit (Lemma 8),

\[
nr^* = \frac{\beta}{1 - \rho \beta} \left\{- (Y^*)^2 + Y^* \left[2 - p_1 - \frac{c}{\beta} - \rho (p_1 - c)\right] + \left(\frac{p_1}{\beta} - 1\right)(1 - p_1)\right\}.
\]

Denote \(F \triangleq \frac{1 - \rho \beta}{\beta} nr^*\). Then

\[
\frac{\partial F}{\partial n} = -2Y^* \frac{\partial Y^*}{\partial n} + \frac{\partial Y^*}{\partial n} \left[2 - p_1 - \frac{c}{\beta} - \rho (p_1 - c)\right] = \frac{\partial Y^*}{\partial n} \left[2 - p_1 - \frac{c}{\beta} - \rho (p_1 - c) - 2Y^*\right]. \tag{37}
\]
As shown above, \( Y^* \) is monotonically increasing in \( n \) for RESE3. Therefore, \( nr^* \) is monotonically decreasing if and only if the square bracket in the last expression is negative. Consider two cases: \( p_1 \leq \frac{c}{\beta} \) and \( p_1 > \frac{c}{\beta} \).

Suppose \( p_1 \leq \frac{c}{\beta} \). By Lemma 4, \( Y^* > 1 - p_1 \) in RESE3, and, therefore, \( 2 - p_1 - \frac{c}{\beta} - \rho(p_1 - c) - 2Y^* < p_1 - \frac{c}{\beta} - \rho(p_1 - c) \leq 0 \).

For \( p_1 > \frac{c}{\beta} \) (\( p_1 \)-lower bound for RESE3 in a monopoly), by monotonicity of \( Y^* \) in \( n \), the RESE3 value of \( Y^* \) for any \( n \) is bounded from below by the RESE3 total supply in a monopoly: \( Y^* \geq 1 - \left( \frac{c}{\beta} + \frac{2p_1 - \rho c}{2 - \rho \beta} \right) \). Therefore,

\[
2 - p_1 - \frac{c}{\beta} - \rho(p_1 - c) - 2Y^* \leq -p_1 - \rho(p_1 - c) + \frac{2p_1 - \rho c}{2 - \rho \beta} - \frac{p_1\rho \beta - \rho(p_1 - c)(2 - \rho \beta) - \rho c}{2 - \rho \beta} = \frac{\rho(p_1\beta - p_1(2 - \rho \beta) + c(1 - \rho \beta))}{2 - \rho \beta} < \frac{\rho(p_1\beta - p_1(2 - \rho \beta) + p_1\beta(1 - \rho \beta))}{2 - \rho \beta} = \rho p_1(\beta - 1) \leq 0.
\]

Thus, the square bracket in (37) is always negative and \( nr^* \) is monotonically decreasing in \( n \).

**Monotonicity of \( nr^* \) in \( \rho \).** By Theorem 1, \( nr^* \) does not depend on \( \rho \) for RESE1 and 2 since there is no intertemporal effect in these cases.

By (10) with \( y^i = \frac{Y^*}{n} \), total profit is \( nr^* = \beta \left[ Y^* - (Y^*)^2 \right] - cY^* + \beta Y^*(1 - v^*) + (p_1 - \beta)(1 - v^*) \) with the derivative

\[
\frac{\partial [nr^*]}{\partial \rho} = \beta(1 - 2Y^*) \frac{\partial Y^*}{\partial \rho} - c \frac{\partial Y^*}{\partial \rho} + \beta(1 - v^*) \frac{\partial Y^*}{\partial \rho} - \beta Y^* \frac{\partial v^*}{\partial \rho} - (p_1 - \beta) \frac{\partial v^*}{\partial \rho} = -\beta \frac{\partial Y^*}{\partial \rho} \left[ 2Y^* - 2 + \frac{c}{\beta} + v^* \right] - \frac{\partial v^*}{\partial \rho} [p_1 - \beta(1 - Y^*)].
\]

For \( n = 1 \), the first square bracket is zero (by Corollary 3), implying \( \frac{\partial [nr^*]}{\partial \rho} < 0 \) since \( p_1 > p_2^* \) and \( \frac{\partial v^*}{\partial \rho} > 0 \) (by part (2) of this proposition).

For \( n > 1 \), let \( R = \left[ 2Y^* - 2 + \frac{c}{\beta} + v^* \right] / \left[ 2Y^* - \frac{n}{n + 1} \left( 2 - v^* - \frac{c}{\beta} \right) \right] \in [0, 1] \) and consider (33) multiplied through by \( \left[ 2Y^* - 2 + \frac{c}{\beta} + v^* \right] \), which yields \( -\frac{\partial Y^*}{\partial \rho} \left[ 2Y^* - 2 + \frac{c}{\beta} + v^* \right] = \frac{\partial v^*}{\partial \rho} \frac{R - 1}{n + 1} \left[ nY^* + (n - 1) \right] \left( \frac{n}{n + 1} - 1 \right) \geq 0 \) (because \( Y^* \geq 1 - p_1 \) by Lemma 4) and \( 0 \leq R < 1 \), we can upper-bound the first term in (38) to obtain

\[
\frac{\partial [nr^*]}{\partial \rho} < \beta \frac{\partial v^*}{\partial \rho} \left[ \frac{n}{n + 1} \left( \frac{p_1}{\beta} - 1 \right) - Y^* - \left( \frac{p_1}{\beta} - 1 \right) \right] = \frac{\beta}{n + 1} \frac{\partial v^*}{\partial \rho} \left[ -Y^* - 2 \left( \frac{p_1}{\beta} - 1 \right) \right] = \frac{2}{n + 1} \frac{\partial v^*}{\partial \rho} \left[ \frac{p_2^* + \beta}{2} - p_1 \right].
\]

Since \( p_2^* = \beta(1 - Y^*) \) and \( Y^* \geq \frac{n}{n + 1} \left( 1 - \frac{c}{\beta} \right) \) in RESE3 by Theorem 1, \( \frac{\partial [nr^*]}{\partial \rho} < 0 \) holds when \( \beta - \frac{n}{2(n + 1)} (\beta - c) \leq p_1 \) which yields the condition of the proposition.

A.18. **Proof of Corollary 7 (\( \sigma_2 \) in \( \rho \)).** The second-period surplus \( \sigma_2 = \rho(\beta v - p_2^*) \) is monotonically non-decreasing in \( n \) because \( p_2^* = \beta(1 - Y^*) \) is non-increasing in \( n \) by part (1) or Proposition 4.

The derivative \( \frac{\partial \sigma_2}{\partial \rho} = \beta v - p_2^* - \rho \frac{\partial p_2^*}{\partial \rho} \) is an increasing linear function of \( v \) that is zero at \( v^0 = \frac{1}{\beta} \left( p_2^* + \frac{\partial p_2^*}{\partial \rho} \right) \). For RESE1, \( v^0 < v^* = 1 \) since \( p_2^* < \beta p_1 < 1 \), by part (1) of Lemma 4, and \( \frac{\partial p_2^*}{\partial \rho} = 0 \).
Minimum valuation of a consumer who purchases in the second period is \( v^2_{\min} = \frac{\beta^2}{\beta} = \max \left\{ \frac{s}{\beta}, 1 - \frac{1}{\beta} \right\} \).

For this consumer, \( \frac{\partial^2 v}{\partial \rho^2} \bigg|_{v=v^2} = -\rho \frac{\partial^2 v}{\partial \rho^2} \), which is nonpositive for RESE3. Thus, \( v^0 \geq v^2_{\min} \). Similarly, we show that \( v^0 < v^* \) because, when \( \rho = 0 \), we have \( \frac{\partial^2 v}{\partial \rho^2} \bigg|_{v=v^*} = \beta p_1 - p_2 > 0 \), and, when \( \rho > 0 \), we know, by Proposition 4, that \( \frac{\partial v^*}{\partial \rho} > 0 \) implying \( \frac{\partial^2 v}{\partial \rho^2} \bigg|_{v=v^*} > 0 \).

A.19. **Proof of Corollary 8** (\( Q_2 \) increases with \( \rho \)). Recall that for a RESE, \( Q_2 = Y^* - (1 - v^*) \), yielding \( \frac{\partial Q_2}{\partial \rho} = \frac{\partial Y^*}{\partial \rho} + \frac{\partial v^*}{\partial \rho} \), which, using (33), is

\[
\frac{\partial Q_2}{\partial \rho} = \left[ \frac{-\frac{n+1}{n+1} Y^* + \frac{n-1}{n+1} (\frac{p_1}{\beta} - 1) + \frac{1}{\beta} \frac{\partial v^*}{\partial \rho} \right] \text{ or }
\frac{\partial Q_2}{\partial \rho} = \frac{\frac{n+1}{n+1} Y^* - \frac{n-1}{n+1} (1 - v^*) - \frac{n-1}{n+1} (\frac{p_1}{\beta} - 1)}{2Y^* - \frac{n}{n+1} (2 - v^* - \frac{c}{\beta})} \cdot \frac{\partial v^*}{\partial \rho}.
\]

Since, by Proposition 4, \( \frac{\partial v^*}{\partial \rho} > 0 \) and, by (34), the denominator of the fraction in the RHS of (39) is positive, the sign of \( \frac{\partial Q_2}{\partial \rho} \) coincides with the sign of the numerator.

For part (1), use \( n = 1 \) and the corresponding \( Y^* = 1 - \frac{1}{2} \left( v^* + \frac{c}{\beta} \right) \) (Corollary 3) in the numerator to get \( \frac{3}{2} Y^* - \left( 1 - \frac{1}{2} \left( v^* + \frac{c}{\beta} \right) \right) = \frac{1}{2} Y^* > 0 \).

For part (2), \( \rho = 0 \) implies \( v^* = p_1 \), and the numerator becomes \( Y^* - 1 + p_1 + \frac{c}{\beta} - \frac{p_2}{\beta} \) as \( n \rightarrow \infty \).

In this case, \( p_1 \)-range for RESE3 is \( c < p_1 < 1 \), and as \( p_1 \rightarrow 1 \), the total supply, given by the larger root of (23), approaches \( Y^* = 1 - \frac{c}{\beta} \) implying that the numerator approaches \( 1 - \frac{1}{\beta} < 0 \).

A.20. **Proof of Proposition 5** (boundary-value gain in RESE3). For \( \beta = 1 \), necessary condition (5) of RESE4 becomes \( \rho < \frac{1 - p}{1 - c} \) and cannot hold as \( \rho \rightarrow 1 \) since \( p_1 > s \). Thus, RESE4 does not exist. RESE2 does not exist because its \( p_1 \)-range is empty.

Part (1) is immediate by part (3) of Proposition 3, because, as \( \rho \) increases from 0 to 1, the switch to RESE1 occurs at \( p_1 = \frac{n+1}{n} \frac{1 - p_1}{1 - c} < 1 \) if \( \frac{1 - p_1}{1 - c} < \frac{n}{n+1} \) or, equivalently, if \( n > \bar{n} \). The resulting equilibrium characteristics are obtained by substituting \( \beta = 1 \) in the description of RESE1. It is immediate to check that the resulting limit of \( p^*_2 \) is below \( p_1 \).

Part (2). When \( n < \bar{n} \), the switch from RESE3 to RESE1 does not occur for any \( \rho < 1 \). Total supply \( Y^* \), which is given by a larger root of (23), is continuous in \( \rho \) near \( \rho = 1 \). We can write (23) for \( \beta = \rho = 1 \) as

\[
Y^2 - \left( (1 - p_1) + \frac{1 - p_1(n-1)}{n} \right) Y + (1 - p_1) \times \frac{(1 - p_1)(n-1)}{n} = \left( Y - \frac{(1 - p_1)(n-1)}{n} \right) (Y - (1 - p_1)) = 0,
\]

resulting in roots \( \frac{(1 - p_1)(n-1)}{n} \) and \( 1 - p_1 \). Thus, \( Y^* \big|_{\rho=1} = 1 - p_1 \) and \( p^*_2 \big|_{\rho=1} = (1 - Y^*) \big|_{\rho=1} = p_1 \).

The necessary condition \( Y^* < 1 - \frac{s}{\beta} = 1 - s \) of RESE3 in Theorem 1 is satisfied for all \( \rho \) sufficiently close to 1 since \( p_1 > s \).

The limit of \( v^* \) is found using

**Lemma 9.** In RESE 3 with \( \beta = 1 \), we have \( \lim_{\rho \rightarrow 1} \frac{\partial v^*}{\partial \rho} \bigg|_{\beta=1} = n(c - p_1) \).
By (21) with $\beta = 1$, we have $v^* = \frac{p_1 - p_1(1 - v^*)}{1 - \rho}$ for all $\rho \in [0, 1)$. Then

$$\lim_{\rho \to 1} v^* = \frac{p_1 - p_1}{1 - 1} = \frac{0 = \lim_{\rho \to 1} \frac{\partial [p_1 - \rho(1 - Y^*)]}{\partial \rho}}{(1 - \rho)} = \frac{-\lim_{\rho \to 1} [-(1 - Y^*) + \rho \frac{\partial Y^*}{\partial \rho}]}{1 - \rho}$$

$$= \lim_{\rho \to 1} \left[ 1 - Y^* - \rho \frac{\partial Y^*}{\partial \rho} \right] = p_1 + n(p_1 - c) \quad \text{(from Lemma 9 and } Y^*|_{\rho \to 1} = 1 - p_1).$$

Using the limiting values of $Y^*$ and $v^*$ in a strict version of condition (a) for RESE3 existence, we obtain the sufficient existence condition of the form (6). Indeed, by continuity, condition (a) is satisfied for all $\rho$ sufficiently close to one.

Using the expression for $r^*$, the limit of the total profit is

$$nr^*|_{\rho \to 1} = \lim_{\rho \to 1} [(p_1 - c)(1 - v^*) + (p_2^* - c)(Y^* - 1 + v^*)] = \lim_{\rho \to 1} [(p_1 - p_2^*)(1 - v^*) + (p_2^* - c)Y^*]$$

$$= (p_1 - p_1)(1 - v^*)|_{\rho \to 1} + (p_1 - c)(1 - p_1) = (p_1 - c)(1 - p_1).$$

To complete the proof of part (2), consider when $(p_1 - c)(1 - p_1) \geq \frac{n(1-c)^2}{(n+1)^2}$. With a change of variables $x = \frac{1 - p_1}{p_1 - c}$, this relation can be represented as $(1 - c)^2(1 - x) \geq \frac{n(1-c)^2}{(n+1)^2}$, or, equivalently, as $(1 - x)x \geq \frac{n}{(n+1)^2}$ resulting in $\frac{1}{n+1} \leq x \leq \frac{n}{n+1}$. This range does not intersect with a feasible range of $x$ for part (2) which is given by $\frac{n}{n+1} < x < 1$ (resulting from $n < \bar{n}$). Thus, for part (2),

$$(p_1 - c)(1 - p_1) < \frac{n(1-c)^2}{(n+1)^2}.$$

Part (3) is immediate since $\bar{n} = \bar{n}$ implies that the limits in parts (1) and (2) are equal.

Part (4) follows from Corollary 4 for $n = \bar{n} = \frac{1 - p_1}{p_1 - c}$. In this case, the condition of the corollary becomes $\frac{1}{n} + 2 < n$, which, after solving for positive integer $n$, yields $n \geq 3$. Moreover, when $n = \bar{n} = \frac{1 - p_1}{p_1 - c}$ and $\rho = 0$, the RESE is of the form 3 because RESE2 is impossible with $\beta = 1$ and the switch to RESE1 occurs only in the limit of $\rho \to 1$. We also have $Y^*|_{\rho = 0} > Y^*|_{\rho \to 1} = 1 - p_1$ because the total supply is (strictly) decreasing in $\rho$, and $p_2^*|_{\rho = 0} < c$ by Corollary 4. Thus, for $\rho = 0$, the total first period profit is $(p_1 - c)(1 - v^*)|_{\rho = 0} = (p_1 - c)(1 - p_1)$, which is exactly the same as the total profit for $\rho \to 1$, while the second-period total profit is $(p_2^* - c)(Y^* - 1 + v^*)|_{\rho = 0} < 0$.

A.21. Analysis of Example 1. Observing that $(p_1 - s)(1 - p_1) \leq \frac{1}{4}(1 - s)^2$ and using relation

$$1 - p_1 = \bar{n}(p_1 - c),$$

we can strengthen (6) to

$$\frac{(n-1)(n-n)}{n} = \frac{p_1 - p_1-c}{4(c-s)} < 1.$$ For RESE3, the range of $n$ is $[1, \bar{n}]$. As a function of $n$, fraction $\frac{n-1}{n-n}$ attains its maximum of $(\sqrt{n} - 1)^2$ in this range at $n = \sqrt{n}$ leading to an even stronger version of the condition, i.e., $(\sqrt{n} - 1)^2 \leq \frac{4}{4(c-s)} < 1$. The LHS of this inequality decreases in $p_1$ and it surely holds if it holds at $p_1 = c$, i.e., if

$$\frac{1-c}{4(c-s)} < 1 \text{ or } c > \frac{1+c}{c-s}.$$ Thus, (6) holds for all $p_1$ and $1 \leq n < \bar{n}$ if $c > \frac{1+4s}{n}$, e.g., if $s = 0$ and $c > 0.2$.

A.22. Proof of Proposition 6 (Monotonicity in RESE4). Part 1. $v^* = \frac{p_1 - p_1}{1 - \rho}$, which is constant in $n$ and increasing in $\rho$ since

$$\frac{\partial v^*}{\partial \rho} = \frac{-s(1 - p_1) + \beta(p_1 - \rho)}{(1 - \rho)^2} = \frac{\beta p_1 - s}{(1 - \rho)^2} \geq 0.$$ Parts (2) and (3) follow directly from part (1) and the formulas for $Y^*$ and $r^*$, given by Theorem 2.

A.23. Proof of Proposition 7 ($Y^* > 1 - c$). Part 1. By Theorem 1, $Y^*|_{\beta = 1} = \frac{n}{n+1}(1 - \frac{s}{\beta})$, which is maximal at $\beta = 1$, and $Y^*|_{\beta = 1} = \frac{n}{n+1}(1 - c) < 1 - c$. Hence, $Y^* > 1 - c$ for any parameters where RESE1 exists.

Part 2. By Theorem 2, $Y^* = \frac{n}{n+1}(1 - v^*)$, which, given other parameters fixed, goes to infinity when $c$ approaches $s$. The condition $Y^* \geq 1 - c$, using $1 - v^* = \frac{1-p_1-\rho(\beta-s)}{1-\rho}$, can be written as $c - s \leq \frac{n-1}{n} p_1 - s \frac{1-p_1-\rho(\beta-s)}{1-\rho^3}$. Since $Y^* = \frac{n}{n+1}(1 - v^*)$, its maximum in
$p_1, \bar{Y}^{*,4}$, can be found from the condition $\frac{\partial Y^{*,4}}{\partial p_1} = 0 = \frac{n-1}{n(1-\rho \beta)((c-s))} \left[1 - p_1 - \rho(\beta - s) - (p_1 - s)\right]$, yielding $\bar{p}_1 = \frac{1}{2}[1-\rho(\beta-s)+s] = \frac{P_4}{1-\rho \beta}$. Since $P_4 = 1-\rho(\beta-s) = 2p_1-s$, we have $1 - v_1^{*}|_{p_1 = \bar{p}_1} = \frac{\bar{p}_1 - s}{1-\rho \beta}$ and $Y^{*,4} = \frac{(n-1)(\bar{p}_1-s)^2}{n(1-\rho \beta)(c-s)}$. Price $\bar{p}_1$ can be in the $p_1$-range of RESE4 since $\bar{p}_1$ is always below $P_4$ — the $p_1$-upper bound ($P_4 > s$), and $\bar{p}_1$ can be greater than $P_2$, which, by Proposition 2, is $p_1$-lower bound in RESE4. Indeed, $\bar{p}_1 > P_2$ holds for any $n \geq 2$ if it holds for $n = 2$ since $P_2$ decreases in $n$. For $n = 2$, inequality $\bar{p}_1 > P_2$ can be written as $(P_4 + s)(1 + \beta) > 4c$, which holds for sufficiently small $c$.

**Part 3.** By Corollary 3, $Y^{*,3} = 1 - \frac{1}{2}(v^* + \frac{c}{\beta})$ if $n = 1$. Then $Y^{*,3} < 1 - c$ is equivalent to $c < \frac{1}{2}(v^* + \frac{c}{\beta})$, which holds for any $\beta \in (c, 1]$ since $v^* \geq p_1 > c$.

By Proposition 4, $Y^{*,3}$ is maximized at $n \to \infty$. By continuity of $Y^*$ at the boundaries, $Y^{*,3} \to 1 - c$ when $p_1 \to P_2|_{n \to \infty} = c$. We will show that there are feasible inputs such that $\frac{\partial Y^{*,3}}{\partial p_1} > 0$. For example, for $n \to \infty$ and $\rho = 0$, equation (23) for $Y$ is $Y^2 - \left[1 - \frac{c}{\beta} + 1 - p_1\right] Y - \left(\frac{P_3}{\beta} - 1\right)(1-p_1) = 0$. Derivative w.r.t. $p_1$ results in $2Y \frac{\partial Y}{\partial p_1} + Y - \left[1 - \frac{c}{\beta} + 1 - p_1\right] \frac{\partial Y}{\partial p_1} - \frac{\partial Y}{\partial p_1} = \frac{1}{\beta}(1 - c - c(1 - \beta))$ yielding $\frac{\partial Y}{\partial p_1} = \frac{1 - c}{c(1 - \beta)} - 1$. The RHS is positive since $1 - c > c(1 - \beta)$.

**A.24. Proof of Proposition 8 (discount).** The proof is similar to the corresponding parts of the proofs of Theorems 1 and 2. The expressions for $Y^*$ result from the symmetric best responses with $r^i = (p_1 - c)q^i + \lambda(p_2 - c)(y^i - q^i)$, $y^i = \frac{Y}{n}$, and $Y^{-i} = \frac{1}{n+1}Y$.

For RESE1, $q^i = 0$ and, using $p_2 = \beta(1 - Y)$, $\frac{\partial r^i}{\partial y^i} = 0 = \lambda [\beta(1 - Y) - c - \beta\frac{Y}{n}]$, yielding $Y = \frac{n}{n+1}\left(1 - \frac{c}{\beta}\right)$ that does not depend on $\lambda$. For RESE2, the result is obvious since $r^i = (p_1 - c)y^i$ does not depend on $\lambda$. For RESE4 with $v^* = \frac{p_1 - p_2}{1 - \rho \beta}$ and $r^i = (p_1 - c)(1 - v^*)\frac{Y}{\lambda} - \lambda(c - s)y^i (1 - \frac{1 - v^*}{Y}) = y^i \left\{\frac{1 - v^*}{Y}[p_1 - c + \lambda(c - s)] - \lambda(c - s)\right\}$, the equation for $Y$ is

$$\frac{\partial r^i}{\partial y^i} = 0 = \frac{1 - v^*}{Y}[p_1 - c + \lambda(c - s)] - \lambda(c - s) - y^i \frac{1 - v^*}{Y^2}[p_1 - c + \lambda(c - s)]$$

$$= \frac{n-1}{n} \frac{1 - v^*}{Y} [p_1 - c + \lambda(c - s)] - \lambda(c - s).$$

This equation yields a unique $Y^{*,4}_{\lambda} = \frac{n-1}{n}(1 - v^*) \left[\frac{p_1 - c}{\lambda(c - s)} + 1\right]$, which decreases in $\lambda$ and $\frac{\partial^2 r^i}{\partial (y^i)^2} = -2 \frac{n-1}{n} \frac{1 - v^*}{Y^2} [p_1 - c + \lambda(c - s)] < 0$. If $\lambda = (1 + \delta)^{-1}$, the relative change in $Y^*$ is

$$\frac{Y^{*,4}_{\lambda} - Y^{*,4}}{Y^{*,4}} = \left[\frac{p_1 - c}{\lambda(c - s)} + 1 - \frac{p_1 - s}{c - s}\right] \frac{c - s}{p_1 - s} = \frac{p_1 - c + \lambda(c - p_1)}{\lambda(p_1 - s)} = \frac{(p_1 - c)(1 - \lambda)}{p_1 - s}.\delta.$$

For RESE3, we have

$$r^i = (p_1 - c)(1 - v^*)\frac{Y}{n + \lambda(p_2 - c)y^i} \left(1 - \frac{1 - v^*}{Y}\right)$$

$$= y^i \left\{\lambda[\beta(1 - Y) - c] + \lambda \beta(1 - v^*) + \frac{1 - v^*}{Y}[p_1 - c(1 - \lambda) - \lambda \beta]\right\},$$
and $Y^*$ results from the first-order optimality condition
\[
\frac{\partial r^i}{\partial y^i} = 0 = \lambda \beta (1 - Y) - \lambda c + \lambda \beta (1 - v^*) + \frac{(1 - v^*)}{Y} [p_1 - c (1 - \lambda - \lambda \beta)] + \frac{Y}{n} \left\{-\lambda \beta - \frac{(1 - v^*)}{Y^2} [p_1 - c (1 - \lambda - \lambda \beta)] \right\}
\]
\[
= -Y \lambda \beta \left(1 + \frac{1}{n}\right) - \lambda c + \lambda \beta (2 - v^*) + \frac{n - 1}{n} \frac{1 - v^*}{Y} [p_1 - c - \lambda (\beta - c)] \tag{40}
\]

as well as the equation $v^* = \frac{p_1 - \rho \beta (1 - Y)}{1 - \rho \beta}$ that links valuation threshold to the rational second-period expectations. The remainder of the proof will formally show that $Y^*$ is decreasing in $\lambda$. The geometric idea behind the proof is provided by a generalized version of the curve $v^*_{min}(Y)$ in (20) that gives valuation threshold for the corresponding stationary point of the profit. Solving (40) for $v^*$, we obtain
\[
v^* = 1 - \frac{Y^2 - Y \frac{n}{n+1} \left(1 - \frac{c}{\beta}\right)}{Y \frac{n}{n+1} + \frac{n-1}{n+1} \left(p_1 - c - \frac{1}{\lambda \beta} - 1 + \frac{c}{\beta}\right)}. \tag{41}
\]

The generalized $v^*_{min}(Y)$ given by the right-hand side shifts down as $\lambda$ increases. Thus, the intersection point of $v^*_{min}(Y)$ and $v^*_{max}(Y)$ (illustrated in Figure 16(a)) shifts to the left as $\lambda$ increases. Since the abscissa of the intersection point is $Y^*$, the claim hold in RESE3 based on this geometric structure.

Formally, (40) multiplied by $-Y \frac{n}{\lambda \beta (n+1)}$, becomes
\[
Y^2 - Y \frac{n}{n+1} \left(2 - v^* - \frac{c}{\beta}\right) - \frac{n}{n+1} (1 - v^*) \left[p_1 - c - \lambda \beta \right] - \left[1 - \frac{c}{\beta}\right] = 0,
\]

implying that, for $n = 1$, $Y$ does not depend on $\lambda$. Substitution for $1 - v^* = \frac{1-p_1-n}{1-\rho \beta}$ yields
\[
Y^2 - Y \frac{n}{n+1} \left(1 - \frac{p_1 - \rho \beta}{1}$

The coefficient in front of $Y^2$ is $1 + \frac{n-\rho \beta}{n+1-\rho \beta}$, and the one in front of $Y$ is
\[
-\frac{1}{n(1+\rho \beta)} \left\{n[1-p_1+\left(1-\frac{c}{\beta}\right)(1-\rho \beta)] - (n-1)\rho \beta \left[p_1 - c - \lambda \beta \right] - \left(1 - \frac{c}{\beta}\right) \right\}.
\]

Multiplying the last equation by $\frac{(n+1)(1-\rho \beta)}{n+1-\rho \beta} > 0$ and denoting $\hat{\lambda} \triangleq \frac{p_1-c}{\lambda} - (\beta - c)$, we obtain
\[
Y^2 - \frac{(\beta - c)n(1-\rho \beta) + \beta(1-p_1)n - \rho \beta(n-1)\hat{\lambda} - (1-p_1)(n-1)}{\beta(n+1-\rho \beta)} \hat{\lambda} Y = 0. \tag{42}
\]

The derivative of (42) w.r.t. $\hat{\lambda}$ is
\[
2Y \frac{\partial Y}{\partial \hat{\lambda}} + Y \frac{\rho \beta (n-1)}{\beta(n+1-\rho \beta)} - \frac{(\beta - c)n(1-\rho \beta) + \beta(1-p_1)n - \rho \beta(n-1)\hat{\lambda}}{\beta(n+1-\rho \beta)} \frac{\partial Y}{\partial \hat{\lambda}} - \frac{(1-p_1)(n-1)}{\beta(n+1-\rho \beta)} = 0.
\]

Multiplication by $\beta(n+1-\rho \beta)$ yields
\[
\frac{\partial Y}{\partial \hat{\lambda}} \left\{2Y \frac{\rho \beta (n+1-\rho \beta)}{\beta(n+1-\rho \beta)} - (\beta - c)n(1-\rho \beta) - \beta(1-p_1)n + \rho \beta(n-1)\hat{\lambda} \right\} = (n-1)(1-p_1-\rho \beta Y).
\]

The RHS is zero for $n = 1$ and, by part (3) of Lemma 4, positive for $n > 1$. It remains to show that $\cdot > 0$ for any $n \geq 1$, implying $\frac{\partial Y}{\partial \lambda} \equiv 0$ for $n = 1$, and $\frac{\partial Y}{\partial \lambda} < 0$ for $n > 1$ because $\hat{\lambda}$ decreases in $\lambda$. Since $\hat{\lambda}$ is minimal at $\lambda = 1$, it can be shown that, for any $\lambda \in (0, 1]$, $Y$ is increasing in $n$. The proof is identical to the corresponding part of the proof of Proposition 4 with the substitution of $\hat{\lambda}/\beta$ for
Then \( \{ \} \geq \beta \left\{ 2(n + 1 - \rho \beta) Y |_{n=1} - \left( 1 - \frac{\rho}{\beta} \right)n(1 - \rho \beta) - (1 - p_1)n + \rho(n - 1)(p_1 - \beta) \right\} = \beta \left\{ n[2 Y |_{n=1} - \left( 1 - \frac{\rho}{\beta} \right)(1 - \rho \beta) - (1 - p_1) + \rho(p_1 - \beta)] + 2(1 - \rho \beta) Y |_{n=1} - \rho(p_1 - \beta) \right\} \), where the square bracket \( \{ \} = 2 Y |_{n=1} - \left( 1 - \frac{\rho}{\beta} \right) - (1 - p_1) + \rho(p_1 - c) \). Since \( Y |_{n=1} \) does not depend on \( \lambda \), we can rewrite, by Corollary 3, the bracket \( \{ \} \) as follows: 

\[
\{ \} = 2 - \frac{\rho}{\beta} - \frac{2p_1 - \rho c}{2 - \rho \beta} - 1 + \frac{\rho}{\beta} - 1 + p_1 + \rho(p_1 - c) = \frac{1 - \rho}{2 - \rho \beta} \left( pc - p_1 \rho \beta + 2p(p_1 - c) - \rho^2 \beta(p_1 - c) \right) = \frac{\rho}{2 - \rho \beta} \left( (p_1 - c)(1 - \rho \beta) + p_1(1 - \beta) \right) \geq 0 \text{ with strict inequality for } \rho > 0. \]

Therefore, the bracket \( \{ \} \) is positive if it is positive for \( n = 1 \). \( \{ \} \) exists and unique in an open neighborhood of \( Y^* \). For \( \rho = 1 \), \( p_1 < 1 - \frac{1}{2} \rho(\beta - c) \), the last bracket \( \{ \} \) > 2 \( 1 - \frac{\rho}{\beta} \rho(\beta - c) \) > 0.

A.25. Proof of Proposition 9 (RESE stability). RESE1,3, and 4, for \( n \geq 2 \), represent a non-degenerate game between retailers that can be reformulated as a one-period game with retailer \( i \)'s payoff \( \pi^i(y, Y^{-i}) = y^i P_i(y, Y^{-i}) - C_i(y^i) \), where \( C_i(y^i) = c y^i \). For RESE1 and 3, by (10), \( P_i(y, Y^{-i}) = \beta(1 - Y) + \beta(1 - v^*), Y^{-i} = (p_1 - \beta)(1 - v^*) \), and for RESE4, by (14), \( P_i(y, Y^{-i}) = s + \frac{(p_1 - \beta)(1 - v^*)}{Y^*} \). By Theorem 3 in al Nowaihi and Levine (1985), a Cournot equilibrium \( Y^* \) is locally asymptotically stable if the following assumptions hold:

(A1) The equilibrium point \( Y^* \) exists and unique in an open neighborhood of \( Y^* \) and \( (y^i)^* > 0, i \in I \) holds by the condition of the proposition and part (2) of Lemma 4.

(A2) \( P_i \) and \( C_i, i \in I \) are twice continuously differentiable functions in an open neighborhood of \( Y^* \) holds for RESE1,3, and 4.

(A3) \( \partial^2 \pi^i / \partial (y^i)^2 < 0 \text{ at } Y^* \) for each \( i \in I \) holds for RESE1,3, and 4 by Lemma 7.

(H1) \( P' < C_i'' \) at \( Y^* \) for each \( i \in I \) holds since \( C_i'' \equiv 0 \) and \( P' < 0 \) for RESE1,3, and 4.

(H2) \( P' + y^i P'' \leq 0 \text{ at } Y^* \) for each \( i \in I \) holds strictly for RESE1,3, and 4. Namely, for RESE1 and 3, this inequality is \( -\beta - \frac{(p_1 - \beta)(1 - v^*)}{Y^*} + 2 Y^* \left( p_1 - \beta \right)(1 - v^*) = -\beta - \frac{(p_1 - \beta)(1 - v^*)}{Y^*} \left( 1 - \frac{1}{n} \right) < 0 \) for any \( n \geq 2 \). For RESE4, the proof is similar.

A.26. Proof of Lemma 5 (total surplus). By the definition of \( v_{min} \), the total consumer surplus in the first period is

\[
\Sigma_1 = \int_{v_{min}}^{1} (v - p_1)dv = \left( \frac{v^2}{2} - p_1v \right)_{v_{min}}^{1} = \left( \frac{1}{2} - p_1 - \frac{(v_{min})^2}{2} + p_1 v_{min} \right) = \frac{1}{2} \left( 1 - (v_{min})^2 \right) - p_1 (1 - v_{min}) = \frac{1}{2} \left( 1 + v_{min} \right) - p_1.
\]

The total surplus in the second period is

\[
\Sigma_2 = \int_{p_2}^{\beta v_{min}} (\tilde{v} - p_2) d\tilde{v} = \left( \frac{\tilde{v}^2}{2} - p_2\tilde{v} \right)_{p_2}^{\beta v_{min}} = \frac{1}{\beta} \left( \beta v_{min} - p_2 \right) \left( \frac{\beta v_{min} + p_2}{2} - p_2 \right) = \frac{\left( \beta v_{min} - p_2 \right)^2}{2\beta}.
\]

Hence, \( \Sigma^* = \Sigma_1 + \Sigma_2 = (1 - v_{min}) \left[ \frac{1}{2} (1 + v_{min}) - p_1 \right] + \frac{\left( \beta v_{min} - p_2 \right)^2}{2\beta} \), with \( v_{min} = v^* \) for a RESE.
A.27. Proof of Proposition 10 (Monotonicity of surplus and welfare). 1. Monotonicity of $\Sigma^*$ in $n$. By Lemma 5, the derivatives $\frac{\partial \Sigma_1}{\partial n}$ and $\frac{\partial \Sigma_2}{\partial n}$ are

$$\frac{\partial \Sigma_1}{\partial n} = \frac{1}{2} \frac{\partial v^*}{\partial n} (1 - v^*) - \frac{\partial v^*}{\partial n} \left( \frac{1}{2} (1 + v^*) - p_1 \right) = - \frac{\partial v^*}{\partial n} (v^* - p_1);$$

$$\frac{\partial \Sigma_2}{\partial n} = \frac{1}{\beta} (\beta v^* - p_2^*) \left( \frac{\partial v^*}{\partial n} - \frac{\partial p_2^*}{\partial n} \right) = \frac{\partial v^*}{\partial n} (\beta v^* - p_2^*) - \frac{\partial p_2^*}{\partial n} \left( v^* - \frac{p_2^*}{\beta} \right).$$

For a RESE, $\frac{\partial \Sigma_1}{\partial n} \leq 0$ since $\frac{\partial v^*}{\partial n} \geq 0$ (Proposition 4) and $v^* \geq p_1$; and $\frac{\partial \Sigma_2}{\partial n} \geq 0$ since, by Lemma 4, $\beta v^* \geq \beta p_1 > p_2^*$ and $\frac{\partial p_2^*}{\partial n} \leq 0$ since, by (3), $p_2^* = s \lor [\beta (1 - Y^*)]$ and, by Propositions 4 and 6, $\frac{\partial \Sigma_2}{\partial n} \geq 0$.

Using the expressions for $\frac{\partial \Sigma_1}{\partial n}$ and $\frac{\partial \Sigma_2}{\partial n}$, we can write

$$\frac{\partial \Sigma}{\partial n} = \frac{\partial v^*}{\partial n} \left[ \beta v^* - p_2^* - (v^* - p_1) \right] - \frac{\partial p_2^*}{\partial n} \left( v^* - \frac{p_2^*}{\beta} \right).$$

(43)

By the definition of $v^*$, the surpluses of a consumer with valuation $v = v^*$ are equal in both periods: $\sigma_1|_{v=v^*} = v^* - p_1 = \sigma_2|_{v=v^*} = \rho (\beta v^* - p_2^*)$. Therefore, since $\rho < 1$, the square bracket in (43) is positive. Then, since $\frac{\partial v^*}{\partial n} \geq 0$ and $\frac{\partial p_2^*}{\partial n} \leq 0$, equation (43) yields $\frac{\partial \Sigma^*}{\partial n} \geq 0$ for a RESE.

By Theorems 1, 2, and Proposition 4, $\Sigma^*$ is constant in $n$ for RESE2 and 4 ($\frac{\partial \Sigma^*}{\partial n} = \frac{\partial p_2^*}{\partial n} = 0$) and monotonically increasing for RESE1 ($\frac{\partial \Sigma^*}{\partial n} = 0, \frac{\partial \Sigma^*}{\partial n} < 0$) and 3 ($\frac{\partial \Sigma^*}{\partial n} > 0, \frac{\partial \Sigma^*}{\partial n} < 0$).

Monotonicity of $\Sigma^*$ in $\rho$. By Theorem 1, $\Sigma^*$ does not depend on $\rho$ for RESE1 and 2 (no intertemporal effects). In general, using the same approach as for $\frac{\partial \Sigma^*}{\partial n}$, we can write for a RESE

$$\frac{\partial \Sigma}{\partial \rho} = - \frac{\partial v^*}{\partial \rho} (v^* - p_1) \leq 0,$$

$$\frac{\partial \Sigma}{\partial \rho} = \frac{\partial v^*}{\partial \rho} \left( \beta v^* - p_2^* - \frac{\partial p_2^*}{\partial \rho} \left( v^* - \frac{p_2^*}{\beta} \right) \right).$$

Due to the side effect of increasing $\rho$ ($\frac{\partial \Sigma}{\partial \rho} \geq 0$), it is not obvious that $\frac{\partial \Sigma}{\partial \rho} \geq 0$. The derivative of total surplus is

$$\frac{\partial \Sigma}{\partial \rho} = \frac{\partial v^*}{\partial \rho} \left[ \beta v^* - p_2^* - (v^* - p_1) \right] - \frac{\partial p_2^*}{\partial \rho} \left( v^* - \frac{p_2^*}{\beta} \right).$$

(44)

For RESE4, we have $\frac{\partial \Sigma}{\partial \rho} = 0$ and, by Proposition 6, $\frac{\partial \Sigma^*}{\partial \rho} > 0$ yielding $\frac{\partial \Sigma^*}{\partial \rho} > 0$.

2. Monotonicity of $W^*$. Recall that for RESE1, $v^* = 1$ and $p_2^* = \frac{1}{n+1} (1 + \rho c)$, yielding, by Lemma 5,

$$W^* = \frac{1}{2} \left( \beta - \frac{1}{n+1} (\beta + nc) \right)^2 = \frac{1}{2} \left[ \frac{n}{n+1} (\beta - c) \right]^2$$

and

$$W^* = \frac{1}{2} \left[ \frac{n}{n+1} (\beta - c) \right]^2 = \frac{1}{2} \left( \frac{n+1}{n} \right)^2 \left( \beta - c \right)^2$$

which is increasing in $n$ and constant in $\rho$.

For RESE2, $v^* = p_1, \Sigma_2 = 0$ (no second period), $W^* = \frac{1}{2} (1 - p_1)^2$, and

$$W^* = \Sigma^* + nr^* = \frac{1}{2} (1 - p_1)^2 + (p_1 - c)(1 - p_1) = \frac{1}{2} (1 - p_1)(1 + p_1 - c),$$

which is constant in both $n$ and $\rho$.

For RESE4, $v^* = \frac{p_1 - p_2}{1 - \rho \beta}, p_2^* = s$, yielding $\Sigma^*$ that is constant in $n$. Then, $W^* = \Sigma^* + nr^* = \Sigma^* + \frac{p_1 - s}{n} (1 - v^*)$ is monotonically decreasing in $n$.

By Lemma 5, $\frac{\partial \Sigma^*}{\partial \rho}$ can be written as follows:

$$\frac{\partial \Sigma^*}{\partial \rho} = \frac{\partial \Sigma^*}{\partial v^*} \frac{\partial v^*}{\partial \rho} = - \frac{1}{2} \left( (v^*)^2 - p_1 (1 - v^*) + \frac{(\beta v^* - s)^2}{2 \beta} \right) \frac{\partial v^*}{\partial \rho} = - \frac{1}{2} \left( v^* + p_1 + \beta v^* - s \right) \frac{\partial v^*}{\partial \rho}.$$
Consider two cases. If \( 1 > \frac{n-1}{n}(p_1 - s)(1 - \beta) \geq 0 \). After collecting the terms with \( \rho \), the latter inequality becomes \( \frac{n-1}{n}(p_1 - s) - (1 - \beta) \geq 0 \). or as \( p_1^* - s - \frac{1}{n}(p_1 - s) \geq \left( p_1^* - s - \frac{1}{n}(p_1 - s) \right) \rho \), which, since \( p_1^* > s \), can be written as

\[
1 - \frac{1}{n} \frac{p_1^* - s}{n} \leq \left( 1 - \frac{\beta}{n} \frac{p_1^* - s}{n} - \frac{1}{n}(p_1 - s) \right) \rho.
\]

A.28. Proof of Corollary 9 (RESE3, non-monotonicity of the total surplus in \( \rho \)). Proof is immediate from the following lemma:

**Lemma 10.** If \( \beta = 1 \), given that RESE is unique,

1. for \( \rho \to 1 \) and \( n < \bar{n} = \frac{1-p_1}{p_1-c} \), \( \frac{\partial \Sigma^*}{\partial \rho} = -n^2(p_1-c)^2 < 0 \);
2. for \( \rho = 0 \),
   
   (2.1) \( \frac{\partial \Sigma^*}{\partial \rho} = \frac{1}{n}(p_1 - c) > 0 \) for \( n = 1 \); and

(2.2) \( \frac{\partial \Sigma^*}{\partial \rho} = \frac{\partial \Sigma^*}{\partial \rho} \left( 1 - (1/p_1) Y^* - (1 - c) \right) > 0 \) for \( n = \infty \), where \( Y^* \) is the larger root of the equation

\[
Y^2 - Y(1 - p_1 + 1 - c) + (1 - p_1)^2 = 0.
\]

**Proof:** By (44) with \( \beta = 1 \), the derivative \( \frac{\partial \Sigma^*}{\partial \rho} \) is

\[
\frac{\partial \Sigma^*}{\partial \rho} = \frac{\partial \Sigma^*}{\partial \rho} \bigg|_{\beta = 1} = \frac{\partial \Sigma^*}{\partial \rho} \left( p_1 - p_2^* \right) - \frac{\partial \Sigma^*}{\partial \rho} \left( p_2^* - p_2^* \right).
\]

**Part (1):** By part (2) of Proposition 5, RESE3 is realized if \( \rho \to 1 \) and \( n < \bar{n} \). Then \( p_2^* \to p_1 \) and, if we show that \( \frac{\partial \Sigma^*}{\partial \rho} < \infty \), equation (47) will become

\[
\frac{\partial \Sigma^*}{\partial \rho} \bigg|_{\beta = 1} = -\frac{\partial p_2^*}{\partial \rho} (v^* - p_2^*),
\]

requiring the expressions for \( \lim_{\rho \to 1} v^* \bigg|_{\beta = 1} \) and \( \lim_{\rho \to 1} \frac{\partial p_2^*}{\partial \rho} \bigg|_{\beta = 1} \). By (33),

\[
\frac{\partial v^*}{\partial \rho} = -\frac{\partial Y^*}{\partial \rho} \cdot \frac{2Y^* - \frac{n}{n+1} \left( 2 - v^* - \frac{c}{\beta} \right)}{\frac{n}{n+1} \left( nY^* + (n - 1) \left( \frac{p_1}{\beta} - 1 \right) \right)},
\]

which, after substituting \( \beta = 1 \), \( \lim_{\rho \to 1} \frac{\partial Y^*}{\partial \rho} \bigg|_{\beta = 1} = -n(p_1 - c) \) (by Lemma 9), canceling \( \frac{1}{n+1} \), and considering the limit as \( \rho \to 1 \), becomes \( \lim_{\rho \to 1} \frac{\partial Y^*}{\partial \rho} \bigg|_{\beta = 1} = n(p_1 - c). \lim_{\rho \to 1} \frac{2Y^*(n+1) - (n-1)(2-v^*-c)}{nY^*+(n-1)(1-p_1)} \bigg|_{\beta = 1} \). Using \( \lim_{\rho \to 1} Y^* \bigg|_{\beta = 1} = 1 - p_1 \), we see that the denominator tends to 1 - \( p_1 \). Thus, \( \lim_{\rho \to 1} \frac{\partial Y^*}{\partial \rho} \bigg|_{\beta = 1} \) is finite for any \( n < \bar{n} \). Using (48) and \( \lim_{\rho \to 1} v^* = p_1 + n(p_1 - c) \) (Proposition 5), we get

\[
\frac{\partial \Sigma^*}{\partial \rho} \bigg|_{\beta = 1} = \left. \frac{\partial p_2^*}{\partial \rho} \left( p_1 + n(p_1 - c) - p_1 \right) \right|_{\beta = 1} = \left. \frac{\partial p_2^*}{\partial \rho} n(p_1 - c) \right|_{\beta = 1} = n(p_1 - c) \lim_{\rho \to 1} \frac{\partial Y^*}{\partial \rho} = -n^2(p_1 - c)^2.
\]
Part (2): Equation (47) with \( \rho = 0 \) (implying \( v^* = p_1 \)) is

\[
\frac{\partial \Sigma^*}{\partial \rho} \bigg|_{\beta=1} = (p_1 - p_2^*) \left[ \frac{\partial v^*}{\partial \rho} - \frac{\partial p_2^*}{\partial \rho} \right] = [Y^* - (1 - p_1)] \left[ \frac{\partial v^*}{\partial \rho} + \frac{\partial Y^*}{\partial \rho} \right].
\]

(50)

The derivative of (21) in \( \rho \) (with \( \beta = 1 \)) results in

\[
\frac{\partial v^*}{\partial \rho} = \frac{1}{(1 - \rho)^2} \left[ -\left(p_2^* + \rho \frac{\partial p_2^*}{\partial \rho}\right) (1 - \rho) + (p_1 - \rho p_2^*) \right] = \frac{1}{(1 - \rho)^2} \left[ p_1 - p_2^* + \rho(1 - \rho) \frac{\partial p_2^*}{\partial \rho} \right],
\]

which, for \( \rho = 0 \), given \( p_2^* = 1 - Y^* \), is \( \frac{\partial v^*}{\partial \rho} \bigg|_{\rho=0} = Y^* - (1 - p_1) \). By (49) with \( \beta = 1 \) and \( \rho = 0 \),

\[
\frac{\partial Y^*}{\partial \rho} = -\frac{\partial v^*}{\partial \rho} \cdot \frac{n Y^* - (n - 1) (1 - p_1)}{2 Y^* (n + 1) - n (1 - p_1 + 1 - c)},
\]

and (50) becomes

\[
\frac{\partial \Sigma^*}{\partial \rho} \bigg|_{\beta=1} = [Y^* - (1 - p_1)]^2 \left[ 1 - \frac{n Y^* - (n - 1) (1 - p_1)}{2 Y^* (n + 1) - n (1 - p_1 + 1 - c)} \right].
\]

(51)

Part (2.1): For \( n = 1 \), Corollary 3 with \( \beta = 1 \) and \( \rho = 0 \) yields \( Y^* = 1 - \frac{1}{2} (c + p_1) \) and (51) is

\[
\frac{\partial \Sigma^*}{\partial \rho} = \left[ \frac{1}{2} (p_1 - c) \right]^2 \left[ 1 - \frac{Y^*}{2 Y^* - [2 - (p_1 + c)] + 2 Y^*} \right] = \frac{1}{8} (p_1 - c)^2.
\]

Part (2.2): After collecting the terms with \( n \) and passing to the limit as \( n \to \infty \), (51) becomes

\[
\lim_{n \to \infty} \frac{\partial \Sigma^*}{\partial \rho} \bigg|_{\beta=1} = [Y^* - (1 - p_1)]^2 \left[ 1 - \frac{Y^* - (1 - p_1)}{2 Y^* - (1 - p_1 + 1 - c)} \right] = \frac{[Y^* - (1 - p_1)]^2 [Y^* - (1 - c)]}{2 Y^* - (1 - p_1 + 1 - c)},
\]

where (by (23) with \( \beta = 1, \rho = 0 \), and \( n \to \infty \)) \( Y^* \) is the larger root of (46), which implies that \( Y^* - (1 - p_1 + 1 - c) = -\frac{(1-p_1)^2}{Y^*} \) and

\[
\lim_{n \to \infty} \frac{\partial \Sigma^*}{\partial \rho} \bigg|_{\beta=1} = \frac{[Y^* - (1 - p_1)]^2 Y^* - (1 - c)}{Y^* - (1 - p_1)^2}
\]

yielding the result of part (2.2). Note also, that by Corollary 4, for \( n = \infty \), the second period is never profitable \( (Y^* > 1 - \xi \Leftrightarrow p_2^* < c) \) implying that for \( \rho = 0 \), \( \lim_{n \to \infty} \frac{\partial \Sigma^*}{\partial \rho} \bigg|_{\beta=1} > 0 \).

A.29. Proof of Corollary 10 (Non-monotonicity of \( W^* \) in \( n \)). By the definition of \( W^* \),

\[
\frac{\partial W^*}{\partial n} = \frac{\partial \Sigma^*}{\partial n} + \frac{\partial \Sigma^*}{\partial v^*} \frac{\partial v^*}{\partial n},
\]

and, using (43) for \( \frac{\partial \Sigma^*}{\partial n} \), (37) for \( \frac{\partial \Sigma^*}{\partial v^*} \), and equalities \( \frac{\partial v^*}{\partial n} = \frac{\rho \beta}{1 - \rho \beta} \frac{\partial Y^*}{\partial n} \) (by (21)) and \( \frac{\partial p_2^*}{\partial n} = -\beta \frac{\partial Y^*}{\partial n} \), we get

\[
\frac{\partial W^*}{\partial n} = \frac{\partial Y^*}{\partial n} \left[ \frac{\rho [\beta v^* - p_2^* - (v^* - p_1)] + (\beta v^* - p_2^*) \left( \frac{1}{\beta} - \rho \right) + 2 - p_1 - c \beta - \rho (p_1 - c) - 2 Y^*}{1 - \rho \beta} \right].
\]

Since, by Proposition 4, \( \frac{\partial Y^*}{\partial n} > 0 \) for RESE3, the sign of \( \frac{\partial W^*}{\partial n} \) coincides with the sign of the curly bracket in the RHS, i.e., \( \frac{\partial W^*}{\partial n} \geq 0 \) is equivalent to

\[
\rho [\beta v^* - p_2^* - (v^* - p_1)] + (\beta v^* - p_2^*) \left( \frac{1}{\beta} - \rho \right) + 2 - p_1 - c \beta - \rho (p_1 - c) - 2 Y^* \geq 0,
\]

which, after canceling like terms and, by (3), substitution \( Y^* = 1 - \frac{p_2^*}{\beta} \), becomes \( \frac{p_2^* - c}{\beta} + [v^* - p_1 - \rho (v^* - c)] \geq 0 \). Then, using \( v^* = \frac{p_1 - \rho p_2^*}{1 - \rho \beta} \), the inequality for \( n W \) (dependence of \( p_2^* \) on \( n W \) is omitted) becomes
The structure and conditions of REE existence. We assume \( v > c \), i.e., the high-cost firm may have second-period consumers with \( v > c \).

REE1: In this case, \( v_{\min} = 1 \) and, using formula (8) for profit with \( c = cH \) and \( y^i = y^H \), the profit of the high-cost retailer is \( r^H = -cH y^H + \beta (1 - y^H - y^L) y^H \) and, similarly, \( r^L = -cL y^L + \beta (1 - y^L - y^H) y^L \). The first-order optimality conditions yield \( \frac{\partial r^H}{\partial y^H} = -cH + \beta (1 - y^H - y^L) \) and, in the same way, \( r^L = -cL + \beta (1 - y^L - y^H) \). When \( y^H \geq 0 \), these two equations give us \( y^H^* = \frac{1}{3} \left[ 1 - (2H - c^H) / \beta \right] \) and \( y^L^* = \frac{1}{3} \left[ 1 - (2L - c^L) / \beta \right] \). Inequality \( y^H \leq 0 \) is equivalent to \( 2cH - c^H \geq \beta \) or \( cH \geq \frac{3}{2}(c^H + \beta) \). In this case, \( y^H^* = 0 \) and retailer \( L \) is a monopolist with \( y^L^* = \frac{1}{2}(1 - c^L / \beta) \).

The expressions for \( y^L^* \) and \( y^H^* \) lead to \( y^* = y^L^* + y^H^* = \frac{2}{3}(1 - \alpha / \beta) \), where \( \alpha = \frac{1}{2}(c^L + cH) \), and \( p^* = \beta (1 - Y^* \beta / \beta) = \frac{1}{3} \left[ \beta + c^H + cL \right] \). Then \( r^H = y^H \cdot p^* = cH \). By Lemma 5, \( \Sigma^* = \beta \alpha / \beta ) \). The existence condition \( \alpha^* = 1 \) is equivalent, by Lemma 1, to \( p_1 - \beta \beta^* - 1 \geq 1 - \beta \beta^* \) is a candidate for REE2, which exists iff (i) there are local maxima of \( y^H \) and \( r^L \) at \( Y = 1 - p_1 \); (ii) this maxima are not less than possible maxima on the interval \( Y > 1 - \beta \).

Condition (i) for \( r^L \) is equivalent to \( \frac{\partial r^L}{\partial y^L} \) and, under REE2, the inventory of the high-cost retailer is separated from zero for any \( c^H < p_1 \) (there are no REE2 equilibria with \( H = 0 \), i.e., high-cost retail is pushed out of the market). Substitution for \( y^H = 1 - p_1 - y^L \) yields \( y^L \leq 1 - p_1 \left[ 1 - \frac{p_1 - \beta}{p_1 - 1} \right] \). In the same way, \( \frac{\partial r^L}{\partial y^L} \) and, using both conditions, \( y^L + y^H = 1 - p_1 \left[ \frac{p_1 - \beta}{p_1 - 1} \right] \) and \( y^L + y^H \leq 1 - \frac{\beta}{p_1 - 1} \). The upper bound for \( y^L \) implies that duopoly REE2 exists only if \( c^L > \beta \).

Condition (ii) follows from \( \frac{\partial r^L}{\partial y^L} \) and, by REE2, the existence of both \( c^L > \beta \) and \( \beta \), which follows from (15) with \( v_{\min} = p_1 \) and \( \beta \), and \( y^L = 1 - \frac{\beta}{p_1 - 1} \) for \( \beta \), by REE2, the existence of both \( c^L > \beta \) and \( \beta \), which follows from (15) with \( v_{\min} = p_1 \) and \( \beta \).
\( p_1 - s \, c^L - \beta p_1 \leq 1 \), which holds since \( \beta p_1 > s \) and \( c^L - \beta p_1 < c^L - s \) \( (\text{decreases in } x) \). Hence, REE2 exists if and only if \( p_1 \leq \frac{2c}{1+\beta} = P_2 \). Moreover, it can be shown that the lower \( p_1 \)-bound for REE1 in the case of monopoly \((y^H, 0) = 0 \) and \( \bar{c} = c^L \) always exceeds \( P_2 \) with \( \bar{c} = \frac{1}{2}(c^L + c^H) \).

REE3: In this case, \( v_{\min}(p_1 - \rho \beta (1-y^H-y^L)) \) and formula (9) for profit with \( c = c^H \) and \( y^L = y^H \) becomes

\[
\begin{align*}
    r^H &= -c^H y^H + p_1 \frac{y^H}{y^H+y^L} \left( 1 - v_{\min} \right) + \beta \left( 1 - y^H - y^L \right) y^H \left( 1 - \frac{1-v_{\min}}{y^H+y^L} \right). \tag{52}
\end{align*}
\]

The first-order conditions, by (12), are

\[
\begin{align*}
    \frac{\partial r^H}{\partial y^H} &= \beta \left( 1 - y^L \right) - c^H + \beta \left( 1 - v_{\min} \right) - 2\beta y^H + (p_1 - \beta)(1-v_{\min}) \frac{y^L}{Y^2} = 0 \tag{52}.
    \\
    \frac{\partial r^L}{\partial y^L} &= \beta \left( 1 - y^H \right) - c^L + \beta \left( 1 - v_{\min} \right) - 2\beta y^L + (p_1 - \beta)(1-v_{\min}) \frac{y^H}{Y^2} = 0. \tag{53}
\end{align*}
\]

Summing up these equations and multiplying by \(-Y/(3\beta)\), we obtain

\[
Y^2 - Y \frac{2}{3} \left( 2 - v_{\min} - \bar{c}/\beta \right) - \frac{1}{3} (p_1/\beta - 1) (1 - v_{\min}) = 0,
\]

which is equation (18) with \( n = 2 \) and \( c = \bar{c} \). Using the same argument as in the symmetric case, the total equilibrium inventory \( Y^* \) is the larger root of this equation where \( v_{\min} = v_{\min}(Y^*) = v^* \) (the same as in the symmetric case). Given \( Y^* \), the expressions for \( y^L, y^H \) follow from subtraction of (53) from (52), which yields

\[
y^L - y^H = \frac{(c^L-c^H)(y^*)^2}{(p_1 - \beta)(1-v^*) + \beta(Y^*)^2},
\]

Since

\[
y^L - y^H + Y^* = 2y^L \quad \text{and} \quad y^H = Y^* - y^L,
\]

we have

\[
y^L = \frac{1}{2} Y^* \left[ 1 + \frac{(c^H-c^L)(y^*)^2}{(p_1 - \beta)(1-v^*) + \beta(Y^*)^2} \right], \quad \text{and} \quad y^H = \frac{1}{2} Y^* \left[ 1 - \frac{(c^H-c^L)(y^*)^2}{(p_1 - \beta)(1-v^*) + \beta(Y^*)^2} \right].
\]

The equation for \( Y^* \) yields \((p_1 - \beta)(1-v^*) = 3\beta(Y^*)^2 - Y^* \frac{2}{3} (2 - v^* - \bar{c}/\beta)\), which leads to equilibrium inventories

\[
y^H = \frac{1}{2} Y^* \left[ 1 - \frac{c^H - c^L}{2\beta(2Y^* - 2 + v^*) + 2\bar{c}} \right] \quad \text{and} \quad y^L = \frac{1}{2} Y^* \left[ 1 + \frac{c^H - c^L}{2\beta(2Y^* - 2 + v^*) + 2\bar{c}} \right]. \tag{54}
\]

Since the denominator of the fraction in the expression for \( y^{H,L} \) decreases in \( \rho \) (it increases in \( Y^* \) and \( \frac{\partial Y^*}{\partial \rho} < 0 \) similarly to the symmetric case, shown in Proposition 4), there may exist such \( \rho^b \in (0,1) \) that

\[
y^H|_{\rho=\rho^b} = 0 \quad \text{while} \quad y^H|_{\rho=0} > 0 \quad \text{given other inputs fixed}.
\]

Condition \( y^H > 0 \) is equivalent to \( |v^*| > 0 \Leftrightarrow c^L > \beta(2Y^* - 2 + v^*) = \beta v^* - 2p_2 \), where \( \beta v^* \) is the highest consumer valuation in the second period.

Considering the case of \( y^H > 0 \) and following the proof of Theorem 1, REE3 exists iff \( \frac{2c}{1+\beta} < p_1 < 1 - \frac{2}{3} \rho(\beta - \bar{c}) \) and, for both retailers, one of the conditions holds: for the high-cost retailer:

(a.H) \( \frac{\partial r^H}{\partial p_1} \big|_{y^H=1-s/\beta-y^L+0} \leq 0 \), which is equivalent to \( y^L \leq \frac{(c^H-s)(1-s/\beta)}{(p_1 - \beta)(1-v^*)} \), or (b.H) condition (a.H) does not hold, \( Y^* < 1 - s/\beta \), and the equilibrium profit \( r^H \) is not less than the profit \( \bar{r}^H \) resulting from the deviation of this retailer from REE3 in such a way that \( \bar{Y}^H = \bar{y}^H + y^L > 1 - s/\beta \), where

\[
\bar{y}^H = \arg \max_{y^H>1-s/\beta-y^L} r^H.
\]

By (15), condition \( \frac{\partial r^H}{\partial p_1} \big|_{y^H=\bar{y}^H} = 0 \) is \( -(c^H-s) + y^L(p_1 - s)(1-v^*)/(\bar{Y}^H)^2 \) yielding

\[
\bar{Y}^H = \sqrt{\frac{y^L(p_1 - s)(1-v^*)}{c^H-s}}. \tag{14}
\]

Substitution of \( \bar{Y}^H \) and \( \bar{y}^H = \bar{Y}^H - y^L \) into (14) leads to \( \bar{r}^H = \bar{y}^H \left[ (p_1 - s)(1-v^*)/\bar{Y}^H - (c^H-s) \right] \).

Similarly, for the low-cost retailer, the conditions are

(a.L) \( Y^* \quad \text{and} \quad r^L \geq \bar{y}^L \left[ (p_1 - s)(1-v^*)/\bar{Y}^L - (c^L-s) \right] \), where \( \bar{y}^L = \bar{Y}^L - y^L \) and \( \bar{Y}^L = \sqrt{\frac{y^L(p_1 - s)(1-v^*)}{c^L-s}}. \)

Under REE3, the high-cost retailer does not supply inventory to the market \((y^H = 0)\) neither resulting in \( p_2 > s \) nor in \( p_2 = s \) if both conditions hold: (i) \( \frac{\partial r^H}{\partial p_1} \big|_{y^H=0} \leq 0 \) and (ii) \( \frac{\partial r^L}{\partial p_1} \big|_{y^H=1-s/\beta-y^L+0} \leq 0 \). By Corollary (3), \( y^L = 1 - \frac{1}{2}(v^{*L} + c^L/\beta) \), where \( v^{*L} = \frac{2c^L - \rho c^L}{2 - \rho^3} \), yielding

\[
y^L = 1 - \frac{1}{2} \frac{2c^L - \rho c^L + 2\rho c^L}{\beta(2 - \rho^3)} = 1 - \frac{c^L - \rho c^L + 2\rho c^L}{2 - \rho^3} = \frac{2c^L - \rho c^L}{2 - \rho^3} \frac{\beta(2 - \rho^3)}{2 - \rho^3}. \tag{55}
\]
A.31. Proof of Proposition 11 (low-cost inventory increasing in \( \rho \)). If \( p_1 > 1 - \frac{2}{3}(\beta - \bar{c}) \), then, similarly to part 3 of Proposition 3, there exists such \( \rho_1 = \frac{3}{2} \frac{1 - p_1}{\beta - \bar{c}} \) that if \( \text{REE3} \) exists, it exists for \( \rho < \rho_1 \) (\( y^{L, \ast} \mid \rho = 0 = y^{L, 3} \mid \rho = 0 \)) while \( \text{REE1} \) exists for \( \rho \geq \rho_1 \) (\( y^{L, 3} \mid \rho = 1 = y^{L, 1} \)). By (54), \( y^{L, 3} \mid \rho = 0 = \frac{1}{2} y^{\ast} \left[ 1 + \frac{c - c L}{2(2 Y^{\ast} + 2 - p_1)} \right] \), where \( y^{\ast} \) is the largest root of \( Y^{\ast}^{2} - Y^{\ast} \left( 2 - p_1 - \bar{c} \right) / \beta - 1 \frac{1}{3} (p_1 / \beta - 1) (1 - p_1) = 0 \), and, by part \( \text{REE1} \) of previous subsection, \( y^{L, 1} = \frac{1}{2} \left[ 1 - \left( 2 c L - c H \right) / \beta \right] \). Then \( y^{L, 3} \mid \rho = 0 < y^{L, 3} \mid \rho = 1 \) \( \Leftrightarrow 1 + \frac{c - c L}{2(2 Y^{\ast} + 2 - p_1) + 2 \bar{c}} < \frac{2(3 + c H - 2 c L)}{3 Y^{\ast}} \) yielding the result.

A.32. Proofs of auxiliary statements.

A.32.1. Proof of Lemma 7 (properties of the profit). Part (1.1) can be shown by direct substitution of \( y^{i} = 1 - \frac{2}{\beta} - Y^{-i} \) (which is strictly positive by the condition of part (1)) into the expressions for...
\[
\frac{\partial r_i}{\partial y_i} \bigg|_{y_i=1-\frac{s}{\beta}-Y^{-i}, 0} = \beta \left(1 - Y^{-i}\right) - c + \beta \left(1 - v_{\text{min}}\right) - 2\beta \left(1 - \frac{s}{\beta} - Y^{-i}\right) + Y^{-i} \frac{(p_1 - \beta)(1 - v_{\text{min}})}{(1 - \frac{s}{\beta})^2} \\
= -c - \beta v_{\text{min}} + 2s + Y^{-i} \left(\beta + \frac{(p_1 - \beta)(1 - v_{\text{min}})}{(1 - \frac{s}{\beta})^2}\right),
\]

\[
\frac{\partial r_i}{\partial y_i} \bigg|_{y_i=1-\frac{s}{\beta}-Y^{-i}+0} = -c + s + Y^{-i} \frac{(p_1 - s)(1 - v_{\text{min}})}{(1 - \frac{s}{\beta})^2}.
\]

These expressions imply that part (1.1) holds if and only if
\[
s - \beta v_{\text{min}} + Y^{-i} \left(\beta + \frac{(p_1 - \beta)(1 - v_{\text{min}})}{(1 - \frac{s}{\beta})^2}\right) < Y^{-i} \frac{(p_1 - s)(1 - v_{\text{min}})}{(1 - \frac{s}{\beta})^2},
\]

which is equivalent to
\[
s - \beta v_{\text{min}} < Y^{-i} \left[\frac{(\beta - s)(1 - v_{\text{min}})}{(1 - \frac{s}{\beta})^2} - \beta\right] = Y^{-i} \left[\frac{\beta(1 - v_{\text{min}})}{1 - \frac{s}{\beta}} - \beta\right] = Y^{-i} \frac{(s - \beta v_{\text{min}})}{1 - \frac{s}{\beta}},
\]

which holds because \(s < \beta v_{\text{min}}\) and, by condition of part (1), \(Y^{-i} < 1 - s/\beta\).

Since \(r^i\) is continuous, i.e. \(r^i(1 - \frac{s}{\beta} - Y^{-i}, 0) = r^i(1 - \frac{s}{\beta} - Y^{-i} + 0)\), and we can show part (1.2) using either (10) or (14). From (14), \(r^i(1 - \frac{s}{\beta} - Y^{-i})\) is
\[
\left(1 - \frac{s}{\beta} - Y^{-i}\right) \left[s + \frac{(p_1 - s)(1 - v_{\text{min}})}{1 - \frac{s}{\beta}}\right] = \left(1 - \frac{s}{\beta} - Y^{-i}\right)(c - s) \left[\frac{(p_1 - s)(1 - v_{\text{min}})}{(1 - \frac{s}{\beta})(c - s)} - 1\right],
\]

which yields the result of part (1.2).

For part (1.3), rewrite the second derivative (13) of \(r^i\) as \(\frac{\partial^2 r^i}{\partial y_i^2} = -2\beta \left[\beta Y^3 + (p_1 - \beta)(1 - v_{\text{min}})Y^{-i}\right].\)

Since \(Y \geq 0\), the RHS of this equation is negative (\(r^i\) is strictly concave) if and only if \(\beta Y^3 + (p_1 - \beta)(1 - v_{\text{min}})Y > 0\). Equality \(Y = 1 - v_{\text{min}}\) holds only at the left boundary of the domain of the profit function. For all other points in the domain \(Y > 1 - v_{\text{min}} \geq 0\) and we have
\[
\beta Y^3 + (p_1 - \beta)(1 - v_{\text{min}})Y > \beta(1 - v_{\text{min}})^2Y + (p_1 - \beta)(1 - v_{\text{min}})Y = [\beta(1 - v_{\text{min}}) + p_1 - \beta](1 - v_{\text{min}})Y = [p_1 - \beta v_{\text{min}}](1 - v_{\text{min}})Y \geq 0
\]

if \(p_1 \geq \beta v_{\text{min}}\) (a sufficient condition for strict concavity of \(r^i\)).

Suppose \(p_1 < \beta v_{\text{min}}\). Although \(r^i\) may be non-concave in this case, \(\frac{\partial^2 r^i}{\partial y_i^2} = -2\beta \left[1 + (p_1 - \beta)(1 - v_{\text{min}})Y^{-i}\right]\) is monotonically decreasing in \(y_i\). Therefore, if \(r^i\) has an inflection point, this point is unique and corresponds to the total supply level \(\bar{Y}\) such that \(\bar{Y}^3 = (1 - \frac{p_1}{\beta})(1 - v_{\text{min}})Y^{-i}\).

Consider an extension \(\tilde{r}^i\) of \(r^i\) in the form (10) to the domain \(y_i \geq (1 - v_{\text{min}} - Y^{-i})^+.\) In terms of the total supply, this domain is equivalent to \(Y \geq (1 - v_{\text{min}}) \vee Y^{-i}.\) We will prove that \(\tilde{r}^i\) is pseudoconcave implying the claim of part (1.3) for the case of \(p_1 < \beta v_{\text{min}}\).

Equation (10), divided through by \(y_i\), implies that \(\tilde{r}^i = 0\) if and only if \(y_i = 0\) or \(\beta (1 - Y) - c + \beta (1 - v_{\text{min}}) + (p_1 - \beta)(1 - v_{\text{min}})Y^{-i} = 0.\) After multiplying by \(-Y/\beta\), this equation becomes
\[
Y^2 - \left(2 - \frac{c}{\beta} - v_{\text{min}}\right)Y + \left(1 - \frac{p_1}{\beta}\right)(1 - v_{\text{min}}) = 0.
\]

Its properties are explored in the following lemma.
Lemma 11. For any feasible values of $c, s, v^\text{min}$, and $p_1 < \beta$, the real roots $Y_{1,2}$ of equation (55) exist and satisfy the conditions: $0 \leq Y_1 \leq 1 - v^\text{min} < Y_2 \leq 2 - \left( \frac{s}{\beta} + v^\text{min} \right)$ with $Y_1 = 1 - v^\text{min}$ only if $v^\text{min} = 1$.

By Lemma 11, the roots $Y_{1,2}$ of (55) always exist and $0 \leq Y_1 \leq 1 - v^\text{min} < Y_2$, where $Y_1 < 1 - v^\text{min}$ unless $v^\text{min} = 1$. Using these roots, we can express $\tilde{r}^i$ as the following function of $Y$: $\tilde{r}^i = -\frac{\beta}{2}(Y - Y^{-i})(Y - Y_1)(Y - Y_2)$. Moreover, by (55), $Y_1Y_2 = \left(1 - \frac{p_1}{\beta}\right)(1 - v^\text{min})$, and the inflection point has the form $\tilde{Y}^\beta = Y_1Y_2Y^{-i}$, i.e., $\tilde{Y}$ is the geometric mean of $Y_1$, $Y_2$, and $Y^{-i}$. Since the second derivative is decreasing, $\tilde{r}^i$ is strictly concave to the right of $\tilde{Y} - Y^{-i}$.

There are three possible locations of $Y^{-i}$ relative to $Y_1 < Y_2$. First, if $Y^{-i} \geq Y_2$, then $1 - v^\text{min} < Y^{-i}$, $\tilde{Y} < Y^{-i}$, and $\tilde{r}^i$ is nonpositive and strictly concave for all $y^i \geq (1 - v^\text{min} - Y^{-i})^\dagger$. In this case, the claim of part (1.3) holds.

Second, if $Y^{-i} \leq Y_1$, then $Y^{-i} \leq 1 - v^\text{min}$, $\tilde{Y} < Y_2$, $\tilde{r}^i$ is nonnegative for $(1 - v^\text{min} - Y^{-i})^\dagger \leq y^i \leq Y_2 - Y^{-i}$ and nonpositive for $y^i \geq Y_2 - Y^{-i}$. Since $r^i$ is concave for $y^i \geq \tilde{Y} - Y^{-i}$ and changes its sign from positive to negative at $Y_2 - Y^{-i} \geq \tilde{Y} - Y^{-i}$, it is also decreasing for all $y^i \geq Y_2 - Y^{-i}$.

Third, if $Y_1 < Y^{-i} < Y_2$, it is still true that $\tilde{Y} < Y_2$, $\tilde{r}^i$ is nonnegative for $(1 - v^\text{min} - Y^{-i})^\dagger \leq y^i \leq Y_2 - Y^{-i}$, and nonpositive, decreasing and strictly concave for $y^i \geq Y_2 - Y^{-i}$. It is also true that, when $(1 - v^\text{min}) \vee Y^{-i} < \tilde{Y}$, $\tilde{r}^i$ is convex in the interval $[(1 - v^\text{min} - Y^{-i})^\dagger, \tilde{Y} - Y^{-i}]$.

We combine the cases two and three by observing that in both of them $\tilde{r}^i$ is nonnegative for $[(1 - v^\text{min} - Y^{-i})^\dagger, Y_2 - Y^{-i}]$ and decreasing as well as concave for $y^i \geq Y_2 - Y^{-i}$. Thus, there is no local minimum for $y^i \geq Y_2 - Y^{-i}$. We complete the proof of part (1.3) using the following lemma.

Lemma 12. If $\tilde{r}^i$ has an internal (local) minimum $(y^i)_{\text{min}}$, then $\tilde{r}^i((y^i)_{\text{min}}) < 0$.

Lemma 12 implies that $\tilde{r}^i$ has no local minimum in the interval $((1 - v^\text{min} - Y^{-i})^\dagger, Y_2 - Y^{-i})$. Thus, $\tilde{r}^i$ has no internal minima in its entire domain, is strictly increasing when it is convex and, therefore, is pseudoconcave.

Parts (1.4) and (2) follow directly from (15).

Part (1.5) immediately follows from parts (1.3) and (1.4). Indeed, condition $\frac{\partial r^i}{\partial y^i}\bigg|_{y^i=1-\frac{s}{\beta}-Y^{-i}+0} \leq 0$ implies that $r^i$ is decreasing for $y^i \geq 1 - \frac{s}{\beta} - Y^{-i}$ (by concavity on this interval). Combining this observation with pseudoconcavity for $y^i \leq 1 - \frac{s}{\beta} - Y^{-i}$, we obtain pseudoconcavity for the entire domain. Similarly, $\frac{\partial r^i}{\partial y^i}\bigg|_{y^i=1-\frac{s}{\beta}-Y^{-i}-0} \geq 0$ implies that $r^i$ is strictly increasing for $y^i \leq 1 - \frac{s}{\beta} - Y^{-i}$, again, leading to pseudoconcavity for the entire domain.

A.32.2. Proof of Lemma 8. The equilibrium profit, using (10) with $y^i = \frac{Y^*}{n}$ and the expression for $v^\text{min} = v^*$, is $r^* = \frac{Y^*}{n}\left[\beta (1 - Y^*) - c + \left(\beta + \frac{p_1 - \beta}{1 - \beta}\right)\left(1 - \frac{(p_1 - \rho\beta)(1 - Y^*)}{n(1 - \rho\beta)}\right)\right]$. After factoring out $\frac{\beta}{n(1 - \rho\beta)}$ and collecting the terms with different powers of $Y^*$, we obtain (30).

A.32.3. Proof of Lemma 9. The expression for $\lim_{\rho \to 1} \frac{\partial Y^*}{\partial \rho}$ at $\beta = 1$ can be found by the implicit differentiation in (23). For brevity, we omit explicit notation indicating $\beta = 1$ throughout the proof. Denote $b_1(\rho) \triangleq \frac{(1-c)n(1-\rho)+((1-p_1)n-(p_1-1)n)(n-1)}{n+1-\rho}$ and differentiate (23) with respect to $\rho$ to obtain $2Y^* \frac{\partial Y^*}{\partial \rho} - \frac{\partial Y^*}{\partial \rho} b_1(\rho) - Y^* \frac{\partial b_1(\rho)}{\partial \rho} + \frac{1-p_1}{\rho} \frac{\partial (1-p_1)^2(n-1)}{n+1-\rho} = 0$ and

$$\frac{\partial Y^*}{\partial \rho} = \frac{2Y^* - b_1(\rho)}{Y^* - b_1(\rho)} = \frac{Y^* \frac{\partial b_1(\rho)}{\partial \rho} - \frac{(1-p_1)^2(n-1)}{(n+1-\rho)^2}}{Y^* - b_1(\rho)} \cdot \frac{n+1-\rho}{(n-1)\rho}.$$

The limits are \( \lim_{\rho \to 1} b_1(\rho) = \frac{(1-p_1)n+(1-p_1)(n-1)}{n} = 2(1-p_1) - \frac{1-p_1}{n} \), resulting in \( \lim_{\rho \to 1} [2Y^* - b_1(\rho)] = 1 \), and for \( \frac{\partial b_1(\rho)}{\partial \rho} = \frac{-(1-c)n+(1-p_1)(n-1)(n+1-\rho)}{n(n+1-\rho)^2} \), it is
\[
\lim_{\rho \to 1} \frac{\partial b_1(\rho)}{\partial \rho} = \frac{1}{n^2} \left\{ -(1-c)n + (1-p_1)(n-1) \right\} n + (1-p_1)n + (1-p_1)(n-1) = \frac{1}{n^2} \left\{ (1-p_1)(n-1) - (p_1-c) \right\}
\]
The limit of the RHS of (56) is \( \frac{1-p_1}{n^2} \left\{ (1-p_1)(n-1) - (p_1-c)n^2 \right\} - \frac{(1-p_1)^2(n-1)}{n^2} = - (1-p_1)(p_1-c) \).

A.32.4. Proof of Lemma 11 (the roots of \( r^i(Y) = 0 \)). The discriminant of (55) is \( D = 2 - \frac{c}{\beta} - v_{\min}^2 - 4 \left( 1 - \frac{p_1}{\beta} \right)(1-v_{\min}) \geq \left( 2 - \frac{c}{\beta} - v_{\min} \right)^2 - 4 \left( 1 - \frac{c}{\beta}(1-v_{\min}) \right) = \left( v_{\min} - \frac{c}{\beta} \right)^2 \geq 0 \), where the first inequality is strict unless \( v_{\min} = 1 \) because \( p_1 > c \), while the second inequality is strict unless \( v_{\min} = \frac{c}{\beta} \). Therefore, \( D > 0 \), the real roots given by \( Y_{1,2} = \frac{1}{2} \left( 2 - \frac{c}{\beta} - v_{\min} \pm \sqrt{D} \right) \) always exist, and \( Y_1 < Y_2 \). Since \( p_1 < \beta \), we have \( 4 \left( 1 - \frac{p_1}{\beta} \right)(1-v_{\min}) \geq 0 \) and \( Y_{1,2} \in \left( 0, 2 - \frac{c}{\beta} - v_{\min} \right) \).

If \( v_{\min} = 1 \), the roots are \( Y_1 = 0, Y_2 = 1 \), and the claim of the lemma holds.

If \( v_{\min} < 1 \), then \( D > \left( v_{\min} - \frac{c}{\beta} \right)^2 \), and an upper bound on \( Y_1 \) is \( Y_1 < 1 - \frac{1}{2} \left( \frac{c}{\beta} + v_{\min} \right) \). The roots are
\[
1 - \frac{1}{2} \left( \frac{c}{\beta} + v_{\min} \right) \leq 1 - v_{\min}, \text{ which, in turn, is a lower bound on } Y_2 : Y_2 > 1 - \frac{1}{2} \left( \frac{c}{\beta} + v_{\min} \right) \geq 1 - v_{\min}.
\]

A.32.5. Proof of Lemma 12. Function \( \tilde{r}^i \), its first and second derivatives are given, respectively, by (9), (11) and (13). If an internal local minimum of \( \tilde{r}^i \) exists, it must satisfy the necessary second-order optimality conditions
\[
\frac{\partial \tilde{r}^i}{\partial y^i} \bigg|_{y^i=(y^i)_{\min}} = 0, \text{ and } \frac{\partial^2 \tilde{r}^i}{\partial (y^i)^2} \bigg|_{y^i=(y^i)_{\min}} \geq 0. \tag{58}
\]

Using condition (57) and the expression for the first derivative of \( \tilde{r}^i \), we obtain
\[
\beta (1-Y) - c + [p_1 - \beta (1-Y)] \frac{1-v_{\min}^2}{Y^2} = - (y^i)_{\min} \beta \left[ -1 + \frac{1-v_{\min}^2}{Y^2} - (p_1 - (1-Y)) \frac{1-v_{\min}^2}{Y^2} \right] \]
\[
= (y^i)_{\min} \beta \left[ 1 + \left( \frac{p_1}{\beta} - 1 \right) \frac{1-v_{\min}^2}{Y^2} \right] \tag{59}
\]

Since the LHS of (59) multiplied by \( y^i \) matches the expression for \( \tilde{r}^i \), it follows that
\[
\tilde{r}^i \bigg|_{y^i=(y^i)_{\min}} = (y^i)_{\min} \beta \left[ 1 + \left( \frac{p_1}{\beta} - 1 \right) \frac{1-y_{\min}^2}{Y^2} \right]. \tag{60}
\]

Condition (58) and the expression for the second derivative of \( \tilde{r}^i \) imply that, at \( y^i = (y^i)_{\min}, \left( \frac{p_1}{\beta} - 1 \right) \left( 1-v_{\min}^2 \right) \leq - \frac{3}{Y^2}. \) Combining this inequality with (60), we obtain \( \tilde{r}^i \bigg|_{y^i=(y^i)_{\min}} \leq (y^i)_{\min} \beta \left[ 1 - \frac{Y^2}{3} \right] < 0 \), which is strict because, here, we consider only \( y^i > 0 \).

Appendix B. First-period demand: general case

This section provides the derivation of the functional form of the first-period demand (2) and examines the robustness of the main results, obtained for \( \gamma = 1 \), with respect to variations in \( \gamma \).
B.1. Model specification. Retailer $i$ demand can be expressed as $d^i(y^i, y^{-i}) = D m^i(y^i, y^{-i})$, where $D$ is the total demand, $m^i(y^i, y^{-i})$ — the market share of retailer $i$, and $y^{-i}$ — the vector of inventories of the others. Since, by the assumptions of §3.1, attractions $a^i(y^i)$ are identical: $a^i(y^i) = a(y^i), i \in I$, and for a non-trivial problem some of $y^i$ are positive, attraction vector $a$ of all $a^i, i \in I$ satisfies four conditions required for the market share theorem (Bell D.E., Keeney R.L., Little J.D.C. (1975). A market share theorem. Journal of Marketing Research, 12(2), 136-141): (A1) $a$ is nonnegative and nonzero: $a^i \geq 0, j \in I$, and there exists $a^i > 0$; (A2) zero attraction leads to zero market share; (A3) any two retailers with equal attraction have equal market share: $a^i(y^i) = a^j(y^j) \Rightarrow m^i(y^i, y^{-i}) = m^j(y^j, y^{-j})$; and (A4) the market share $m^i$ of any retailer decreases on the same amount $\Delta^i$ if the attraction $a^j$ of any other retailer $j$ is increased by a fixed amount ($\Delta^i$ does not depend on $j \neq i$). Then, by the market share theorem applied to this symmetric case, $m^i$ has the following functional form:

$$m^i(y^i, y^{-i}) = \frac{a(y^i)}{\sum_{j \in I} a(y^j)}.$$  

Using (61), the homogeneity of $m^i$ (follows, by Assumption 1, from the homogeneity of $d^i$ and $D$), and the continuity of $a(y^i)$ (Assumption 3), Lemma 2 specifies the functional form of attraction: $a(y) = a(1)(y)^\gamma$. By choosing the scale of attraction so that $a(1) = 1$, we obtain functional form (2) for demand $d^i$.

A feasible range for $\gamma$ results from the observation that retailer $i$ can choose $y^i$ either not to enter the market: $y^i = 0 = a(0) = d^i(0, y^{-i})$, to sell only in the first period: $y^i = \bar{y}^i \triangleq d^i(\bar{y}^i, y^{-i})$, or in both: $y^i > d^i(y^i, y^{-i}) \geq \bar{y}^i$ (the last inequality is strict when $d^i$ is strictly increasing in $y^i$). These properties hold if $d^i$ is concave in $y^i$. In extreme cases, $d^i$ as a function of $y^i$, can be a straight line with a slope less than one ($\gamma = 1$) if $y^i \geq \bar{y}^i$ or, as an opposite case, a constant if all $y^i$ are positive and any changes in $y^i$ are not supported by the correspondent changes in market efforts or consumers completely ignore these efforts ($\gamma = 0$).

In this model, $\gamma$ is the inventory elasticity of attraction: $E_y(a) \triangleq \frac{\partial d^i}{\partial y^i} = \gamma (y)^{\gamma - 1} \frac{y^i}{(y^i)^\gamma} = \gamma, or the inventory elasticity of the first-period demand, normalized by the market share of other retailers:

$$E_{y^i}(d^i) \triangleq \frac{\partial d^i}{\partial y^i} d^i = D \left[ \frac{\gamma (y^i)^{\gamma - 1} - (y^i)^\gamma \gamma (y^i)^{\gamma - 1}}{\sum_{j \in I} (y^j)^\gamma} \right] \frac{(y^i)^\gamma D \sum_{j \in I} (y^j)^\gamma}{\sum_{j \in I} (y^j)^\gamma} \gamma \left[ 1 - m^i \right] = \gamma \frac{\sum_{j \neq i} (y^j)^\gamma}{\sum_{j \in I} (y^j)^\gamma},$$

where $\sum_{j \neq i} (y^j)^\gamma / \sum_{j \in I} (y^j)^\gamma$ is the market share of other retailers.

The following results use some supplementary material, provided in §B.3.

B.2. Changes in RESE structure with $\gamma$. This section shows the effect of changing $\gamma$ on the main results of this paper. For $\gamma = 1$, the structure of RESE coincides with the one described in Theorems 1 and 2. This structure continuously changes with $\gamma$ by continuity of demand (2). In particular, changes in $\gamma$ lead to the following effects.

I. RESE1 does not depend on $\gamma$ because this RESE, by the same argument as in the proof of Theorem 1, exists only when the first-period demand is zero ($v_{min} = 1$) due to a combination of relatively high $p_1$, the difference $\beta - c$, the level of competition $n$, and strategic behavior $\rho$; namely, when $p_1 \geq P_1 = 1 - \frac{n}{n+1}\rho(\beta - c)$.

II. The area of RESE2 is decreasing in $\gamma$, which follows from a necessary condition of existence of RESE2 that requires the profit of a deviator from $Y^{*2} = 1 - p_1$ be not increasing in $y^i$: $\frac{\partial v^i}{\partial y^i} \bigg|_{y^i=\frac{1-p_1}{n+1}} \leq 0$. This inequality ($\S B.3.1$) is equivalent to $p_1 \leq \frac{n c}{\gamma(n+1)+\beta n(1-\gamma)+\gamma} \triangleq P_2(\gamma)$. This bound decreases in $\gamma$ from $P_2(0) = \frac{c}{\beta}$ to $P_2(1) = \frac{n c}{n-1+\beta}$ coinciding with $P_2$ given in Theorem 1.

III. The area of RESE3 is (a) increasing in $\gamma$ along the boundary with RESE2 and (b) decreasing along the boundary with RESE4. Part (a) follows from the $p_1$-range for RESE3: $P_2(\gamma) < p_1 < P_1$,
which results from the same geometric argument as in the proof of Theorem 1 because $Y^{*,3} (\gamma)$ is still a larger root of a quadratic equation with coefficients depending on $\gamma$ (equation (68)). Part (b), for $\gamma = 0$, follows from the lack of incentive for the retailers to deviate to salvage, which is expressed in $\frac{\partial v}{\partial \rho} \bigg|_{Y \geq 1 - \frac{s}{\beta}} = s - c < 0$ (§B.3.1), i.e., a sufficient condition, corresponding to condition (a) in part RESE3 of Theorem 1 always holds. The intuition is that, for $\gamma = 0$, retailers share evenly the first-period demand regardless of the inventories. Therefore, any increase in inventory does not increase the first-period market share, and possible second-period sales below cost only reduce total two-period profit. For $0 < \gamma < 1$, part (b) is checked numerically and illustrated in Figure 17 for $\gamma \in \{0, 0.4, 1\}$.

IV. The area of RESE4 is increasing in $\gamma$. §B.3.1 provides a unique

\[ Y^{*,4} (\gamma) = \frac{n - 1}{n} p_1 - s \gamma (1 - v^{*,4}), \]  

(62)

where $v^{*,4} = \frac{p_1 - \rho s}{1 - \rho \beta}$. This expression for $Y^{*,4} (\gamma)$ implies a sufficient condition of RESE4 existence, namely, $\frac{n - 1}{n} Y^{*,4} (\gamma) \geq 1 - \frac{s}{\beta}$ (salvaging is forced on retailers), which is

\[ \gamma \geq \tilde{\gamma} \Delta \left(1 - \frac{s}{\beta}\right) \left(\frac{n}{n - 1}\right)^2 \frac{c - s}{p_1 - s} \frac{1 - \rho \beta}{1 - \rho (\beta - s)}, \]

where $\tilde{\gamma}$ can be sufficiently small for any feasible $p_1, \rho, \beta$, and $s$ if $c$ is sufficiently close to $s$, i.e., RESE4 can exist for small $\gamma$ but does not exist for $\gamma = 0$ (Figure 17). On the other hand, inequality $Y^{*,4} < 1 - \frac{s}{\beta}$, combined with (62), gives a sufficient condition of RESE4 non-existence. Since $Y^{*,4}$ is increasing in $n$ and decreasing in $\rho$ ($v^{*,4}$ is increasing in $\rho$), RESE4 does not exist for given $\gamma$ and any $n$ and $\rho$ if $Y^{*,4} < 1 - \frac{s}{\beta}$ for $\rho = 0$ and $n \to \infty$, which is $\frac{p_1 - s}{c - s} (1 - p_1) < 1 - \frac{s}{\beta}$ or $\gamma < \tilde{\gamma} \Delta \left(1 - \frac{s}{\beta}\right) (c - s) \frac{1 - \rho \beta}{p_1 - s (1 - \rho \beta)}$.

The scatterplots in Figure 17 were constructed by checking $p_1$-boundaries for RESE1 and 2, and, for RESE3 and 4, using the direct comparison of equilibrium profits with the profit of a potential deviator, according to the definition of RESE.

Since the first-period demand (2) is continuous and monotonic in $\gamma$, the case $\gamma = 0$ for RESE3 is of a special interest as opposing to $\gamma = 1$. Although, a complete independence of market share from inventory may be an idealization for many practical settings, this case illustrates the robustness of the results of this paper and shows the direction and amplitude of the changes with respect to variations in the demand patterns. This assumption about first-period market share was used, e.g., in Liu and van Ryzin (2008), §4.4.

**Proposition 12.** For $\gamma = 0$, a unique RESE 3 with $v^* = \frac{p_1 + n (p_1 - \rho c)}{1 + n (1 - \rho \beta)}, p_2^* = c + \frac{\beta p_1 - c}{1 + n (1 - \rho \beta)}, Y^* = \frac{1 - p_1 + n (1 - \frac{c}{\beta}) (1 - \beta \rho)}{1 + n (1 - \rho \beta)}$, and $r^* = \frac{1}{n} [(p_1 - c) (1 - v^*) + (p_2^* - c) (Y^* - 1 + v^*)]$ exists if and only if $\frac{c}{\beta} < p_1 < P_1$; no other equilibria exist in this area. Moreover,

1. $p_2^* \to c + 0$ with $n \to \infty$ for any $p_1 \in \left(\frac{c}{\beta}, P_1\right)$ or with $p_1 \to \frac{c}{\beta} + 0$ for any $n \geq 1$;
2. $v^*, p_2^*, Y^*$, and $r^*$ are continuous at the boundaries; monotonicity of $v^*, Y^*$ in $n$ and $\rho$, and $nr^*$ in $n$, stated in Proposition 4 for $\gamma = 1$ hold;
3. $nr^*$ is decreasing in $\rho$ if and only if either $n = 1$ or $p_1 \geq c + \frac{2n (\beta - c)}{(n + 1) \beta}$ for any $n > 1$;
4. $nr^*$ attains minimum in $\rho$ at $\rho^0 \Delta \frac{(n + 1)^2 (p_1 - c) - 2n (p_1 - c)}{\beta n + 1 (p_1 - c)}$ for any $n \in (1, n^0)$, where $n^0 \Delta \frac{\beta p_1 + p_1 (\beta - c) (\beta + c - 2p_1)}{p_1 c (\beta - c) (p_1 - c)} > 2$, if and only if $p_1 < \frac{5c + 4 \beta}{9}$;
5. When $\beta = 1$, $nr^* |_{p_1 = 1} < nr^* |_{p_1 = 0}$ for any RESE 3 inputs; the minimum possible value of $\rho^0 = \frac{n^2 + 1}{n (n + 1)}$ is $\tilde{\rho}^0 \Delta \rho^0 |_{n = 2} = \rho^0 |_{n = 3} = \frac{5}{6}$.
Figure 17. The \((\rho, p_1)\)-scatterplots of the areas where a particular RESE exists for \(n = 10, c = 0.1, s = 0.05\), and given \(\gamma\) and \(\beta\)

\[
\begin{array}{c}
\begin{array}{c}
\beta = 0.75, \gamma = 1.0 \\
\beta = 0.75, \gamma = 0.4 \\
\beta = 0.75, \gamma = 0.0 \\
\beta = 0.25, \gamma = 1.0 \\
\beta = 0.25, \gamma = 0.4 \\
\beta = 0.25, \gamma = 0.0
\end{array}
\end{array}
\]
Proposition 12 shows that, for $\gamma = 0$,
(i) RESE exists for all feasible model inputs since RESE3 boundaries ($\frac{c}{D} < p_1 < P_1$) complement the boundaries of RESE1 and 2;
(ii) the second-period price is always above the cost for $n < \infty$;
(iii) a closed-form necessary and sufficient condition shows when $nr^*$ is decreasing in $\rho$; 
(iv) there exist closed-form expressions for $\rho^0$, the unique minimum of $nr^*$ in $\rho$ (part 4), and for $n^0$, the upper boundary of $n$-range where $nr^*$ is non-monotonic in $\rho$;
(v) there is no effect of “boundary-value gain” (part 5); this result supports the conclusion, formulated in the discussion of Proposition 5, that under this effect, the maximum level of strategic behavior prevents the second-period sales at loss under competitive pressure ($n \geq 3$). As shown in part 1, the second-period sales are always profitable for $\gamma = 0$ because retailers have no incentive to compete for the first-period market by increasing inventories.

Thus, when the inventory elasticity of attraction $\gamma$ decreases, the “boundary-value gain” in $\rho$ becomes weaker (Figures 4 and 18 a) and disappears at $\gamma = 0$ (Figure 18 b); the “discontinuous gain” in $\rho$ caused by the switch from RESE4 to RESE3 emerges at lower $\rho$ (Figures 5 b and 17) and disappears at $\gamma = 0$ due to non-existence of RESE4; the “continuous gain” in $\rho$ (Figure 4) exists even for $\gamma = 0$. The last effect becomes less pronounced because decreasing $\gamma$ weakens the first-period inventory competition and decreases the correspondent second-period losses. The point of minimum profit, $\rho^0$, decreases in $\gamma$ (Figures 4 and 18).

B.3. Appendix B supplement.

B.3.1. Profit function, its properties and inventory decisions.
Retailer $i$ has no sales in the second period. In this case, the general formula (1) for profit becomes $r^i = (p_1 - c)y^i$, which yields a unique profit-maximizing inventory $y^i = \hat{y}^i = d^i$. Unlike $\gamma = 1$, other retailers may have sales in the second period, implying that, in general, $\bar{\alpha} \neq 0$ and $v^{\min} \geq p_1$. Using (2) with $y^j = \sum_{j \neq i}^{n^*} (y^j)^\gamma$ and $D = 1 - v^{\min}$, $\hat{y}^i$ is a root of a non-linear equation: $y^i = \frac{(1-v^{\min})(y^i)^\gamma}{(n-1)(Y^*/n)^\gamma + (y^i)^\gamma}$. After dividing by $y^i$, which eliminates the extraneous root $y^i = 0$, this equation can be written as $(n-1) \left( \frac{Y^*}{n} \right)^\gamma + (y^i)^\gamma = (1-v^{\min}) (y^i)^\gamma - 1$ or $(n-1) \left( \frac{Y^*}{n} \right)^\gamma = (y^i)^\gamma - 1 - v^{\min} - y^i$, which, for $n = 1$, yields $\hat{y}^i = 1 - v^{\min}$ for any $\gamma \in [0, 1]$. When $n > 1$, this equation can be written as

$$
(y^i)^{1-\gamma} = \frac{1}{n-1} \left( \frac{n}{Y^*} \right)^\gamma (1 - v^{\min} - y^i), \tag{63}
$$
which, for $\gamma = 1$, yields $\bar{y}^i = 1 - v^\min - \frac{n-1}{n} Y^*$. For $\gamma < 1$, this equation has a unique positive root since the LHS is zero at $y^i = 0$ and increasing in $y^i$, and the RHS is a decreasing linear function in $y^i$, which is positive at $y^i = 0$. For $\gamma = 0$, equation (63) results in $\bar{y}^i = \frac{1 - v^\min}{\rho}$, which is the maximum $\bar{y}^i$ in $\gamma$ by the following lemma.

**Lemma 13.** The solution of (63), $\bar{y}^i$, is decreasing in $\gamma$ if $\bar{y}^i < \frac{y^*}{n}$.

**Proof.** Equation (63) can be written as $\exp \left[ (1 - \gamma) \ln(\bar{y}^i) \right] = \exp \left[ \gamma \ln \left( \frac{n}{Y^*} \right) \right] \frac{1 - v^\min - \bar{y}^i}{n-1}$. The derivative of this equation in $\gamma$ is $(\bar{y}^i)^{1-\gamma} \left[ \frac{1 - v^\min - \bar{y}^i}{n-1} \right] \ln(\bar{y}^i) = \ln \left( \frac{n}{Y^*} \right) \frac{1}{n-1} (\frac{n}{Y^*})^{-\gamma} (1 - v^\min - \bar{y}^i) - \frac{1}{n-1} (\frac{n}{Y^*})^\gamma \partial \bar{y}^i = \ln(\bar{y}^i) (\bar{y}^i)^{-\gamma}$, which can be written as $\partial \bar{y}^i = [1 - (\bar{y}^i)^{1-\gamma}] \ln(\bar{y}^i)$, where the bracket $[\gamma]$, in the LHS is positive and the RHS, by (63), becomes $(\bar{y}^i)^{1-\gamma} \ln(\bar{y}^i) - \ln \left( \frac{n}{Y^*} \right)$, which is negative, leading to $\partial \bar{y}^i < 0$.

Retailer $i$ has sales in the second period, $p_2 > s$. Profit (1) with $q^i$, given by Lemma 3, becomes $r^i = p_1d^i + p_2(y^i - d^i) - c\bar{y}^i$, which, with $y^i = \frac{y^*}{n}$, $j \neq i$, $v^\min = v^*$, and $d^i$ from (2), can be written as

$$r^i = \left[ p_1 - p_2(y^i) \right] \frac{(1 - v^*)}{(n-1) \left( \frac{Y^*}{n} \right)^\gamma + (y^i)^\gamma} + \left[ p_2(y^i) - c \right] y^i$$

(64)

where, by (3), $p_2(y^i) = \beta \left[ 1 - \frac{n-1}{n} Y^* - y^i \right]$. The derivative, after simplifications, is

$$\frac{\partial r^i}{\partial y^i} = \frac{\beta(1 - v^*) (y^i)^{\gamma}}{(n-1) \left( \frac{Y^*}{n} \right)^\gamma + (y^i)^\gamma} + \left[ p_1 - p_2 \right] \frac{(1 - v^*) \gamma (y^i)^{\gamma-1} \left( \frac{Y^*}{n} \right)^\gamma}{\left[ (n-1) \left( \frac{Y^*}{n} \right)^\gamma + (y^i)^\gamma \right]^2}$$

$$- 2\beta y^i + \beta \left[ 1 - \frac{n-1}{n} Y^* \right] - c.$$  

(65)

When $v^* = 1$, RESE takes the same form of RESE1 as for $\gamma = 1$ (Theorem 1) because the first-period demand is zero.

When $\alpha = 0$ and $v^* = p_1$ (no second-period sales), the necessary condition of RESE2 existence, namely, $\frac{\partial r^i}{\partial y^i} \bigg|_{y^i = \frac{1-p_1}{n}+1} \leq 0$, using formula (65) with $v^* = p_1$, becomes $\beta \frac{1-p_1}{n} + p_1(1 - \beta) \frac{2n-1}{n} - 2\beta \frac{1-p_1}{n} + c - \beta \frac{2n-1}{n} (1-p_1) \leq 0$, which, multiplied by $n$, can be written as $p_1 [(1-\beta)\gamma(n-1) + n\beta] \leq nc$ or $p_1 \geq \frac{\gamma(n-1) + n\beta}{p_1 - \beta}$. When $\alpha = 1$, $p_1 \leq v^* < 1$, a candidate for RESE3 results from two conditions: $v^* = v^*(Y^*)$ and $\frac{\partial r^i}{\partial y^i} \bigg|_{y^i = \frac{Y^*}{n}} = 0$, which, using Lemma 1 and (65), are $v^* = \frac{p_1 - \beta}{1 - \rho \beta}$ and

$$\beta \frac{1 - v^*}{n} + [p_1 - \beta(1 - Y^*)] \frac{(1 - v^*) \gamma (n-1)}{n Y^*} + \beta \left[ 1 - \frac{n+1}{n} Y^* \right] - c = 0.$$  

(66)

After multiplication by $-\frac{n Y^*}{\beta(n+1)}$ and collection of terms with $Y^*$, this equation becomes

$$Y^* = \frac{n+1}{n} \left[ \frac{1 - v^*}{n} (1 + \gamma(n-1)) + 1 - \frac{c}{\beta} \right] - \frac{n-1}{n+1} \gamma \left( \frac{p_1}{\beta} - 1 \right) (1 - v^*) = 0,$$

(67)
which, for \( \gamma = 1 \), coincides with (18). Substitution for \( 1 - v^* = \frac{n_1 - \rho \beta Y^*}{1 - \rho \beta} \) and collection of terms with \( Y^* \) leads to \( (Y^*)^2 a_2 + Y^* a_1 + a_0 = 0 \), where

\[
a_1 = - \frac{[1 + \gamma(n - 1)](1 - p_1) + n \left( 1 - \frac{\rho}{\beta} \right) (1 - \rho \beta) - (n - 1) \gamma \left( \frac{p_1}{\beta} - 1 \right) \rho \beta}{(n + 1)(1 - \rho \beta)}, \text{ and}
\]

\[
a_0 = - \frac{n - 1}{n + 1} \gamma \left( \frac{p_1}{\beta} - 1 \right) \frac{1 - p_1}{1 - \rho \beta}.
\]

After division by \( a_2 \), the last quadratic equation becomes

\[
(Y^*)^2 - \frac{(\beta - c) n(1 - \rho \beta) + \beta[1 + \gamma(n - 1)](1 - p_1) - \gamma (p_1 - \beta) \rho \beta(n - 1)}{\beta n \rho \beta(g - 1) + n + 1 - \gamma \rho \beta} Y^* - \frac{\gamma (p_1 - \beta)(1 - p_1)(n - 1)}{\beta(n + 1)(1 - \rho \beta)} = 0, \tag{68}
\]

which, for \( \gamma = 1 \), coincides with (23). The equilibrium inventory is the larger root of this equation since, multiplying (68) by \(-a_2 \frac{\beta(n+1)}{n} < 0\), we obtain the original equation (66) with substituted \( v^*(Y^*) \) and multiplied by \( Y^* > 0 \). The LHS of this resulting equation is a quadratic function with a negative coefficient in front of \( (Y^*)^2 \), i.e., the LHS decreases in \( Y^* \) at the larger root, which corresponds to the maximum of profit.

Retailer \( i \) has sales in the second period, \( p_2 = s \). By (64) with \( p_2 = s \),

\[
r^i = (p_1 - s) \frac{(1 - v^*)(y^i)^\gamma}{(n - 1) \left( \frac{Y^*}{n} \right)^\gamma + (y^i)^\gamma} + (s - c)y^i \quad \text{and} \quad \frac{\partial r^i}{\partial y^i} = (p_1 - s) \frac{(1 - v^*)\gamma (y^i)^{\gamma-1} \left( n - 1 \right) \left( \frac{Y^*}{n} \right)^\gamma}{\left[ \left( n - 1 \right) \left( \frac{Y^*}{n} \right)^\gamma + (y^i)^\gamma \right]^2} + s - c. \tag{69}
\]

Profit (69) is concave in \( y^i \) since \( (y^i)^\gamma \) is concave, function \( \frac{A_z}{B+z} \) is concave in \( z \) for any positive \( z, A \), and \( B \) (first term of \( v^i \)) and \( (s - c)y^i \) is concave.

A candidate for RESE4 results from conditions: \( \frac{\partial r^i}{\partial y^i} \big|_{y^i=\frac{Y^*}{n}} = 0 \) and \( v^*,4 = \frac{p_1 - \rho s}{1 - \rho \beta} \). The latter implies the same \( p_1 \)-upper bound as for \( \gamma = 1 \). Namely, \( v^*,4 < 1 \) (there are sales in the first period) is equivalent to \( p_1 < P_4 \triangleq 1 - \rho (\beta - s) \). The former yields \( Y^*,4 : (p_1 - s) \left( \frac{1 - v^*(-1)Y^*/n}{n} \right)^{\gamma-1} + s - c = 0, \) which, multiplied by \( \frac{\gamma - s}{p_1 - s} Y^*n = (1 - v^*)\gamma(n - 1) \) or \( Y^*4(\gamma) \) in the form of (62).

### B.3.2. Proof of Proposition 12.

For \( \gamma = 0 \), equation (68) becomes

\[
Y^* = \frac{\beta(1-p_1)+(\beta-c)n(1-\rho \beta)}{\beta[1+n(1-\rho \beta)]} = 0 \quad \text{yielding a unique } Y^* > 0. \]

Substitution of \( 1 - Y^* = \frac{p_1 + n(1-\rho \beta)}{1+n(1-\rho \beta)} \) into \( p_2^* \) yields \( p_2^* = \beta(1 - Y^*) \) and \( v^* = \frac{p_1 - \rho \beta s}{1 - \rho \beta} \) results in the corresponding expressions.

Condition \( v^* < 1 \) (there are sales in the first period) is \( p_1 + n(p_1 - \rho c) < 1 + n(1 - \rho \beta) \) or \( p_1(n + 1) < 1 + n[1 - \rho (\beta - c)] \) yielding \( p_1 < P_1 \) — the boundary with RESE1. Condition \( v^* \geq p_1 \) is \( p_1 + n(p_1 - \rho c) \geq p_1 + np_1(1 - \rho \beta) \) or \( \rho c \leq p_1 \rho \beta \), which holds for \( \rho = 0 \). For \( \rho > 0 \), it becomes \( p_1 \geq \frac{\gamma}{\rho} \).

RESE3 exists if and only if any retailer \( i \) has no incentive to deviate neither to (i) sales in both periods with \( p_2 = s \) nor to (ii) sales only in the first period. Part (i) holds since, by (70), \( \frac{\partial r^i}{\partial y^i} \big|_{y^i=\frac{Y^*}{n}} = s - c < 0 \) for any \( y^i \) leading to \( p_2 = s \). Part (ii) is equivalent to \( \frac{\partial r^i}{\partial y^i} \big|_{y^i=\frac{1-Y^*}{n}} > 0, \)
which, by (65) with \( \gamma = 0 \), is 
\[-2\beta \frac{1-v^*}{n} + \beta \frac{1-v^*}{r} + \beta \left(1 - \frac{n-1}{n} Y^*\right) > c.\]
Multiplication by \( \frac{n}{\beta} \) leads to
\[n(1-c/\beta) - (n-1)Y^* > 1 - v^*,\]
and, after the substitutions of \( Y^* \) and
\[1 - v^* = \frac{1 - p_1 + n[1 - p_1 - \rho(\beta-c)]}{1 + n(1-\rho\beta)},\] (71)
the last inequality, multiplied by \( 1 + n(1-\rho\beta) > 0 \), becomes
\[n(1-c/\beta) + n^2(1-c/\beta)(1-\rho\beta) - (n-1)(1-p_1) - n(n-1)(1-c/\beta)(1-\rho\beta) > 1 - p_1 + n[1 - p_1 - \rho(\beta-c)] + n[1-c/\beta] + n\rho(\beta-c) > 2n(1-p_1) - n\rho(\beta-c) > 0,\]
which, after simplifications, yields \( p_1 > \frac{n}{\beta} = P_2(0) \). This inequality implies that \( v^* = p_1 \) only if \( \rho = 0 \). Since the \( P_1 \)-boundaries of RESE3 are the negations of the boundaries of RESE1 and RESE2, and, for \( \gamma = 0 \), RESE4 does not exist, no other equilibria exist in the area \( \frac{c}{\beta} < p_1 < P_1 \).

Continuity of \( Y^*, v^*, p_2^*, \) and \( r^* \) can be shown directly by substitution of the boundaries to the correspondent formulas. For example, \( Y_{n,3}\big|_{P_1=c/\beta} = \frac{1-c/\beta+n[1-c/\beta]}{1+n(1-\rho\beta)} = 1 - c/\beta = 1 - p_1 = Y_{n,2}^* \).

Monotonicity of \( v^*, Y^*, \) and \( p_2^* \), \( \frac{\partial v^*}{\partial n} = \frac{\partial v^*}{\partial (1+n(1-\rho\beta))} \{ -nc[1 + n(1-\rho\beta)] + n[p_1 + n(p_1 - p_c)] \}, \) where \( \{ \} = (\beta p_1 - c)(n+1) > 0 \) for any RESE3 inputs.

\[\frac{\partial Y^*}{\partial n} = \frac{1}{1+n(1-\rho\beta)^2} \{ (1-c/\beta)(1-\rho\beta) + n[1-n(1-\rho\beta)] - (1-\rho\beta)[1 - p_1 + n(1-c/\beta)(1-\rho\beta)] \},\]
where \( \{ \} = p_1 - c/\beta > 0 \).

The results for \( Y^* \) imply that \( p_2^* = \beta(1 - Y^*) \) is decreasing in \( n \) and increasing in \( \rho \).

Monotonicity of \( nr^* \) in \( n \). By (64) with \( \gamma = 0 \),
\[nr^* = (p_1 - p_2^*)(1 - v^*) + (p_2^* - c)Y^*.\]
Then \( \frac{\partial(nr^*)}{\partial n} = -\frac{\partial p_2^*}{\partial (1+n(1-\rho\beta))} \{ (1-c/\beta)(1-\rho\beta) + n[1-n(1-\rho\beta)] - (1-\rho\beta)[1 - p_1 + n(1-c/\beta)(1-\rho\beta)] \},\]
and \( \frac{\partial v^*}{\partial n} = \frac{\beta(1-v^*) - \rho(1-\rho\beta)(p_1 - p_2^*)}{1+n(1-\rho\beta)} \). Using (71) and
\[p_1 - p_2^* = \frac{p_1(1 - \beta) + n(1 - \rho\beta)(p_1 - c)}{1 + n(1-\rho\beta)},\] (73)
\[p_2^* - c = \frac{\beta p_1 - c}{1 + n(1-\rho\beta)},\] (74)
the bracket \( \{ \} \), multiplied by \( [1 + n(1-\rho\beta)] \), becomes
\[\beta(1 - p_1) + n(1 - \rho\beta)(p_1 - c) + \rho(1 - \rho\beta)(p_1 - p_1) + \beta(p_1 - c) - \beta(1 - p_1) - n(1 - \rho\beta)(1 - \rho\beta) + \frac{\beta(1 - v^*)}{1+n(1-\rho\beta)} \{ (1-c/\beta)(1-\rho\beta) + n[1-n(1-\rho\beta)] - (1-\rho\beta)[1 - p_1 + n(1-c/\beta)(1-\rho\beta)] \},\]
which is decreasing in \( n \). For \( n = 1 \), this expression is \( -\rho\beta(p_1 - c) - \frac{\rho p_1(1-\beta)}{1-\rho\beta} \leq 0 \). Therefore, \( nr^* \) is decreasing in \( n \) for any \( n \geq 1 \) since \( \frac{\partial(nr^*)}{\partial n} = 0 \) only for \( n = 1 \) and \( \rho = 0 \).

The conditions of monotonicity of \( nr^* \) in \( \rho \). Using (72),
\[\frac{\partial(nr^*)}{\partial \rho} = -\frac{\partial p_2^*}{\partial \rho} (p_1 - v^*) + \frac{\partial Y^*}{\partial \rho} (p_2^* - c) + \frac{\partial p_2^*}{\partial \rho (p_1 - p_2^*)} Y^*,\]
which, using \( \frac{\partial p_2^*}{\partial \rho} = -\beta \frac{\partial v^*}{\partial \rho}, \) can be written as
\[\frac{\partial(nr^*)}{\partial \rho} = \frac{\partial p_2^*}{\partial \rho} \left[ 2Y^* + \frac{c}{\beta} - 2 + v^* \right] -
\[ \frac{\partial \nu^*}{\partial \rho} [p_1 - \beta(1 - Y^*)]. \] The first bracket \([\cdot]\) is zero for \(n = 1\) because, by (67) for \(\gamma = 0\) and \(n = 1\), \(Y^* = \frac{1}{2} \left(2 - v^* - \frac{c}{\beta}\right)\), whereas the second bracket \([\cdot]\) > 0. Therefore, for \(n = 1\), \(\frac{\partial (nr^*)}{\partial \rho} < 0\).

For \(n > 1\) and \(\gamma = 0\), equation (67) yields \(Y^* = \frac{1}{n+1} \left[ v^* + \frac{n}{n+1} \left(1 - \frac{c}{\beta}\right)\right]\), which can be written as

\[ 1 - Y^* = \frac{1}{n+1} \left[ v^* + \frac{n}{n+1} \left(1 - \frac{c}{\beta}\right)\right] \text{ or } \rho^*_n = \frac{\beta v^* + nc}{n+1}. \tag{75} \]

Since \(\frac{\partial \nu^*_n}{\partial \rho} = \frac{\beta}{n+1} \frac{\partial \nu^*}{\partial \rho} \cdot \frac{\partial (nr^*)}{\partial \rho} = \frac{\partial \nu^*}{\partial \rho} \frac{1}{n+1} \left\{ \beta \left[2Y^* + \frac{c}{\beta} - 2 + v^*\right] - (n+1) \left[p_1 - \beta(1 - Y^*)\right]\right\}, \] which means that \(\frac{\partial (nr^*)}{\partial \rho} \leq 0\) is equivalent to \({\cdot}\) ≤ 0 or \(p_1 \geq \beta(1 - Y^*) + \frac{1}{n+1} \left[c + \beta v^* - 2\beta(1 - Y^*)\right]\) = \(p^*_n + \frac{c + \beta v^* - 2p^*_n}{n+1}\) or, using (75),

\[ p_1 \geq c + \frac{2\beta v^* - p^*_n}{n+1} \Leftrightarrow p_1 \geq c + \frac{2n}{n+1} (p^*_n - c). \tag{76} \]

The last inequality always holds for \(n = 1\) and never holds when \(\rho \beta \rightarrow 1\) (leading to \(p^*_n \rightarrow p_1\)) and \(n > 1\) since \((n+1)(p_1 - c) < 2n(p_1 - c)\) for any \(n > 1\).

Condition (76) is only sufficient for monotonicity of \(nr^*\) under RESE3 because violation of this condition may take place outside the area of RESE3 inputs, and inside this area, \(nr^*\) can be monotonic. Namely, by part 3 of Proposition 3, which holds for \(\gamma = 0\), RESE3 exists only for \(\rho < \rho^1 = \frac{n+1}{n} \frac{1-p_1}{\beta - c}\), where \(\rho^1\) can be less than one for large \(n\). In order to take into account this bound, condition (76) can be written in terms of inputs using (74):

\[ p_1 \geq c + \frac{2n}{n+1} \frac{\beta p_1 - c}{1+n(1-\rho \beta)}. \tag{77} \]

The RHS of this inequality is increasing in \(\rho\); therefore, given other inputs fixed, \(\frac{\partial (nr^*)}{\partial \rho} < 0\) for all \(\rho\) under RESE3 if and only if (77) holds for \(\rho = \rho^1\). With this \(\rho\), condition (77) becomes

\[ p_1 \geq c + \frac{2n}{n+1} \frac{(\beta p_1 - c)(\beta - c)}{(\beta - c)(1-p_1)} \Leftrightarrow p_1 \geq c + \frac{2n}{(n+1)^2}(\beta - c). \tag{78} \]

The RHS of (78) decreases in \(n\) to \(c\) with \(n \rightarrow \infty\). Therefore, there exists \(n^0\) such that \(\frac{\partial (nr^*)}{\partial \rho} < 0\) for all \(n > n^0\). On the other hand, since \(\frac{\partial (nr^*)}{\partial \rho} < 0\) for \(n = 1\) and (78) is necessary and sufficient for \(\frac{\partial (nr^*)}{\partial \rho} < 0\), \(nr^*\) attains minimum (since the RHS of 77 increases in \(\rho\)) for any \(n \in (1, n^0)\), where \(n^0 > 2\) if and only if (78) does not hold at least for \(n = 2\), i.e., \(p_1 < c + \frac{4}{9}(\beta - c) = \frac{5\beta + 4\beta^2}{9}\).

\(n^0\) can be found, e.g., from the negation of (78) bearing in mind that non-monotonicity holds for \(n < n^0\), where \(n^0\) is the larger root of the equation, corresponding to \((p_1 - c)(n^2 + 2n + 1) < 2n(\beta - c)\). The equation is \(n^2 - 2n\frac{\beta p_1}{p_1 - c} + 1 = 0\), where \(\frac{\beta p_1}{p_1 - c} > 0\) since \(p_1 \in (c, \beta)\). The discriminant

\[ D = 4 \left[ \left(\frac{\beta p_1}{p_1 - c}\right)^2 - 1\right] > 0 \text{ because } \beta - p_1 > p_1 - c \text{ is equivalent to } p_1 < \frac{\beta + c}{2}, \text{ where the RHS is greater than } \frac{5\beta + 4\beta^2}{9}. \]

Then the larger root is \(n^0 = \frac{\beta p_1 + \sqrt{(\beta - p_1)^2 - (p_1 - c)^2}}{p_1 - c} = \frac{\beta p_1 + \sqrt{(\beta - c)(\beta + c - 2p_1)}}{p_1 - c}\).

The expression for \(\rho^0\) can be found from (77) when it holds as an equality: \((p_1 - c)(1+n(1-\rho \beta)) = \frac{2n}{n+1}(\beta p_1 - c) \Leftrightarrow 1 - \rho \beta = \frac{2}{n+1} \frac{\beta p_1 - c - 1}{n} \Leftrightarrow \rho^0 = \beta p_1 - c \left[ 1 + \frac{4}{9} - \frac{2}{n+1} \frac{\beta p_1 - c}{p_1 - c} \right] \), yielding

\[ \rho^0 = \frac{(n+1)^2(p_1 - c) - 2n(\beta p_1 - c)}{\beta n(n+1)(p_1 - c)}. \tag{79} \]
When $\beta = 1$, $nr^{*,3}$, using (72) and the expressions (73), (71), and (74), is

$$nr^{*,3}|_{\beta=1} = \frac{1}{[1 + n(1 - \rho)]^2} \left\{ n(1 - \rho)(p_1 - c)(1 - p_1) + n^2(1 - \rho)(p_1 - c)(1 - p_1 - \rho(1 - c)) \right. \\
+ (1 - p_1)(p_1 - c) + n(1 - \rho)(p_1 - c)(1 - c) \right\},$$

$$\lim_{\rho \to 1} nr^{*,3}|_{\beta=1} = (1 - p_1)(p_1 - c) < nr^{*,3}|_{\beta=1} = \frac{1}{(1 + n)^2} \left\{ n(p_1 - c)(1 - p_1) + n^2(p_1 - c)(1 - p_1) \right. \\
+ (1 - p_1)(p_1 - c) + n(p_1 - c)(1 - c) + n(p_1 - c)(1 - p_1) \left\} \\
= (1 - p_1)(p_1 - c) + \frac{n(p_1 - c)^2}{(1 + n)^2}.$$

Formula $\rho^0|_{\beta=1} = \frac{n^2 + 1}{n(n+1)}$ results from (79) with $\beta = 1$. Minimum of $\rho^0$ in $n$ can be found from

$$\frac{\partial \rho^0}{\partial n} = 0 = \frac{2n^2(n+1) - (2n+1)(n^2+1)}{n^2(n+1)^2}$$

which is equivalent to $n^2 - 2n - 1 = 0$ with the roots $n_{1,2} = 1 \pm \sqrt{2}$. The relevant root is $n_2 = 1 + \sqrt{2}$; direct calculation yields $\rho^0|_{n=2} = \frac{5}{6} = \frac{10}{12} = \rho^0|_{n=3}$. 