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Optimal Dynamic Pricing of Perishable Items by a Monopolist Facing Strategic Consumers

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We introduce a dynamic pricing model for a monopolistic company selling a perishable product to a finite population of *strategic consumers* (customers who are aware that pricing is dynamic and may time their purchases strategically). This problem is modeled as a stochastic dynamic game in which the company's objective is to maximize total expected revenues, and each customer maximizes the expected present value of utility. We prove the existence of a unique subgame-perfect equilibrium pricing policy, provide equilibrium optimality conditions for both customer and seller, and prove monotonicity results for special cases. We demonstrate through numerical examples that a company that ignores strategic consumer behavior may receive much lower total revenues than one that uses the strategic equilibrium pricing policy. We also show that, when the initial capacity is a decision variable, it can be used together with the appropriate pricing policy to effectively reduce the impact of strategic consumer behavior. The proposed model is computationally tractable for problems of realistic size.

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1. Introduction

The recent growth in internet-based marketing has stimulated widespread experimentation with *dynamic pricing*—the practice of varying prices for the same goods over time or across customer classes in an attempt to increase total revenues for the seller. In this paper, we examine a particular version of dynamic pricing that has been widely applied to sales of perishable items over a finite time period. Familiar examples include passenger transportation tickets, hotel bookings, automobile rentals, end-of-season style goods, and expiring models of electronic equipment or other high-value items.

Particularly notable successes of dynamic pricing have been reported in the airline industry, which has, perhaps, the longest history of controlled dynamic pricing through their *revenue management* (RM) systems (traditional RM is more properly viewed as inventory control across price classes; however, the distinction between RM and dynamic pricing is becoming increasingly blurred as the use of online booking expands). There is an extensive literature on RM and related practices. For a review of this area, see Phillips (2005) and Talluri and van Ryzin (2004).

In dynamic pricing, it is crucial to take into account key characteristics of the market such as consumer behavior and the availabilities of goods and services. Classical dynamic pricing models often assume that consumer behavior is *myopic* – a consumer makes a purchase as soon as the price drops below his/her valuation for the product while ignoring future product availability and purchasing opportunities. This assumption leads to tractable models, significantly simplifies analysis, and can lead to attractive structural properties for pricing policies. But many real-life consumers are *strategic*; that is, they attempt to "get more for less" by delaying purchase in anticipation of price reductions. When supplies are limited, such strategic consumers must also take into account product availability and the strategic behavior of other consumers. Thus, a comprehensive model of strategic consumer behavior needs to capture interactions between the seller and the consumers as well as among the consumers.

1.1. Survey of Existing Literature

Dynamic pricing models for an infinite population of myopic customers include those of Feng and Xiao (2000), Gallego and van Ryzin (1994), and Zhao and Zheng (2000). In their book, Talluri and van Ryzin (2004) discuss myopic vs. strategic consumer models (Section 5.1.4.1). They point out that the myopic assumption provides an acceptable approximation when customers are relatively spontaneous in making decisions (typical with low-price products) or when they do not have enough time or information to behave strategically. However, for more expensive and durable products and when information is available, this assumption becomes less realistic. They argue that failure to account for strategic customer behavior could significantly reduce expected revenues from dynamic pricing.

The literature on consumer behavior issues in RM for perishable products is growing rapidly; for a survey see Shen and Su (2007). Here, we only review models that are the most relevant in terms of their assumptions and conclusions relative to the present work. In general terms, any analysis of individual strategic consumer behavior requires simplifying assumptions. For example, many studies in this area assume deterministic demand, and, even in the more realistic stochastic case, most articles restrict the dynamics of price changes to markdowns or consider problems with only two decision periods. Each of these make additional specific assumptions on consumer behavior.

An early deterministic model of dynamic pricing with strategic consumers is presented in Besanko and Winston (1990). They show that the subgame-perfect equilibrium pricing policy for the firm is to lower prices over time in a manner similar to *price-skimming*. The selling capacity in this model is unlimited. Su (2007) considers a deterministic demand model with customers partitioned into four segments according to their valuation level and waiting costs, and derives conditions relating to when the seller should use markdowns or markups. The company controls both the price and the fill rate in this work.

Aviv and Pazgal (2008) study the optimal pricing of fashion-like seasonal goods in the presence of strategic customers who arrive according to a Poisson's process but have deterministically declining valuations over time. This work considers subgame-perfect Nash equilibria between a seller and strategic consumers for the cases of inventory-contingent discounting strategies and announced fixed discount strategies. The paper finds that the seller cannot effectively avoid the adverse impact of strategic consumer behavior even under low levels of initial inventory, and that fixed discounting strategies may outperform contingent pricing strategies. Elmaghraby et al. (2008) study optimal markdown mechanisms in the presence of strategic consumers who have fixed valuation throughout the selling season.

A number of authors consider fully dynamic pricing but under specific behavioral assumptions. Xu and Hopp (2004) study a continuous-time model with strategic customers who arrive according to a Poisson process and whose price sensitivity varies with time. In this model, customers commit to a purchasing time which is determined from an open-loop (state independent) decision process. One of their results is that the optimal prices follow a submartingale if the price sensitivity is constant or decreasing, and supermartingale if the price sensitivity is increasing.

Papers which study two-period pricing models with strategic consumers include Liu and van Ryzin (2008) who use quantity decisions (rather than pricing) as means to induce early purchases. Zhang and Cooper (2008) also consider the effect of strategic customers in a two-period model with regular and clearance prices and an option of restricting product availability in the second period. Cachon and Swinney (2007) study the value of an additional product procurement, termed "quick response," in a two-period model where the second period price is always a markdown. The model considers three segments of customers: myopic, bargain hunting (those who only purchase in the second period), and strategic. The paper finds that "quick response" offers more value in the presence of strategic consumers.

A two-channel model is studied by Caldentey and Vulcano (2007) who consider a setting in which strategic consumers, who arrive according to a Poisson process, decide between buying immediately at a (fixed) posted price and buying later by entering their bids to a sealed-bid multi-unit uniform price auction. Two cases are studied: the single-auction channel case where the company only manages the auction, and the dual-channel case where the company manages both the auction and the fixed-price channel. The authors discuss the equilibrium and a simple, asymptotically optimal, heuristic rule for its approximation.

Several articles study the effects of customer response to RM over multiple selling seasons; see, for example, Ovchinnikov and Milner (2007). In this work, the market is treated in an aggregated form, and some fraction of customers learns to expect end-of-season discounts based on previous interactions with the seller. Within each season, given the full and sale prices, the seller selects the number of units available at the end-of-season discount. The article provides analytical results for the explicit multi-season model. Gallego et al. (2008) investigate a similar class of models with multiple selling seasons and two-period pricing within each season. Customers update their expectations about the probability of acquiring an item at a discount based on previous interaction with the seller. Both of these papers show the existence of complex dynamic patterns of consumer–seller interactions.

1.2. Inherently Stochastic Markets

In a deterministic setting, the company does not experience uncertainty in demand and, given its pricing/sales policy, can always accurately predict consumer behavior and evaluate the performance of the policy. However, most markets in which dynamic pricing is employed are *inherently stochastic* in the sense that outcomes (sales paths) are not wholly determined by company pricing decisions but also depend on other random events. In our view, customer valuations or propensities to consume can be uncertain up to the very time of consumption, particularly for advance purchases. Appropriate examples include discretionary hotel reservations, vacation packages, and sports and entertainment events whose value may depend on uncertain weather and time availability for the consumer. Uncertainty in valuations is also present for a wide variety of services that need to be reserved in advance (housekeeping, gardening, snow removal, etc.).

An intention to purchase at a given time or price point may not result in a purchase. For example, a customer who feels that a price is acceptable may not act immediately for a number of reasons, including procrastination, congestion delay, or terms and conditions of purchase. Also, many customers exhibit ''wavering'' around a decision, particularly for discretionary purchases – deciding one moment that they will purchase and the next that they really shouldn't. It can also take more than a single inquiry before a consumer settles on a purchase. (Uncertainty in individual choices, even under seemingly identical conditions, has been reported in empirical studies [see, e.g., Tversky 1972]. Moreover, Xie and Shugan indicate that ''buyers are nearly always uncertain about their future valuations for most services'' [Xie and Shugan 2001, p. 219].) All of these behaviors are difficult to capture with fixed valuation assumptions.

Demand uncertainty often brings product availability into consideration for both the company and its customers. As a result, the pricing policy and consumer response will, in general, depend on the remaining capacity level and remaining time. Finally, implementation of a deterministic policy assumes that the company can credibly commit to a price path, an assumption that does not always hold in practice. The issue of credible price commitment is not important when consumers are myopic and, in that case, a deterministic policy can be asymptotically optimal Gallego and van Ryzin (1994), However, as shown by Aviv and Pazgal (2008), this becomes an important issue in the presence of strategic consumers.

In this paper, we present a dynamic pricing model for a finite population of strategic consumers in a market which is inherently stochastic in its operation. The seller is a monopolist firm that contingently prices a fixed stock of items over a finite time horizon, and the consumers exercise choice based on the strategic factors discussed above. Our consumer choice model is similar in spirit to that of Besanko and Winston (1990), but we extend that work by taking demand uncertainty into consideration and allowing for capacity restrictions. In contrast to the deterministic model of Su (2007), which also treats capacity restrictions, we do not assume that the firm has explicit control of the fill rate. Dynamic explicit control of the fill rate may not be easy to implement since selective rejection of purchase attempts over time can alienate customers, and it may be hard to keep a customer from making another purchase attempt if the original attempt is rejected. Our model also differs from that of Su (2007) in the description of strategic consumer behavior, since the latter introduces explicit waiting costs for the consumers. In contrast to many prior stochastic models, we do not limit the dynamics of prices to two time periods, and we do not limit the model to decreasing price paths (markdowns). While markdown policies are appropriate in some retail settings, they are not universally applicable. These two aspects allow us to study much richer dynamics of price and sales processes than more restricted models permit.

In models with deterministic valuations (or random valuations that are realized at the beginning of the sales season and remain fixed thereafter), homogeneous consumers will all complete purchases simultaneously when ''conditions are right.'' This, of course, does not accord with the behavior of real markets. Firms are much more likely to see relative increases or decreases in the *pace* of sales as prices and inventories change. This motivates a model in which individual consumers respond to prices and other market conditions by controlling the intensity of their demand processes. In this way, actual completions of sales will be distributed over time stochastically, and consumers will have to take this uncertainty into account in their decision making. Such a strategic response of consumers in the form of shopping intensity is a distinctive feature of this paper.

This approach to consumer behavior removes the need for explicit rationing controls for the firm because, much like in real life, random rationing of the product to consumers occurs as a natural consequence of market operation. This model also obviates the assumption that the choice behavior of consumers is predetermined at the beginning of the sales season.

Such generality comes with a cost – the model requires an assumption that consumers re-sample their valuations at each decision point, independently of previous valuations. While this can be viewed as a limiting model for consumer ''wavering,'' a more satisfactory model would allow dependency across time; for example, with a random-walk valuation process. This is an interesting topic for future research.

1.3. Chief Contributions

The model described here is a stochastic dynamic game in which the company controls prices contingent on time and present market conditions to maximize total expected revenues, and each customer controls demand intensity to maximize the expected present value of utility. In the presence of imperfect information, such stochastic games are notoriously difficult to analyze. Therefore, we make some simplifying assumptions which guarantee that this is a perfect information game, and its participants can rationally anticipate the behavior of others. In this aspect, our game structure is similar to that of Besanko and Winston (1990).

We prove that a unique subgame-perfect equilibrium pricing policy exists, and that the ''strategicity'' of consumers affects the choice of an optimal pricing policy. We also provide equilibrium optimality conditions for both consumer and seller, and study properties of the equilibrium policy. The equilibrium obtained here is a direct generalization of the optimal pricing policy for myopic consumers which would be obtained in a discrete-time, finite population version of the classical model of Gallego and van Ryzin (1994).

Practical insights derived from this work relate to the structure of pricing policies and their relation to capacity decisions. We show that, when consumer rationality is limited, the pricing policy exhibits natural monotonic properties: price decreases as remaining capacity increases or remaining time decreases. Such properties are familiar from the analysis of Gallego and van Ryzin (1994) in the myopic case. When consumers are fully rational and strategic, prices decrease monotonically in time in the special case of high product supply. This result is also in agreement with Besanko and Winston (1990) in the deterministic case. However, these authors also find that price approaches marginal cost as the discount factor approaches one, whereas in our case price remains strictly bounded away from marginal cost. This difference arises because of the stochastic nature of the sales process.

We also demonstrate that monotonic properties do not hold in general. This contrasts with well-known results in the theory of dynamic pricing for the myopic case. In practical terms, if strategic customers are only partially rational and are not aware of the present state of the system or cannot take it into account, then a familiar monotonic property holds: price increases as inventory decreases. In contrast, if the customers are rational and aware of the current state of the system, then the company may have to counteract their strategic behavior with policies that deviate from monotonicity. For example, there must be a credible threat to increase prices even in the absence of sales.

Numerical simulations of pricing policies in the general case also indicate that price paths are markedly different for the cases of myopic and strategic consumers. In the myopic case, the average level of prices starts high and decreases over time, whereas the average level is more constant in the strategic case. Also, for strategic consumers, pricing flexibility decreases significantly with an increase in supply level when compared with the myopic case.

The model described here can be used as a benchmark for the importance of strategic consumer behavior. For example, it can be used to estimate the losses a company can incur due to strategic consumer behavior, both when this behavior is ignored and when it is optimally anticipated in the company's pricing policy. Numerical examples demonstrate that a company that ignores strategic consumer behavior may receive much lower total revenues than one that uses the strategic equilibrium pricing policy, and that losses are much more profound when supply levels are high.

While the focus of our paper is on contingent pricing policies, the model can be appropriately modified to evaluate consumer response to pre-announced, deterministic, pricing policies. We numerically evaluate the effects of two-period pre-announced pricing policies in comparison with fully dynamic, contingent, ones and find that the latter perform better when the supply level is low and consumer behavior is closer to myopic. However, contingent policies may underperform when the capacity level is high. This agrees with the findings of Aviv and Pazgal (2008) obtained under different modelling assumptions.

Finally, the expected revenue as a function of initial capacity provided by our model can also be used in a newsvendor-type formulation in which the initial procurement decision is followed by dynamic pricing in a strategic consumer market. This is made possible by the stochastic market feature of the model. We show that, when the initial capacity is a decision variable, its proper use together with dynamic pricing is crucial for effective reduction of the effects of strategic consumer behavior. In short, dynamic pricing may not be able to compensate for inappropriate capacity decisions. Also, the appropriate capacity level is lower for the case of strategic consumers. On the computational side, the proposed model is tractable for realistic-size problems. In summary, the chief contributions of this paper are:

- pricing policies (based on a game between a monopolist and strategic consumers) that generalize classical optimal pricing policies for the myopic case in an unrestricted dynamic setting,
- monotonic properties of pricing policies in the presence of strategic consumers, and
- an analysis of relations between and effects of appropriate/inappropriate procurement decisions and dynamic pricing policies in the presence of strategic consumers.

These results are restricted to the case of monopolistic firms that possess knowledge of consumer valuation distributions. Such results are much harder to obtain in more complex market settings. In a companion paper, Levina et al. (2009), we relax the assumption of knowledge of the valuation distribution and allow for learning of consumer behavior as the sales season unfolds. That model cannot explicitly capture the dynamic aspect of the game between the company and the consumers. A game-theoretic model of interactions between the company and strategic consumers in the presence of consumer and company learning is an interesting direction for future research. In Levin et al. (2009), we examine dynamic pricing in an oligopoly with strategic consumers in multiple market segments. The complexity of the models in both of these papers does not permit detailed study of monotonic properties of pricing policies and the importance of capacity decisions.

The organization of this paper is as follows. We present the notation and game-theoretic model in Section 2 and show existence of the unique subgame-perfect equilibrium in Section 3. We explore monotonicity results of the equilibrium pricing policies in Section 4, and show the effect of strategic consumer behavior on the performance of pricing policies and capacity decisions in Section 5. We conclude and indicate several directions for further research in Section 6. Additional discussions of the modeling assumptions, a glossary of notation, and mathematical proofs are provided in the supporting information Appendix S1.

2. Model Description

We describe the model in three stages. First, in Section 2.1, we specify general model assumptions with particular attention to the demand process, which is characterized by the *shopping intensity* of consumers. In Section 2.2, we describe the consumer choice model and define the stochastic dynamic game that captures company–consumer interactions. Finally, in Section 2.3, we derive the objective functions for consumers and the company and the equilibrium response for consumers.

2.1. General Modeling Assumptions

Consider a product with limited availability sold by a monopolistic company over a finite planning horizon $[0, T]$. A common model of demand from a population of consumers is a counting process with intensity generally dependent on time and price (see, e.g., Gallego and van Ryzin 1994). Since we want to describe behavior of individual consumers, this assumption takes the following form:

Demand process:

- The population demand process is a sum of counting demand processes originating from a finite number of individual consumers. The population demand intensity is the sum of consumer demand intensities.
- Consumers control the intensity of their demand processes (which we call shopping intensity) in response to price and other market conditions.
- There exists a common upper bound $\bar{\lambda}$ for the shopping intensity of each consumer.

In this interpretation of demand, consumers cannot control the precise timing of their purchases, but respond to more favorable prices and other market conditions by increased eagerness to purchase, which translates to a higher expected frequency of purchases. This assumption is motivated by two practical considerations. First, in a real marketplace, all interested consumers cannot purchase at exactly the same time because both the capacity of the purchasing channel and the timing of access by consumers to that channel are constrained. Second, as pointed out by Su (2006), real-world consumers have a tendency to procrastinate. The total demand intensity is bounded in practice: even if the product is available for free, there will be a maximum number of customers per unit of time who can purchase the product. Since each consumer contributes a fraction of the total intensity, it is reasonable that the individual intensity is also bounded. The value $\bar{\lambda}$ captures transaction uncertainties in the market as well as consumer tendency toward procrastination, and has a precise interpretation as the reciprocal of the expected time for an eager customer to purchase the product. It can also be interpreted as the intensity of shopping opportunities, and the quantity $\bar{\lambda}T$ is the expected number of shopping opportunities for a customer who is eager throughout the sales season.

Finally, to simplify the model we take the approach employed in Talluri and van Ryzin (2004) and discretize the time into short time steps which we call *decision periods*. The number of decision periods is sufficiently large that any continuous-time counting process in the model can be well approximated by its discrete-time analogue. Under such a choice of discretization, the probability of more than one event occurring in a decision period has to be relatively small, and we can assume that at most one event in any of the processes can occur per decision period. For simplicity of notation, we assume that each decision period has a length of exactly one time unit. Then the shopping intensity of a customer is equivalent to the probability that this particular customer purchases an item in the next decision period. Consequently, the demand intensity of the entire market is equivalent to the probability that any consumer purchases an item in the given decision period. The purchase probabilities retain the additive property of intensities: the probability of a purchase occurring is the sum of purchase probabilities of individual customers. We summarize this as:

Representation of intensity as probability: Consumer shopping intensity decisions are expressed as purchase probabilities in the decision periods of the discrete-time model.

Consumer behavior, discussed in detail in the next section, can be summarized as follows. In each period t, each customer draws a valuation according to a given distribution. Draws are independent across periods and are exchangeable random variables across customers. Each individual customer then uses his draw to select a shopping intensity from the interval $[0, \bar{\lambda}]$. To ensure that our time discretization is sufficiently fine, we must select the time step (time units) so that the upper bound $\bar{\lambda}$ on demand intensity is less than the reciprocal of the total maximum market size. The individual customer's decision will generally also depend upon the posted price, the time remaining in the horizon, the remaining inventory, and the customer's assessment of the expected value of delaying purchase. Once all customers have simultaneously selected their probabilities, then at most one customer will, conditionally independent of the past, make a purchase in period t according to the selected probabilities.

Next, we introduce notation, describe how consumers decide between shopping now and waiting – the consumer choice model – and how the company can influence this behavior.

2.2. The Game

The goal of our model is to capture the intertemporal behavior of customers who strategically attempt to time their purchase to periods with lower price. They indirectly control timing of their purchases by moderating their shopping intensity relative to that of myopic customers. Customer shopping intensity is selected dynamically in response to the company's pricing policy, which is also dynamic. The resulting model belongs to the general class of stochastic dynamic games.

For convenience, the notation introduced here and throughout the paper is summarized in a glossary in Appendix S1. Recall that there are T decision periods in our model. The initial inventory of the product at time 0 is Y, with no replenishment possible during the planning period. At time T, the product expires and all unsold items are lost.

The product is offered to a finite number N of customers, each of whom can purchase at most one item. To ensure that probabilities in our discrete-time model adequately represent intensities of the demand processes, we choose time discretization so that $N\bar{\lambda}$ is much <1. Without loss of generality, we assume that the initial number of customers $N \geq Y$ (the case of $N < Y$ can be reduced to $N = Y$ since any excess items cannot be sold and should be disregarded. The case of $N = Y$ is appropriate for a company offering a make-to-order product to strategic customers). All customers are present from the beginning and can monitor the market as well as make purchases during the entire planning horizon.

With regard to the information structure of the game, we assume by default:

Perfect market information: All game participants have perfect information about the market including its parametric, distributional, and dynamic characteristics (except for random valuation realizations).

A perfect information assumption implies that the remaining inventory level $y \leq Y$ and the number of customers who have not yet acquired the product, $n \leq N$, are public information. Because the customer population is homogeneous, it is unimportant which particular customer has not yet acquired the product, and the state of the system in decision period $t \in \{0, 1, \ldots, T\}$ can be described by the pair y, n. However, since the current inventory level can be recovered from n as $y = Y - (N - n)$, we can drop it from the state description. A transition from state n to $n-1$ represents a sale of one unit of the product to one of the customers. Given maximum shopping intensity $\bar{\lambda}$, the value of n, together with the remaining time, allows us to quantify the risk that a consumer will fail to complete the purchase (rationing risk). This is how a randomized sales process can replace explicit rationing controls considered in some models. We view this as an advantage of the model since, as pointed out in the introduction, explicit rationing controls may be hard to implement when the sales process is truly dynamic.

The key modeling issue is how strategic consumers compare a current purchase with a possible purchase in the future. One of the common approaches is based on random utility models (see Section 3.2 of Anderson et al. 1992). The use of such models is pervasive in economics, marketing, and other fields. In a dynamic setting under linear random utility, a consumer values a purchase at price p at time $t \in \{0, 1, \ldots, T-1\}$ as $a_t + \varepsilon_t - p$, where a_t is a known constant describing consumer perception of quality or value of the product, and ε_t is a random variable with mean zero (usually assumed to be continuously distributed). The quantity $p'_t = a_t + \varepsilon_t$ formally corresponds to valuation of the product at time t. One modeling option would be to assume that for each consumer the valuation is drawn from some known distribution at the beginning and then remains constant throughout the selling season. However, in reality, individual consumer valuations cannot be measured precisely and are unlikely to be certain. In general, an individual consumer valuation can be modelled over time as a stochastic process. For example, one may assume it is a continuous-state random walk (Markov) process. By changing the degree of statistical dependence between consecutive states of this process, we get a range of possible models for valuations. At one end of this spectrum, we have a fixed (deterministic) model – an assumption made, for example, in deterministic models of Besanko and Winston (1990), Su (2007), where it is completely appropriate. In a model like ours, a finite-population model with a stochastic sales process, the fixed valuation model will lead to a discontinuous total demand intensity response to posted prices (intuitively, as the price increases, customers will switch from shopping to non-shopping one by one based on their fixed utility thresholds). Moreover, as the company prices its product, it will also have to learn the unknown and uncertain demand intensity profile. Such a problem falls into the class of incomplete information control problems. In such a formulation, the state description would have to include the histories of price and sales processes, which would make the resulting control problem very difficult. Consumer evaluation of a possible future purchase would have to depend on initial valuation as well as price and sales history, resulting in a major complication for a strategic consumer behavior model. An intermediate level of statistical dependence of consecutive valuations is more realistic, but it does not resolve the issue of dealing with the demand-learning problem. At the other end of the spectrum – complete independence between valuations at different time – the demand-learning problem can be avoided if the valuation distribution is known. This leads to the following:

Strategic choice model: Suppose that the current decision period is t, and the price announced by the company is p; then,

- The consumer values the purchase that can occur in the current decision period as $u(p' - p)$, where p' is a possible valuation level and $u(\cdot)$ is a strictly increasing utility function with an inverse $u^{-1}(\cdot)$. An arbitrary utility reference value is chosen so that $u(0) = 0$.
- Consumer valuations are distributed according to $F_t(p)$. Valuations of different customers at time t are exchangeable random variables.
- Valuations are independent random variables for different t.
- Each consumer's value of future purchase opportunities as perceived in the next decision period is the same known function of time and state, $U(t+1, n)$. Consumers may assess this value exactly or heuristically depending on their rationality level.
- There is no customer "disutility" associated with failing to purchase an item, and a customer without an item at time T derives a utility of zero.
- The utility of buying an item in the future is discounted by a factor $\beta \in [0, 1]$ per decision period. The customer values future opportunities as the expected value of $\beta U(t+1, \cdot)$ over all possible states resulting from the current one.

The *degree of strategicity* of a customer is determined by the parameter $\beta \in [0, 1]$. Indeed, the value $\beta = 0$ means that the customer completely disregards the possibility of a future purchase; that is, the customer is myopic. The value $\beta = 1$ means that the customer values the current purchase the same as a purchase at any point in the future and will exhibit fully strategic behavior. Intermediate values of β determine how long customers can postpone their purchase without excessive loss of utility. Our use of identical characteristics to describe different customers: β, $u(\cdot)$, $F_t(p)$ and $U(\cdot)$ implies that the pool of customers is homogeneous (in a stochastic sense). In practice, these parameters could be estimated from such sources as past sales history and marketing surveys. In e-commerce applications, where it is often possible to track behavior of individual customers, these parameters can be estimated from consumer responses to prices over time. The maximum shopping intensity parameter $\bar{\lambda}$ could be estimated from the number of individual consumer inquiries. Some approaches to learning of customer behavior in this way are discussed in a separate paper: Levina et al. (2009).

The company's objective is to maximize the expected total revenues by selecting the pricing policy in the class of state-feedback policies: $p = p(t, n)$ for state n at time t. A customer's objective is to maximize the expected present value of utility of acquiring the product. The objectives of the company and its customers determine the payoff functions in a stochastic dynamic game which unfolds over the decision periods $t \in \{0, \ldots, T-1\}$. If, at time $t < T$, the system is in state $n > N - Y$, the company announces the price $p = p(t, n)$ (move of the company), the customers observe their valuation realization p' (which is different for each consumer and unknown by other consumers), and respond by choosing their shopping intensity from the interval $[0, \bar{\lambda}]$. The equilibrium shopping intensity response function of a customer is denoted as $\lambda^U(t, n, p, p')$, and its average over realizations of p' (the average response that the company would expect in the present state) as $\lambda^U(t, n, p)$. The game ends either at time T regardless of the state, or if the company sells out prior to time T (that is, in state $n = N - Y$). At the end of the game all unsold items (if any) expire, and all customers still in the market obtain a utility of 0.

We now turn to the nature of equilibrium in this game.

2.3. Equilibrium

To arrive at a strategic decision concerning shopping intensity, the customer needs to know the expected present value of utility from a possible future purchase. This value may be computed exactly or approximately. The exact computation requires the customers to know the price which will be used in every possible state in the future. This is possible if we suppose that customers are fully rational and can compute the optimal pricing policy used by the company or if the customers can resort to third-party services. An alternative is to assume partial rationality: customers use a given heuristic rule to estimate the expected present value of utility from a possible future purchase. In either case, we assume

Rationality: The company is fully rational and knows how customers evaluate future purchasing possibilities. We further assume that all participants of the game assume the optimality (in an exact or a heuristic sense) of the future strategies of all opponents (prices and customer responses), and each customer assumes that the concurrent responses of other customers are exactly optimal.

The sequence of moves at each t described in the previous subsection corresponds to a hierarchical equilibrium structure with the company as the leader and the customers as the followers. The equilibrium at each t can be described as a Stackelberg equilibrium between the company and the customers and a Nash equilibrium with asymmetric information (since each customer knows their own current valuation but not those of others) between the customers. According to our rationality assumptions, the behavior of the company is always time consistent. We can make the customer response time-consistent as well under full customer rationality. Thus, under full rationality, we have a subgame perfect equilibrium with asymmetric information. The equilibrium between the customers at each t can also be viewed as a variation of a *quantal response equilibrium*, see McKelvey and Palfrey (1995, 1998), because the payoff of each customer is modulated by a random perturbation drawn from the valuation distribution, and decisions of each customer can be simplified to a binary choice between shopping now (with the maximum possible intensity) or waiting (that is, shopping with intensity 0).

We now derive the objective function for a customer and the equilibrium decision in terms of the shopping intensity. This derivation is for a simpler case in which valuation draws across different customers are independent and identically distributed instead of exchangeable (which admits some dependency). Under this assumption, it is easy to see that the expected payoffs and equilibrium responses of all customers are identical. We present the formal treatment in the general case of exchangeable draws in Appendix S1.

Suppose that the current decision period is t and consider a particular customer. A future purchase can only occur in one of the decision periods $t+1, \ldots, T-1$. The expected value of utility at time $t+1$ from a possible future purchase, given that the state at time $t+1$ is n, is assessed by each customer as $U(t+1, n)$. Given strategicity parameter β, the expected present utility (at time t) from a possible future purchase is then $\beta U(t+1, n)$. At the time of decision, the customer has also observed the posted price p and his/her own valuation level p'. We denote the decision variable as $\lambda \in [0, \bar{\lambda}]$. Since valuation realizations of all other customers are identically distributed and independent of the value observed by the customer, their shopping intensities can be estimated by the customer at the expected value $\lambda^U(t, n, p)$. Given t, n, p, p', the value function $V^U(t, n, p, p')$ is the expected present value of utility over all possible purchase events at time t (recall that $\bar{\lambda}f$ and $\lambda^U(t, n, p)$ are probabilities):

$$V^U(t,n,p,p') = \max_{0\le\lambda\le\bar{\lambda}} \{\lambda u(p'-p) + \beta(n-1)\lambda^U(t,n,p) \times U(t+1,n-1) + \beta(1-\lambda-(n-1) \times \lambda^U(t,n,p))U(t+1,n)\}.$$

The first term in the above expression refers to the certain utility that will be realized if the customer completes the purchase now. The other terms represent the expected present value of utility when a purchase by another customer occurs (second term) or does not occur (third term). After collecting terms, this expression can be rewritten as

$$V^U(t,n,p,p') = \max_{0\le\lambda\le\bar{\lambda}} \{\lambda(u(p'-p) - \beta U(t+1,n))\} + \beta(n-1)\lambda^U(t,n,p)(U(t+1,n-1) - U(t+1,n)) + U(t+1,n).$$

From the linearity of the objective in λ, we conclude that the customer optimal shopping intensity is given by the expression

$$\lambda^U(t,n,p,p') = \bar{\lambda} I[u(p'-p) \ge \beta U(t+1,n)]. \quad (1)$$

The interpretation of this result is straightforward: the hypothetical customer will shop with the maximum possible intensity of $\bar{\lambda}$ whenever the utility of the valuation/price difference is greater than or equal to the present value of utility of a possible future purchase. The condition inside the indicator function can also be expressed as $p' \ge p + u^{-1}(\beta U(t+1, n))$, where $u^{-1}(\cdot)$ is the inverse of $u(\cdot)$.

We thus have a simple rule for how customers choose their shopping intensities after observing their private valuations, but we must also specify how shopping intensity can be assessed without knowledge of those valuations. This is needed by the company, by customers considering other customers (in the fully rational case), and by customers assessing their own behavior in the future. To accomplish this, we take expectations of the optimal shopping intensity and the corresponding objective value over the valuation distribution $F_t(p)$ and obtain the following proposition (where we let $V^U(t,n,p) = E_{p'}[V^U(t,n,p,p')]$):

PROPOSITION 1. *Given the evaluation of the expected utility from a possible future purchase $U(t+1,\cdot)$, which is identical for all remaining customers, the expected equilibrium shopping intensity of any customer observing price p in state $n = 1,\ldots,N$ at time t is given by*

$$\lambda^U(t,n,p) = \bar{\lambda}\bar{F}_t\big(p + u^{-1}(\beta U(t+1,n))\big). \qquad (2)$$

The expected present value of utility for each remaining customer is given by

$$\begin{aligned} V^U(t,n,p) =& \bar{\lambda}\int_0^{+\infty}\{(u(p'-p)-\beta U(t+1,n))\}^+ dF_t(p') \\ &+ \beta((n-1)\lambda^U(t,n,p)(U(t+1,n-1) \\ &- U(t+1,n)) + U(t+1,n)), \end{aligned} \qquad (3)$$

where, for any real x, its positive part $\{x\}^+ = x$ if $x \geq 0$ *and $\{x\}^+ = 0$ if $x < 0$.*

An important observation from this analysis is that the customer reaction to price given in Equation (2) depends on the state n only through his/her evaluation of the future given by $U(t+1,n)$. Thus, for an exogenously specified U, the result contained in Equation (2) can be generalized to the case in which the customer is uncertain about state information: we would replace $U(t+1,n)$ by the expected value over n. We do not pursue this generalization here but note that this would allow extension to a variety of limited-rationality cases.

The company can use the equilibrium consumer shopping intensity in its policy optimization. The probability of a sale occurring in the current decision period and state n is equal to $n\lambda^U(t,n,p)$. The expression for the optimal expected revenue of the company is obtained in the same fashion as the expression for the optimal expected customer's utility. The company's equilibrium pricing strategy $p^U(\cdot)$ is chosen so that $p^U(t,n)$ delivers the maximum expected future revenues:

$$\begin{aligned} R(t,n) =& R(t+1,n) + \max_p \\ &\times \{n\lambda^U(t,n,p)(p + R(t+1,n-1) - R(t+1,n))\}, \\ & n = N-Y+1,\ldots,N,\ t\in\{0,\ldots,T-1\}, \end{aligned} \qquad (4)$$

with the terminal conditions

$$R(T,n) = 0,\ n = N-Y+1,\ldots,N, \text{ and} \qquad (5)$$

$$R(t,N-Y) = 0,\ t\in\{0,\ldots,T-1\}. \qquad (6)$$

Given the rule $U(\cdot)$ for consumer evaluation of expected utility from a possible future purchase, $p^U(\cdot)$ and $\lambda^U(\cdot)$ describe the game *equilibrium*.

One particularly interesting case results when $U(\cdot)$ corresponds to fully rational customers. It can be obtained by

$$\begin{aligned} U(t,n) =& V^U(t,n,p^U(t,n)),\ n = N-Y+1,\ldots,N, \\ & t\in\{0,\ldots,T-1\} \end{aligned}$$

and the terminal conditions on the customer utility

$$U(T,n) = 0,\ n = N-Y+1,\ldots,N \qquad (7)$$

$$U(t,N-Y) = 0,\ t\in\{0,\ldots,T-1\}. \qquad (8)$$

We will denote the expected utility for this case as $U^*(\cdot)$ and the corresponding equilibrium policies as $p^*(\cdot)$ and $\lambda^*(\cdot)$.

If, in addition to the pricing policy, the company can control its initial capacity (procurement) decision, it can use the optimal revenue at time 0 for fixed N but different values of Y to make the optimal capacity choice. For the time being, however, we assume that Y is fixed and fully concentrate on the pricing decisions of the company. We return to the question of the optimal capacity decision in Section 5 where we numerically examine the effects of customer strategicity.

3. Existence and Uniqueness of Equilibrium

A typical approach in the analysis of dynamic pricing models is to regard the aggregate demand rather than the price as a decision variable for the company. In our case, it is also convenient to change the decision variable of the company to a quantity closely related to the aggregate demand. We note that a particular price p in state n at time t results in the following *eagerness to purchase* averaged over the valuation distribution:

$$w = \bar{F}_t\big(p + u^{-1}(\beta U(t+1,n))\big).$$

The purchase intensity corresponding to w is $\lambda^U(t,n,p) = \bar{\lambda}w$.

To use eagerness w as a policy variable, we require $\bar{F}_t(\cdot)$ to have an inverse function $g_t(\cdot)$. The value $g_t(w)$ of the inverse function specifies the quantile of the valuation distribution at time t corresponding to the right-tail probability w. To this end, we will make the following assumption regarding $g_t(w)$:

ASSUMPTION (A). *Let the valuation probability density function be strictly positive and continuous on a (possibly infinite) interval $(\bar{p}, \bar{\bar{p}})$, and zero outside the interval.*

Then the tail distribution function $\bar{F}_t(\cdot)$ is decreasing and has an inverse function $g_t(\cdot)$, which is continuously differentiable and decreasing. The domain of $g_t(\cdot)$ is $(0, 1)$, and its range is $(\bar{p}, \bar{\bar{p}})$.

If Assumption (A) holds, then we can express p as the following continuously differentiable function of w:

$$p = g_t(w) - u^{-1}(\beta U(t+1, n)).$$

In the sequel, we use the following abbreviated notations for the quantities marginal in n:

$$\Delta R(t,n) = R(t,n) - R(t,n-1),$$
$$\Delta U(t,n) = U(t,n) - U(t,n-1),$$
$$\Delta u^{-1}(\beta U(t,n)) = u^{-1}(\beta U(t,n)) - u^{-1}(\beta U(t,n-1)).$$

The dynamic programming recursion (4) for the expected future revenues of the company can now be rewritten in terms of eagerness w as follows:

$$\begin{aligned} R(t,n) =& R(t+1,n) + n\bar{\lambda} \max_{0 \le w \le 1} w \\ & \times \left[g_t(w) - u^{-1}(\beta U(t+1,n)) - \Delta R(t+1,n)\right], \\ & n = N - Y + 1, \ldots, N, \; t \in \{0, \ldots, T-1\}. \end{aligned} \tag{9}$$

The value of the expression under the maximum is defined at 0 and 1 in the limiting sense. An optimal policy value $w^U(t,n)$ delivers the maximum in this expression. Note the role of the $u^{-1}(\beta U(t+1,n))$ term. When the customers are myopic, this term is equal to zero, and the price corresponding to w is given by $p = g_t(w)$. The quantity $u^{-1}(\beta U(t+1,n))$ can then be viewed as a state and time-dependent transaction cost. This is a portion of the total surplus the company has to give up to the consumer due to his/her strategicity.

Next, we use Equation (3) and the unique customer response (2) to derive an expression for the equilibrium expected present value of utility $V^U(t,n)$ in terms of $w^U(t,n)$:

$$\begin{aligned} V^U(t,n) =& V^U(t,n,p^U(t,n)) \\ =& \bar{\lambda} \int_0^{+\infty} \{u(p' - g_t(w^U(t,n)) + u^{-1}(\beta U(t+1,n))) \\ & - \beta U(t+1,n)\}^+ dF_t(p') \\ & - \beta\bar{\lambda}(n-1) w^U(t,n) \Delta U(t+1,n) + \beta U(t+1,n). \end{aligned}$$

Defining

$$H_t(w,v) = \int_0^{+\infty} \left\{u(p' - g_t(w) + u^{-1}(v)) - v\right\}^+ dF_t(p'),$$

we obtain the following relation

$$\begin{aligned} V^U(t,n) =& \bar{\lambda} H_t\left(w^U(t,n), \beta U(t+1,n)\right) \\ & - \beta\bar{\lambda}(n-1) w^U(t,n) \Delta U(t+1,n) + \beta U(t+1,n), \\ & n = N - Y + 1, \ldots, N, \; t \in \{0, \ldots, T-1\}. \end{aligned} \tag{10}$$

This relation becomes recursive in the fully rational consumer case where $U(t,n) = U^*(t,n) = V^{U^*}(t,n)$ for all $n = N - Y + 1, \ldots, N$ and $t = 0, \ldots, T-1$, and the terminal conditions are given in (7) and (8).

Even under full rationality, the computational effort (required to find the optimal policy in our strategic-customer model in this parametrization) is only modestly greater than the computational effort needed in a time-discretization of the myopic-customer model of Gallego and van Ryzin (1994) with the same initial inventory level. The increase results entirely from the need to solve recursion (10) and is modest as long as the function $H_t(w,v)$ can be evaluated efficiently. Thus, our strategic customer model is computationally tractable for problems of realistic size. This result could be achieved because of our reliance on time-consistency and a hierarchical structure of the equilibrium.

Using the dynamic programming recursion (9) for the company, we arrive at the following existence and uniqueness result (the proof of this and all subsequent formal statements is provided in the Appendix S1). The result applies to both the full $U = U^*$ and limited consumer rationality cases (exogenous U). It also gives the equilibrium pricing policy of the company. We make two additional assumptions:

ASSUMPTION (B). *$wg_t(w)$ is strictly concave for all t, and*

ASSUMPTION (C). $\lim_{w \to 0} wg_t(w) = 0$ *for all t,*

which correspond to the requirements of concavity and regularity of the revenue rate in Gallego and van Ryzin (1994). These assumptions are satisfied for such common distributions as exponential $\bar{F}_t(p) = \exp(-p)$, and power $\bar{F}_t(p) = (1+p)^{-\gamma}$, $\gamma > 1$ (in these cases, $g_t(w) = -\ln(w)$ and $g_t(w) = w^{-1/\gamma} - 1$, respectively).

PROPOSITION 2. *Suppose that the valuation distribution satisfies assumptions (A), (B) and (C). Then there exists a unique equilibrium such that, for every (t,n), the customer eagerness to purchase, $w^U(t,n)$, is*

- *zero if* $\lim_{w \to 0}(w g'_t(w) + g_t(w)) \le u^{-1}(\beta U(t+1,n)) + \Delta R(t+1,n)$,

- *one if* $\lim_{w\to 1}(w g'_t(w) + g_t(w)) \geq u^{-1}(\beta U(t+1,n)) + \Delta R(t+1,n)$,
- *and, otherwise, satisfies the equation*

$$w^U(t,n)g_t'(w^U(t,n)) + g_t(w^U(t,n)) = u^{-1}(\beta U(t+1,n)) + \Delta R(t+1,n). \tag{11}$$

The corresponding optimal price is

$$p^U(t,n) = g_t(w^U(t,n)) - u^{-1}(\beta U(t+1,n)). \tag{12}$$

Given the customer perception of the future specified by U, the equilibrium eagerness value $w^U(t,n)$ specifies the optimal customer reaction to the equilibrium price $p^U(t,n)$. When assessing their reaction, customers generally take into account how many items are still available as well as competition from other customers as summarized by the state description n. Thus, the game developed here is a dynamic Stackelberg game between the company and a homogeneous population of individual consumers. If customers are unaware of the state, the equilibrium takes a somewhat simpler form in which the company plays Stackelberg games with customers who make decisions independently from each other. However, for the company, these games are linked through the limited inventory. To interpret (11), observe that, on the left-hand side, $wg'_t(w) + g_t(w)$ is the marginal rate of revenue per customer while, on the right-hand side, marginal expected revenue per item $\Delta R(t+1,n)$ can be viewed as an opportunity cost of capacity. A standard interpretation of optimality conditions in dynamic pricing is that the marginal revenue rate equals the opportunity cost of capacity (see, for example, the interpretation of equation (5.13) on page 203 of Talluri and van Ryzin (2004)). In our model, this is modified by addition of the certainty equivalent from a possible future purchase $u^{-1}(\beta U(t+1,n))$, which we interpreted as a cost of transaction with strategic customers.

The following corollary immediately follows from the optimality conditions described in the above proposition and the (strictly) decreasing property of $wg'_t(w) + g_t(w)$ in w. It relates monotonic properties of the optimal policy to those of $u^{-1}(\beta U(t+1,n)) + \Delta R(t+1,n)$ (the sum of the certainty equivalent of purchasing in the future and the marginal future expected revenues).

Corollary 1. *The relation*

$$u^{-1}(\beta U(t+1,n)) + \Delta R(t+1,n) \leq (\geq) \times u^{-1}(\beta U(t+1,n-1)) + \Delta R(t+1,n-1)$$

holds if and only if $w^U(t,n) \geq (\leq)\, w^U(t,n-1)$. *In addition, if* $g_t(\cdot)$ *is stationary (independent of* t*) then the relation*

$$u^{-1}(\beta U(t+1,n)) + \Delta R(t+1,n) \leq (\geq)\, u^{-1}(\beta U(t+2,n)) + \Delta R(t+2,n)$$

holds if and only if $w^U(t,n) \geq (\leq)\, w^U(t+1,n)$.

This result sheds some light on the consequences of consumer rationality for monotonic properties of the company's policy. One of the common monotonicity results in dynamic pricing models is that the marginal revenues decrease in capacity; that is, $\Delta R(t+1,n) \leq \Delta R(t+1,n-1)$. On the other hand, intuition suggests that, for rational consumers, inequality $U(t+1,n) \geq U(t+1,n-1)$ should typically hold since the customer competition for remaining items becomes more acute when there is one less item available (even though there is also one less customer). Because this inequality for utility is in the opposite direction to the inequality for marginal revenues, we conclude that consumer rationality works against typical monotonicity properties. Some exceptions to this are possible if $U(t+1,n)$ is independent of n. The reader will see that this intuitive conclusion is confirmed in the structural results and numerical experiments to follow. As a final remark about Proposition 2, we show that certain monotonic properties of prices (in contrast to the result of Corollary 1 which is in terms of eagerness) can be established under the following assumption:

Assumption (D). $wg'_t(w)$ *is increasing for all* t.

This assumption is also satisfied for exponential and power distributions that we mentioned earlier (after Assumptions (B) and (C)).

Corollary 2. *Let Assumption (D) hold in addition to (A), (B), and (C). Then the pair of inequalities*

$$\begin{cases} \Delta R(t+1,n) \leq (\geq) \Delta R(t+1,n-1), \\ w^U(t,n) \geq (\leq) w^U(t,n-1), \end{cases}$$

implies $p^U(t,n) \leq (\geq)\, p^U(t,n-1)$. *If, moreover,* $g_t(\cdot)$ *is stationary (independent of* t*), then the pair of inequalities*

$$\begin{cases} \Delta R(t+1,n) \leq (\geq) \Delta R(t+2,n), \\ w^U(t,n) \geq (\leq) w^U(t+1,n), \end{cases}$$

implies $p^U(t,n) \leq (\geq)\, p^U(t+1,n)$.

To explain this result, we rewrite Equation (12) using Equation (11) as $p^U(t,n) = \Delta R(t+1,n) - w^U(t,n)\, g_t'(w^U(t,n))$. Since, under strategic consumer behavior, $w^U(t,n)$ is generally time and state dependent, appropriate monotonic relations on both $\Delta R(t+1,n)$ and $w^U(t,n)$ are needed to guarantee corresponding monotonic relations between prices. On the other hand, in the case of myopic consumers it suffices to have monotonic relations on $\Delta R(t+1, n)$. In managerial terms, an interpretation of the first claim of the corollary is that, if under equilibrium policies a marginal value of an extra sale opportunity (one extra item plus one extra customer) decreases and the shopping intensity of an individual customer increases with n, then the price decreases with n. Similar interpretations apply to the second claim and also if inequalities are reversed.

As a reality check of our model, we have also examined what happens if the original problem is scaled to an arbitrarily large population size, with capacity proportionally large but still smaller than the population. Under such scaling, we also need to take the number of decision periods to infinity. Through the law of large numbers, the demand process becomes increasingly deterministic from the point of view of the company, but each consumer still faces a potential rationing risk because of insufficient capacity. In Appendix S1, we show that in the limit the price optimality conditions in our discrete-time model correspond to a Hamilton–Jacobi–Bellman equation for a deterministic continuous time control problem. Thus, we find that the stochastic discrete-time model has a natural deterministic continuous-time analog.

4. Monotone Properties of the Optimal Policy

In this section, we study monotone properties of the optimal policy. We show that the optimal price and marginal revenues possess monotone properties in two special cases: first, when customers have limited rationality and evaluate the expected utility from future purchases independently of the state (this includes myopic case), and, second, when customers are strategic but do not need to compete for the items since the inventory is sufficient to satisfy *all* customers. Our results show that under these conditions the monotonic properties well-known in the infinite population myopic case hold in our model as well. We also present a numerical illustration which shows that the price may not be monotone in general. This is in contrast with the results in dynamic pricing literature for the myopic consumer case. In practical terms, this suggests that the company may have to deviate from monotonic pricing policies to counteract strategic consumer behavior. All results of this section are obtained under Assumptions (A)–(C).

We start our discussion with basic observations which apply to the most general form of our model. First, note that the optimal expected future revenues are decreasing in time: $R(t, n) \geq R(t+1, n)$, since the maximum of expression (28) under maximization of the recursive relation (9) for revenues is nonnegative. Given that $U \geq 0$, it is also straightforward to establish that the equilibrium customer utility is nonnegative: $V^U(t, n) \geq 0$, since we can regroup the terms in Equation (10). For U^*, we can establish the nonnegativity by reverse induction from terminal conditions (7) and (8).

4.1. Special Case: State Unaware, Limited Rationality Consumers

We analyze the case of a finite population of limited rationality customers whose evaluation of the future is such that $\beta U(t, n)$ is independent of the state n for every t. Note that myopic consumer behavior, which is a special case of our model when $\beta = 0$, satisfies this restriction. The following proposition establishes that the marginal revenues (with respect to the number of remaining customers) are decreasing with respect to n and t. These properties generally coincide with those found in many myopic infinite population models (see, for example, Proposition 5.2 on page 203 of Talluri and van Ryzin (2004)). We also remark that this monotonicity result provides a bound on the decrement of $\Delta R(t, n)$ with respect to t:

PROPOSITION 3. *Let U be such that $\beta U(t, n)$ does not depend on n for each t. Then we have*

$$\Delta R(t, n) \leq \Delta R(t, n-1),\ t = 0, \ldots, T,\ n = N - Y + 2, \ldots, N, \tag{13}$$

$$\begin{aligned}\Delta R(t, n) \geq \Delta R(t+1, n) + \bar{\lambda} w^U(t, n)\big[g_t(w^U(t, n)) \\ -u^{-1}(\beta U(t+1, n)) - \Delta R(t+1, n)\big], \\ t = 0, \ldots, T-1,\ n = N - Y + 1, \ldots, N.\end{aligned} \tag{14}$$

The first inequality can also be restated as $\Delta_2 R(t, n) \leq 0$, and implies concavity of the expected revenues in n.

The following corollaries establish monotonic properties of the policy variables in the state unaware, limited rationality case. The first corollary shows that the customer eagerness to purchase increases with respect to n, and that prices pursue the opposite behavior:

COROLLARY 3. *If U is such that $\beta U(t, n)$ does not depend on n for each t, then the following inequalities hold for all $t = 0, \ldots, T-1$, and all $n = N - Y + 2, \ldots, N$:*

$$w^U(t, n) \geq w^U(t, n-1), \tag{15}$$

$$p^U(t, n) \leq p^U(t, n-1). \tag{16}$$

Recall that smaller n also means lower inventory level y; thus, this property is natural because, under otherwise identical market conditions, our intuition suggests increasing price if there are fewer items left to sell. Our model provides an additional insight that the price increases in spite of the fact that the market is also smaller after a sale.

The next corollary establishes monotonic properties of the pricing policy with respect to time in the case of myopic customers and a stationary valuation distribution. An additional assumption made is that the consumer evaluation of expected utility from a possible future purchase decreases in time (a natural property for a heuristic utility evaluation rule).

COROLLARY 4. *If U is such that $\beta U(t,n)$ does not depend on n, $\beta U(t,\cdot) \geq \beta U(t+1,\cdot)$ for all t, and $g_t(\cdot) = g(\cdot)$, then the following inequality holds for all $t = 0,\ldots,T-2$ and $n = N-Y+1,\ldots,N$*

$$w^U(t,n) \leq w^U(t+1,n) \tag{17}$$

with inequality being strict if $w^U(t+1,n) > 0$. In addition, under Assumption (D) or in the myopic case of $\beta = 0$

$$p^U(t,n) \geq p^U(t+1,n). \tag{18}$$

This corollary asserts that, under otherwise identical market conditions, consumer eagerness to purchase increases when there is less time remaining. It is much less obvious that the price is monotonic since there are generally two opposite effects: $g(w^U(t,n)) \geq g(w^U(t+1,n))$ and $-u^{-1}(\beta U(t,n)) \leq -u^{-1}(\beta U(t+1,n))$, and it is not clear which one dominates. When $\beta = 0$, the last inequality holds as an equality which implies monotonicity of prices.

4.2. Special Case: Fully Rational Strategic Consumers Under High-Supply Market

Next, we analyze the case of fully rational strategic consumers in a "high product supply" situation, when there are enough items in inventory to satisfy the purchasing requirements of all customers. The results imply that the game at each time t and state n decomposes into n identical games played simultaneously between the company and each of the remaining customers individually. This is possible because customers are statistically identical.

PROPOSITION 4. *In the case of $N = Y$, the following equalities hold*

$$w^*(t,n) = w^*(t,n-1),\ t = 0,\ldots,T,\ n = N-Y+2,\ldots,N, \tag{19}$$

$$p^*(t,n) = p^*(t,n-1),\ t = 0,\ldots,T,\ n = N-Y+2,\ldots,N, \tag{20}$$

$$\begin{aligned}\Delta R(t,n) = {} & \Delta R(t+1,n) + \bar{\lambda} w^*(t,n)[g_t(w^*(t,n)) \\ & -u^{-1}(\beta U^*(t+1,n)) - \Delta R(t+1,n)], \\ & t = 0,\ldots,T-1,\ n = N-Y+1,\ldots,N,\end{aligned} \tag{21}$$

$$\Delta R(t,n) = \Delta R(t,n-1),\ t = 0,\ldots,T,\ n = N-Y+2,\ldots,N, \tag{22}$$

$$U^*(t,n) = U^*(t,n-1),\ t = 0,\ldots,T,\ n = N-Y+2,\ldots,N. \tag{23}$$

The result of this proposition is that the equilibrium price, the consumer reaction, and the expected utility are independent of the state, while the expected revenues increase linearly with the state (thus, the revenue is concave in the state here as well). Such equilibrium structure is natural since there is no competition between consumers, and they only need to take into account time remaining to the end of the planning horizon. Equation (21) also implies that the marginal expected revenues decrease in time.

Based on the above proposition, we also obtain a time monotonicity result for the equilibrium eagerness to purchase and price in the special case of fully strategic customers.

COROLLARY 5. *In the case of $\beta = 1$ $N = Y$ and $g_t(\cdot) = g(\cdot)$, for all $t = 0,\ldots,T-2$* and *$n = 1,\ldots,N$*

$$w^*(t,n) \leq w^*(t+1,n). \tag{24}$$

In addition, under Assumption (D)

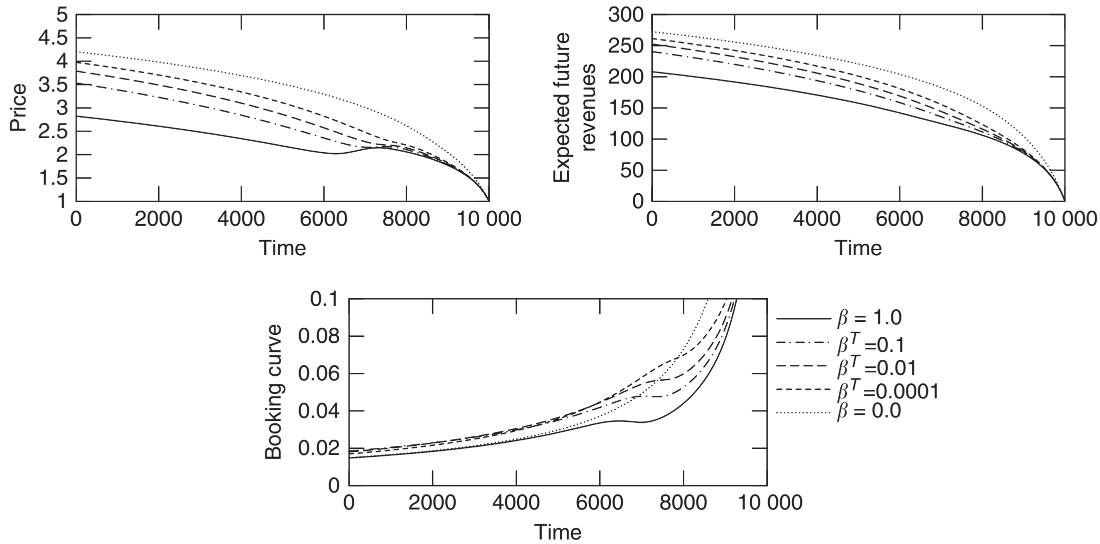
$$p^*(t,n) \geq p^*(t+1,n). \tag{25}$$

4.3. General Case: Numerical Illustrations

The monotonicity properties discussed above for special cases do not hold for more general strategicity or inventory level conditions. We illustrate this with numerical counterexamples below. In these examples, we use a risk-neutral utility $u(p'-p) = p'-p$ and a stationary exponential tail distribution of the valuation $\bar{F}_t(p') = \exp(-p')$. We solve the model for $T = 10{,}000$ decision periods, $N = 100$ customers and maximum individual shopping intensity $\bar{\lambda} = 0.005$. The purpose of choosing a large value for T is primarily to provide a sufficiently close approximation to continuous time.

4.3.1 Monotonicity with Respect to Time. Our first numerical experiment concerns the non-monotonicity of price in time when the number of customers is fixed. The initial inventory level for this experiment is fixed at $Y = 75$ items. Five values of the strategicity parameter β are examined: zero (myopic), one (fully strategic), and three intermediate values such that the value of a purchase after 10,000 time steps is 0.1, 0.01, and 0.0001 of the value of a purchase if it is made

Figure 1 The Optimal Price $p^*(t, 100)$, Future Expected Revenues $R(t, 100)$ and the Booking Curve $\bar{\lambda}100w^*(t, 100)$ as Functions of Time for $Y = 75$, $N = 100$ and Different Values of β

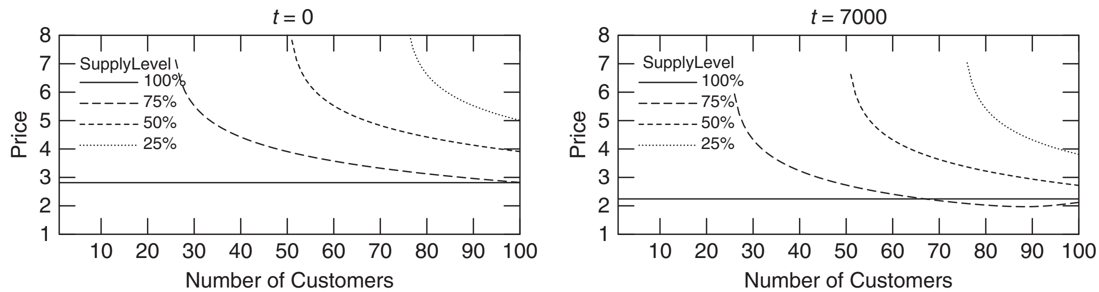


immediately. These values were chosen because of the logarithmic effects of strategicity on the optimal policies, and produce graphs which fill the visual gaps between myopic and fully strategic customers quite well. Figure 1 shows the optimal price $p^*(t, 100)$, the optimal expected revenues $R(t, 100)$ and the "booking curve" for the state with 100 customers and all five values of β. The graph of the optimal price for the myopic case is, of course, monotone decreasing, in full agreement with Corollary 4. However, as the value of β increases, we see a tendency towards non-monotonic behavior. For $\beta = 1$, the change in direction can be seen very clearly and takes place around time steps $t = 6200\ldots7200$. A possible explanation of this non-monotonic behavior is that the supply level $Y/N = 75\%$ is relatively high. Therefore, in the beginning of the interval, the state $n = 100$ will provide a plentiful opportunity for the customer to buy. As the expiration date gets closer, the customers will have fewer possibilities to purchase. This allows the company to increase prices even if all other parameters of the model stay the same. Note that this is possible only because the customers are strategic and aware of the reduction in future purchase opportunities. Fully rational behavior of the company ensures that a price increase is a credible threat, thus stimulating consumers to shop earlier.

Another observation from this plot is that the optimal price tends to decrease in β. Naturally, the same can be expected from the optimal expected revenues. We can see this type of behavior of $R(t, 100)$ in Figure 1. Recall that the total shopping intensity of n customers at time t is given by $\bar{\lambda}nw^*(t, n)$. This can be viewed as a "booking curve" over time. Non-monotonic behavior in time is seen here as well although not as prominently as for $p^*(t, 100)$ (see Figure 1). Another interesting feature observed from this plot is that the shopping intensity is *not* monotonic in β. In the beginning of the interval, the shopping intensity is almost the same for $\beta = 0$ and 1, while being slightly larger for the intermediate values of β. A possible explanation is that both the myopic and fully strategic ($\beta = 1$) customers in the beginning of the planning interval do not care when exactly their purchases occur. On the other hand, customers with intermediate values of β effectively have less time than T to make sure the item is acquired than the customer with $\beta = 1$ who can take advantage of the whole planning interval.

4.3.2 Monotonicity with Respect to *n*. The second experiment examines the monotonic behavior of prices with respect to n at fixed time t. The *initial* inventory Y is fixed, which implies that for smaller values of n we look at respectively smaller values of the *remaining* inventory $y = Y - (N - n)$. Four values of Y were examined corresponding to the supply levels Y/N of 25%, 50%, 75%, and 100%. The strategicity parameter β is fixed at 1. The graphs of the optimal price $p^*(t, n)$ as functions of n are shown in Figure 2 for fixed values of $t = 0$ (the decision period near which $p^*(t, n)$ is decreasing in t, a natural monotonic behavior) and $t = 7000$ (where we observed some deviations from monotonic behavior). The value of $p^*(t, n)$ for the supply level of 100% remains constant in agreement with Proposition 4. For lower values of the supply level, the value of $p^*(t, n)$ is decreasing in n at the beginning of the planning period ($t = 0$). This is not the case for $t = 7000$ and the supply level of 75%. We recall that the price is also non-monotone in t for

Figure 2 The Optimal Price $p^*(t, n)$ as a Function of the Number of Remaining Customers n for β = 1, fixed t = 0, t = 7000 and Different Values of Initial Supply Level



this supply level around $t = 7000$. For supply levels of 25% and 50%, the prices are monotonic for both values of t.

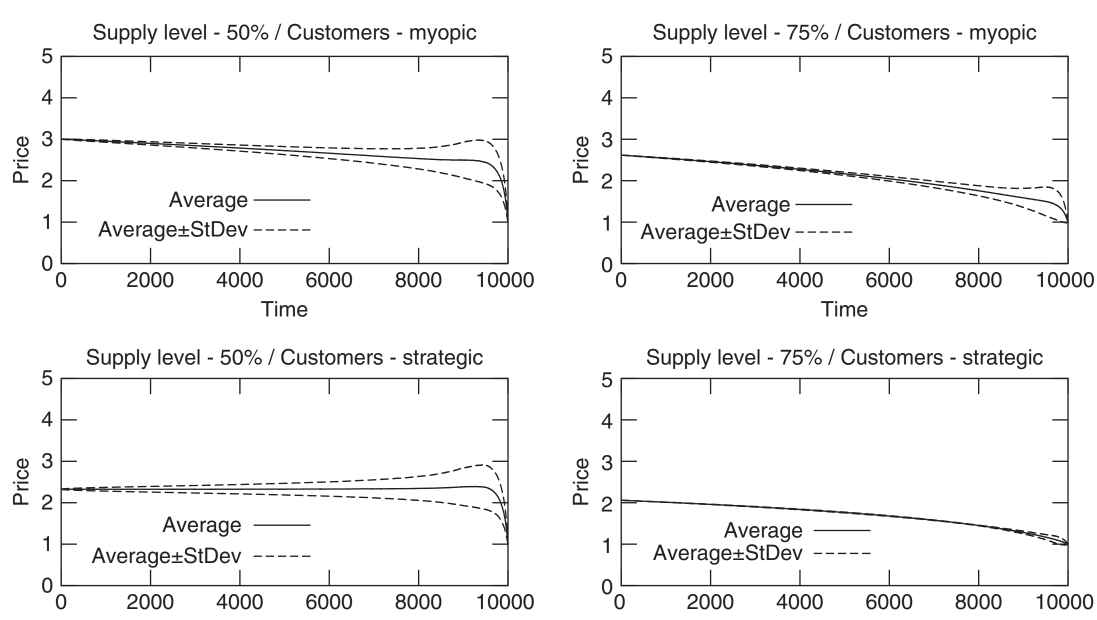
4.3.3 Typical Price Path Realizations. In addition to monotonic properties of policies themselves, it is interesting to examine typical realizations of these policies – the price paths, since this is how policies are perceived in the market place. Therefore, we have simulated 10,000 realizations of pricing policies for myopic and fully strategic ($\beta = 1$) consumers at supply levels of 50% and 75%. The averages of simulated prices plus/minus one standard deviation for all four combinations are shown in Figure 3. In the myopic case, increased supply level leads to reduced price flexibility (as evidenced by a smaller standard deviation) and significantly lower prices in the second half of the planning horizon. When consumers are strategic, the effect of increased supply level is even more dramatic. At $s = 50\%$, there is relatively steady price level of about 2.3 with progressively larger price fluctuations as the end of selling season gets closer. At $s = 75\%$, we see a steady decline in prices. Even the starting price is just slightly above 2 and there is practically no price fluctuation around the average price path. This suggests that appropriate choice of capacity level is especially important in the presence of strategic consumers.

The non-monotonicity of prices for the high supply level also suggests that we may discover interesting behavior in expected revenues when the values of Y and N are close. A numerical experiment addressing this and other issues is described in the next section.

5. Effects of Strategic Consumer Behavior on Company Performance and Decisions

In this section, we examine the interplay between strategicity and company performance and policies.

Figure 3 Averages of Simulated Price Paths for Myopic/Fully Strategic Consumers, Supply Levels 50%/75%, N = 100 and T = 10,000



We start by assuming that the initial capacity Y is specified exogenously (for example, a flight capacity is determined by the aircraft following a pre-determined schedule) and, in Section 5.1, study strategicity effects on the pricing policies, expected revenues, and utilities. In particular, we examine performance of pricing policies obtained under the assumption that customers are myopic in a situation when customers are in fact fully rational and strategic. This examination is done by testing the effect of company deviation from the equilibrium decisions. We assume that the customers are aware of the fact that the company is behaving non-optimally, can compute their actual expected utility in this situation exactly, and adjust their response to the observed prices accordingly. Major observations made in this subsection include much lower or even negative marginal revenue in terms of extra product supply for supply levels close to 100%, possibly high costs of assuming that customers are myopic when they are strategic, and a description of conditions when these costs are low.

In Section 5.2, we relax the assumption of exogenous Y and study the effects of strategicity on the initial capacity decisions. In particular, we observe importance of appropriate initial capacity reduction to counteract strategic consumer behavior, possibly significant profit reduction even if strategic behavior is correctly taken into account, and truly disastrous effects on profits if strategicity is ignored in procurement decisions. We also study contribution of pricing vs. capacity decisions to the proper company response to strategicity.

5.1. Effects on Revenues and Pricing Policies

In the study of pricing decisions, we keep all parameters of the model fixed except for the values of β (the strategicity parameter) and Y. It is convenient to present the results in terms of the supply level $s = Y/N$. We denote by $p^*_{\beta,s}(\cdot)$ the pricing policy that is optimal for fixed values of β and s, and by $R_{\beta,s}(\cdot)$ and $U_{\beta,s}(\cdot)$ the corresponding (optimal) future expected revenues and expected present values of customer utility in all states, respectively. The pricing policy which is optimal for myopic customers (under the assumption that $\beta = 0$) is $p^*_{0,s}(\cdot)$. If the company uses this policy in a situation when $\beta > 0$, the behavior of the customers will be different from the one expected by the company, and this policy will no longer be optimal. Given that the company applies the policy $p^*_{0,s}(\cdot)$, let the actual (exact) utility values for customers with strategicity parameter β be $U^0_{\beta,s}(\cdot)$. The eagerness to purchase in response to $p^*_{0,s}(\cdot)$ will be denoted by $w^0_{\beta,s}$, and is computed via Equation (2) where we use $U = U^0_{\beta,s}$ resulting in

$$w^0_{\beta,s}(t,n) = \bar{F}_t\Big(p^*_{0,s}(t,n) + u^{-1}(\beta U^0_{\beta,s}(t+1,n))\Big).$$

The utility values are computed recursively according to

$$\begin{aligned} U^0_{\beta,s}(t,n) =& \bar{\lambda} H_t\Big(w^0_{\beta,s}(t,n), \beta U^0_{\beta,s}(t+1,n)\Big) \\ & - \beta\bar{\lambda}(n-1) w^0_{\beta,s}(t,n)\Delta U^0_{\beta,s}(t+1,n) + \beta U^0_{\beta,s}(t+1,n), \\ & n = N - Y + 1, \ldots, N,\ t \in \{0, \ldots, T-1\}. \end{aligned} \tag{26}$$

The expected future revenues in the situation when the myopic policy $p^*_{0,s}(\cdot)$ is used for customers with arbitrary β is denoted by $R^0_{\beta,s}(\cdot)$, and can be computed via

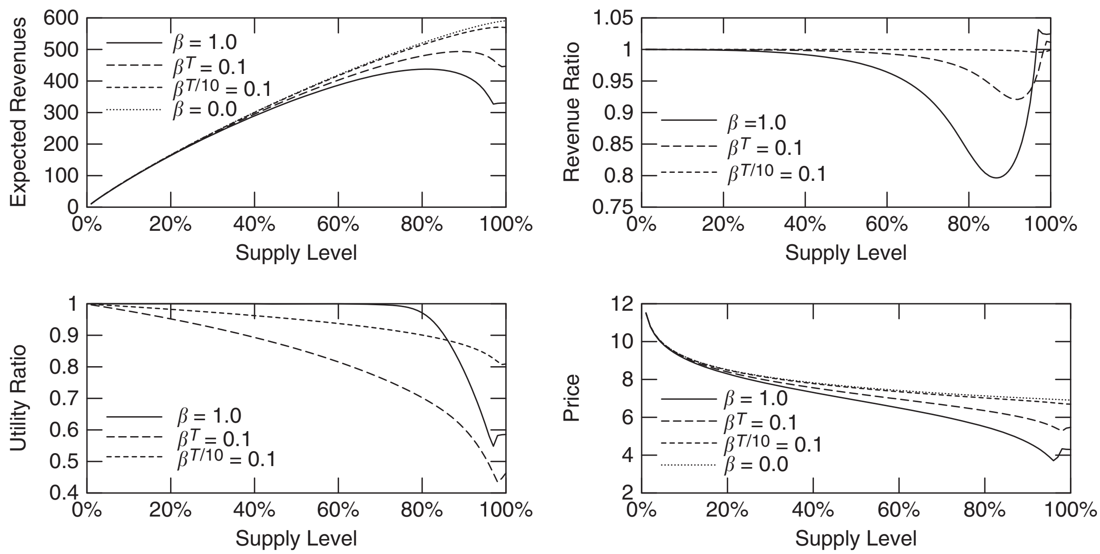
$$\begin{aligned} R^0_{\beta,s}(t,n) =& R^0_{\beta,s}(t+1,n) + n\bar{\lambda} w^0_{\beta,s}(t,n) \\ & \Big[g_t(w^0_{\beta,s}(t,n)) - u^{-1}\Big(\beta U^0_{\beta,s}(t+1,n)\Big) \\ & - \Delta R^0_{\beta,s}(t+1,n)\Big],\ n = N - Y + 1, \ldots, N, \\ & t \in \{0, \ldots, T-1\}. \end{aligned}$$

The derivation of these recursions is straightforward and closely follows that of Equations (9) and (10). The boundary conditions for $U^0_{\beta,s}(\cdot)$ and $R^0_{\beta,s}(\cdot)$ are identical to those for $U_{\beta,s}(\cdot)$ and $R_{\beta,s}(\cdot)$ (equations of the same form as (7), (8), and (5), (6), respectively).

For the numerical experiments described below, we use a risk-neutral utility $u(p' - p) = p' - p$ and a stationary exponential tail distribution of the valuation $\bar{F}_t(p') = \exp(-p')$. We solve the model for $N = 100$ customers, the maximum individual shopping intensity of $\bar{\lambda} = 0.001$ per time step and a time horizon of $T = 1{,}000{,}000$. In addition to $\beta = 0$ and $\beta = 1$, we examine two values of β which correspond to a purchase after T and $T/10$ time steps being valued at 10% of utility for an immediate purchase.

5.1.1 Optimal Expected Total Revenues as a Function Supply Level. Figure 4 shows the optimal expected total revenues $R_{\beta,s}(0,100)$ as a function of the supply level $s = Y/N$ (as well as the ratio of the expected total revenues $R^0_{\beta,s}(0,100)/R_{\beta,s}(0,100)$, the ratio of the expected present value of consumer utilities $U^0_{\beta,s}(0,100)/U_{\beta,s}(0,100)$, and the optimal price $p^*_{\beta,s}(0,100)$). The plot of the expected revenues (top left) contains four graphs which correspond to different values of β. The most prominent feature is that, although in the case of myopic customers the revenues are monotonically increasing in s, this is not generally true in the presence of strategic customers. *As β increases, the graph of $R_{\beta,s}(0,100)$ acquires a maximum in s.* This drop in revenue after a certain point can be explained by a decrease in competition between customers for the remaining inventory. We also see that the optimal revenues for all values of s are decreasing in β. Moreover, the slope of the graphs

Figure 4 The Optimal Expected Total Revenues $R_{\beta,s}(0, 100)$, the Ratio of the Expected Total Revenues $R^0_{\beta,s}(0, 100)/R_{\beta,s}(0, 100)$, the Ratio of the Expected Present Value of Consumer Utilities $U^0_{\beta,s}(0, 100)/U_{\beta,s}(0, 100)$ and the Optimal Price $p^*_{\beta,s}(0, 100)$ as Functions of the Supply Level $s = Y/N$ for Different Values of β and T = 1,000,000



(essentially, marginal revenues in Y) is decreasing. Therefore, marginal analysis would suggest a lower initial stocking level when customers are strategic compared to the myopic case. The final observation regarding these plots is a plateau-type behavior for the highest supply levels. This behavior occurs because the company will be less likely to sell its entire inventory when the customers are strategic if the initial inventory level is too high.

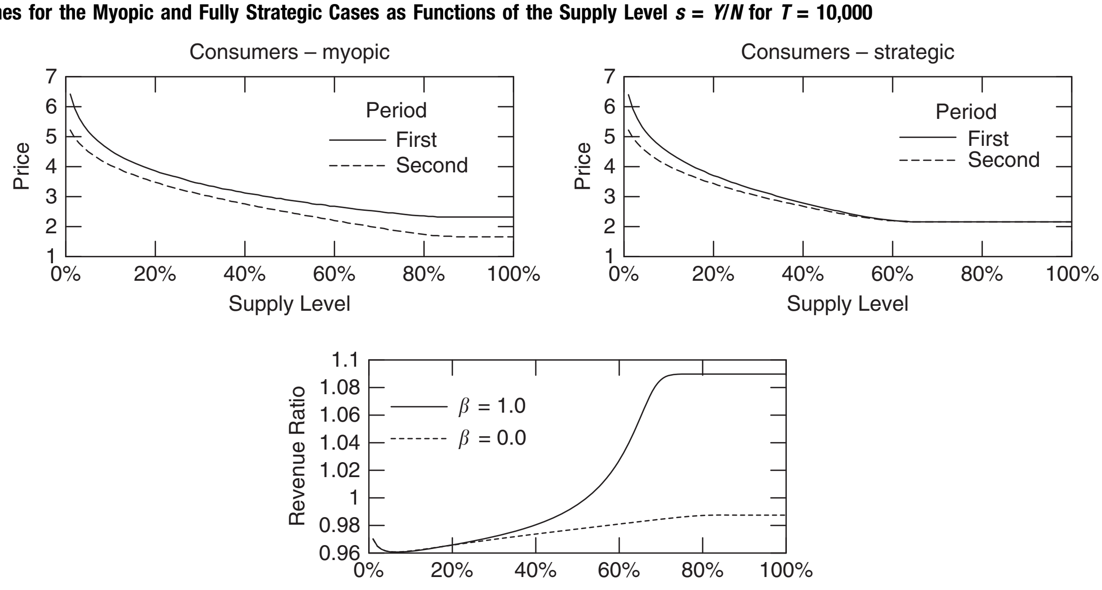
5.1.2 Relative Cost of Assuming Myopic Behavior. Next, we consider the ratio of the expected total revenues $R^0_{\beta,s}(0, 100)/R_{\beta,s}(0, 100)$, which represents a relative cost to the company of its assumption that the customers are myopic in a situation where they are not. The ratio is shown in the top right plot of Figure 4 as a function of s. A few observations are in order. First, *the loss in revenues due to ignoring the strategic nature of the customers can be very significant*. For $\beta = 1$ the drop is as high as 20% for the supply level near 87%. Other numerical trials showed that for a time horizon of $T = 100{,}000$ the maximum loss is smaller and is approximately 13%, for even shorter time horizons the maximum loss decreases further. Therefore, taking into account the strategic nature of consumers may not be as important if customers only have a few shopping opportunities. Another useful observation from the plots is that, for the supply levels under about 30%, the loss in revenues does not exceed 1%. We also remark that the loss in revenues becomes very small (well under 1%) for all supply levels if the customer utility of a purchase after 1/10 of the planning period is 10% of an immediate purchase, i.e., $\beta^{T/10} = 0.1$. This suggests that *taking into account the strategic nature of consumers is unnecessary if customers' strategicity is such that they prefer to make their purchases within much shorter time spans than the planning period of the company*. Thus, we see that even if the exact value of β is unknown, this model may allow a company to assess whether or not strategic behavior is important. If company losses from disregarding strategic behavior for a wide range of strategicity parameter values are minor, then the company may stick with a pricing policy which assumes myopic behavior. The validity of this conclusion may be affected if the capacity is not costless and the performance is measured in terms of profit. We examine this issue further in Section 5.2.

5.1.3 Relative Boost in Revenues Resulting from Myopic Behavior Assumption for Supply Levels near 100%. An additional observation about Figure 4 concerns a somewhat surprising boost in revenues resulting from the assumption that the customers are myopic when they are, in fact, strategic for supply levels close to 100% (also observed for shorter time horizons). This type of phenomenon is not unusual in game theory in general. It occurs in this case, since the pair of company-customer strategies under consideration is no longer an equilibrium (indeed, the company assumes in its pricing policy that the customers are myopic, while the actual *strategic* customers are aware of that and adjust their behavior accordingly). In the well-known prisoner's dilemma, for example, both players gain if they *simultaneously* deviate from the equilibrium. We can confirm that the prisoner's dilemma-type situation occurs here by looking at the plots of utility ratios $U^0_{\beta,s}(0,100)/U_{\beta,s}(0,100)$ presented in the bottom left plot of Figure 4. We see that the customers *gain* (at any supply level) if the company uses the correct value of β instead of assuming myopic behavior ($\beta = 0$) since the ratio never exceeds 1. To conclude, we remark that the company can avoid this non-Pareto-optimal equilibrium, if the supply level is a decision variable, and it is known that the customers are strategic. Indeed, there is no gain in revenue from bringing the supply level close to 100%. The company will always work with smaller values of s as suggested by the plot of the expected revenues.

5.1.4 Optimal Price as a Function of Supply Level for Different β's. We also examine the optimal price $p^*_{\beta,s}(0,100)$ as a function of s in the bottom right plot of Figure 4. The optimal price in case of strategic consumers is decreasing for smaller-to-medium values of s but may exhibit an upward jump at a higher value of s. The optimal price for higher values of β is smaller. Incidentally, this explains why the consumer utility increases if the company knows that the customers are strategic – because the price goes down.

5.1.5 Comparison of Fully Dynamic Contingent and Pre-announced Pricing Policies. Some businesses operate under deterministic pricing policies rather than contingent ones, which are the focus of this paper. In practice, such policies are often pre-announced, as in the case of retail chain Filene's Basement with its automatic 25–50–75% price reduction scheme. However, even if a deterministic policy were not pre-announced, fully rational consumers would be able to reconstruct it from the available information. Therefore, we can assume that the deterministic policy is known to the consumers, and substitute it into the utility recursion (a similar computation was done in Equation (26) for the pricing policy which assumes that consumers are myopic). Since the expected consumer utility for each time, state, and each pre-announced policy is available, the company can use this information to optimize pre-announced prices. We have implemented such calculation for the case of two prices, the time horizon of $T = 10{,}000$, and a price switch at time $t = 5000$. The best prices were selected using a search over the price grid with step 0.02. The resulting two-price policies for the cases of myopic and fully strategic consumers are shown in the top two plots of Figure 5. Similarly to the fully dynamic case, we observe that the first period prices in a market with strategic consumers are lower. However, the company prefers to keep prices higher in the second period when supply levels are high. This can be explained by the desire of the company to discourage waiting on the part of strategic consumers. We also observe very little price flexibility for supply levels above 40%. The ratio of expected revenues of the pre-announced policies over fully dynamic ones is shown in the bottom left plot of Figure 5. When consumers are myopic, we see that pre-announced policies always perform worse than fully dynamic ones (up to 4% for low supply levels and more than 1% for high supply levels). For the case of strategic consumers, we also see that pre-announced pricing policies underperform when supply levels are low. However, if supply levels are higher than

Figure 5 Prices in Preannounced Two-Price Policy and the Ratios of Expected Revenues of the Preannounced Two-Price Policies Over the Fully Dynamic Ones for the Myopic and Fully Strategic Cases as Functions of the Supply Level $s = Y/N$ for $T = 10{,}000$



approximately 50%, the pre-announced policy outperforms the fully dynamic one (up to 9% for supply levels above 70%). This finding is in agreement with those of Aviv and Pazgal (2008), who also observed that announced pricing policies can be advantageous to the seller compared with contingent pricing schemes (although this conclusion was obtained under different assumptions). A study of pre-announced pricing strategies in a model setting similar to ours is an interesting topic for future research.

5.2. Implications of Strategic Consumer Behavior on Optimal Initial Capacity Decisions

In this subsection, we assume that the initial market size N is fixed but the company can select the amount of initial capacity Y at the variable cost c per unit. The optimal Y is then selected to maximize the expected present value of profit at time 0:

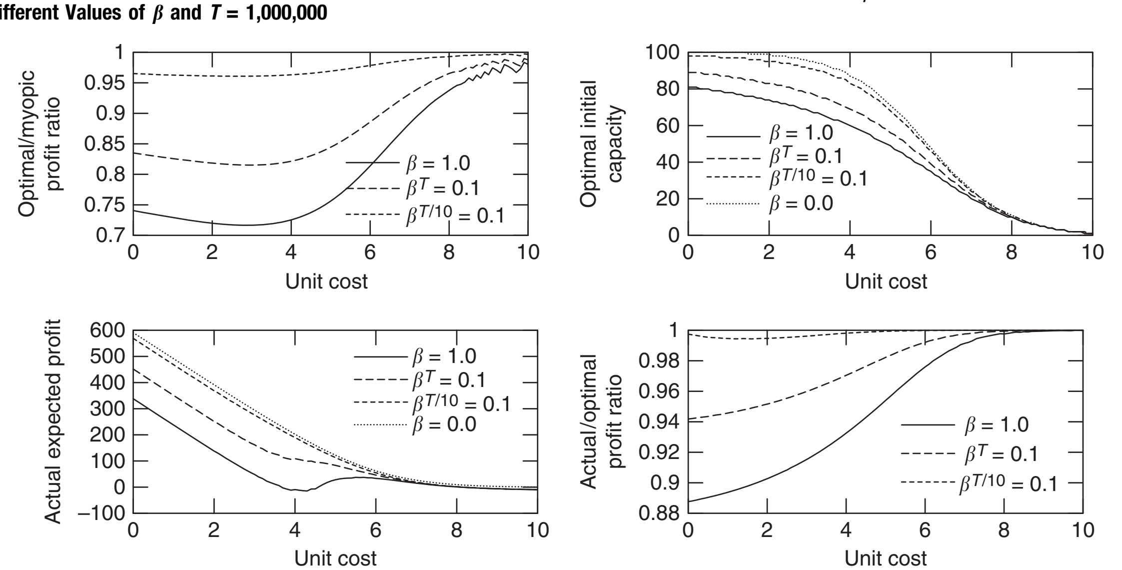
$$\max_{Y=0,\ldots,N} \{R_{\beta,Y/N}(0,N) - cY\}. \tag{27}$$

Different values of c will result in different optimal solutions, which we denote as $Y_\beta(c)$. The corresponding optimal profits are denoted as $\pi_\beta(c)$. All results discussed below use the same parameter settings as in the previous subsection.

5.2.1 Effects of Strategicity on Profits and Optimal Procurement Decision When it is Correctly Taken into Account. The top left plot of Figure 6 shows the ratios of the optimal expected profit $\pi_\beta(c)$ for different values of strategicity parameter β to the optimal expected profit $\pi_0(c)$ in the myopic case as functions of $c \in [0, 10]$. We examine the same values of β as in the previous subsection. The main conclusion is that the *impact of consumer strategicity can be quite substantial*, exceeding 25% of profits if customer behavior changes from myopic to fully strategic, *even if the company can correctly take it into account*. On the other hand, when customers are only moderately strategic ($\beta^{T/10} = 0.1$), then the impact is $<5\%$ of profits in the entire range of c. The strategicity impact is stronger for the lower values of unit cost c, but becomes smaller as c approaches 10. As suggested in the previous subsection, *the correct way for the company to respond is to decrease the initial capacity,* thus limiting strategic consumer behavior. We can see this from the top right plot of Figure 6, which shows the optimal initial capacity for different values of strategicity parameter β as functions of $c \in [0, 10]$. For lower values of c the optimal initial capacity in the strategic case is up to approximately 30% lower than in the myopic case. However, when c is high and the optimal Y is small to start with, the difference between myopic and strategic case also becomes small since customers have only limited opportunities for strategic behavior.

5.2.2 Possible Loss of Profitability if Strategicity is Ignored. If the company ignores strategic consumer behavior both in pricing and capacity decisions, the impact on profits can be disastrous. The bottom left plot of Figure 6 shows the actual expected profits $R^0_{\beta,Y_0(c)/N}(0,N) - cY_0(c)$ resulting from using the pricing policy $p^*_{0,s}(\cdot)$ and the initial capacity decision $Y_0(\cdot)$ (both optimal for the myopic case) when the customers are in fact strategic as functions of c for different values of β. Some of the worst situations – *near zero or negative profit* – are when the unit cost is in the range [3,5] and near 10. One may be interested in how much of this dramatic drop in profit is due to pricing.

Figure 6 The Ratio of Optimal Expected Profits $\pi_\beta(c)/\pi_0(c)$, the Optimal Initial Capacity $y_\beta(c)$, the Actual Expected Profit Resulting from Using the Myopic Case Policies when Customers are Strategic, and the Ratio of the Actual and Optimal Expected Profits $\pi^0_\beta(c)/\pi_\beta(c)$ as Functions of Unit Cost c for Different Values of β and T = 1,000,000



5.2.3 Contribution of Capacity vs. Pricing Decisions in Proper Response to Strategicity. We also examine the actual expected profit for the case when the capacity is selected optimally under the realization that the pricing policy will result in revenues $R^0_{\beta,Y/N}(0,N)$ rather than $R_{\beta,Y/N}(0,N)$:

$$\pi^0_\beta(c) = \max_{Y=0,\ldots,N}\left\{R^0_{\beta,Y/N}(0,N) - cY\right\}.$$

In fact, the company would be unable to realize that the expected revenue is $R^0_{\beta,Y/N}(0,N)$ under its incorrect myopic model of demand, but the ratio $\pi^0_\beta(c)/\pi_\beta(c)$ can be used to gauge how much of the optimal profit must be attributed to using the correct pricing policy. This ratio is shown in the bottom right plot of Figure 6 as a function of c for different values of β. When the unit cost is low, the pricing policy correctly accounting for the strategic behavior contributes up to around 10% of the optimal profit in the fully strategic case. When the unit cost is high, a correct capacity decision appears to be more important since the profit ratio attained by it is close to 1. This can be explained by generally low levels of the optimal capacity when the unit cost is high.

Overall, the results of this subsection suggest that taking into account strategic behavior of consumers is very important in capacity decisions. Moreover, a proper capacity choice can partially compensate for consumer strategicity.

6. Conclusions and Directions for Future Research

The paper presents a new, inherently stochastic, game-theoretical dynamic pricing model for a finite population of strategic customers. This model explicitly incorporates the strategic nature of consumers in the decision-making process. The model falls within the general class of stochastic dynamic games but generalizes stochastic consumer behavior in a manner that, to our knowledge, has not previously been achieved in the literature. We demonstrate the existence of a unique subgame-perfect equilibrium pricing policy under reasonable assumptions, provide optimality conditions, and explore some of the structural properties of solutions. We also discuss implications of strategic consumer behavior for overall performance as well as pricing and procurement decisions. The proposed computational procedure allows evaluation of this model on problems of realistic size.

The future research related to this model may include the following topics:

(1) considering situations when the parameters n and/or y are not known to the company or customers exactly, and describing the situations in which it would be beneficial for the company to reveal the information about n and/or y to the customers; and
(2) a dynamic game model of interactions between the company and strategic consumers in the presence of consumer and company learning.

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Supporting Information

Additional supporting information may be found in the online version of this article:

Appendix S1. Assumptions, Notation, and Proofs

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