Product Upgrades with Stochastic Technology Advancement, Product Failure, and Brand Commitment

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Abstract

The optimal upgrade strategy in the presence of stochastic technology has previously been characterized as a threshold policy based on the incumbent product's level of technology. By considering the impact of brand commitment in a model of product upgrades, we analyze an alternative policy based on the level of pent-up demand for the next generation product. When there is no risk of product failure, the two policies are proven to be equivalent and optimal. However, if there is a risk of a product failure, the threshold policy based on pent-up demand is a more robust approach to managing upgrades. In circumstances where a product failure is likely to result in significant lost sales, the structure of policy can change from upgrading to offering a price reduction when pent-up demand exceeds the threshold value. We perform numerical experiments to examine the impact of brand commitment on upgrade frequency and profitability under different scenarios related to the pace and magnitude of technology. The implications of the findings are discussed in the context of the smartphone industry.

1. Introduction

In many technology intensive industries, consumer demand for a durable product is shaped by stochastic advancements in technology driven by multiple types of component suppliers. Most component suppliers operate in competitive markets where they are under pressure to continually improve upon the capabilities of their products. This creates an environment in which the technology horizon is constantly expanding and where firms can purchase leading-edge technology at any point in time to upgrade their incumbent product.

Thus, firms in these industries must constantly assess the trade-off between continuing to offer their current product and releasing an upgrade with leading-edge technology. Continuing to sell a product without leading-edge technology results in sales erosion and prevents a firm from capturing incremental sales associated with the latest advancements. On the other hand, upgrading incurs a significant cost associated with introducing the next-generation product. In situations where the firm decides not to upgrade, it can consider whether to offer a temporary price reduction (TPR) to slow sales erosion associated with the technology gap. In order to choose the best option in any given period, the firm must take account of the stochastic nature of technological advancement and its impact on sales erosion and market growth.

Product upgrade strategies are further complicated by a number of firm specific factors. Firms face uncertainty regarding whether a product launch will be successful, due to the relatively high occurrence of design, manufacturing, or supply chain problems (Fernandes and Paunov 2014). This is particularly true when firms rely on multiple component vendors in addition to contract manufactures. To a large extent, the probability of a failed launch is dependent upon the firm's competencies in all facets of the upgrade process, but can escalate with the size and complexity of the upgrade in terms of bridging a technology gap. (Meyer et al. 1997; Tatikonda and Rosenthal 2000; Bhaskaran and Ramachandran 2011). In the current digital age, the risk of lost sales from launch problems or product defects can escalate due to the wide dissemination of reviews from professional and social media sites coinciding with the launch. (Chen and Xie 2005). Given the ramifications of a failed product launch on the firm's profits, the risk of this occurrence needs to be accounted for in the upgrade strategy.

Firms must also consider how customers respond to the technology gap of the incumbent product to accurately assess whether or not to upgrade. While firms suffer erosion in sales due to the advancement of technology, not all lost sales from an incumbent product necessarily result in lost sales to the firm. Sales erosion of customers committed to a brand may often be recaptured, since committed consumers may wait for a future product release rather than exit the firm's market. This form of brand commitment is fairly robust for firms with dominant brands resulting from factors related to brand identity and social influence. However, it may also occur for other firms due to network effects and switching costs usually present in technology products.

Brand commitment has important implications for modeling the upgrade decision, since eroding sales may be converted into demand for a future product launch. We characterize the strength of a firm's brand commitment as the willingness of consumers to wait for an upgrade. Consequently, high brand commitment enables a firm to aggregate substantial *pent-up demand* for their next generation product. Unlike sales erosion which only increases with the technology gap, pent-up demand is a dynamic process. Customers' vary in terms of their willingness to wait for a future product release and over time may lose patience and exit the firm's market.

In this paper we develop a Markov decision process (MDP) that governs a firm's decision on when to upgrade and when to offer a TPR given exogenous stochastic advancements in technology. In the baseline case where TPRs are ineffective at slowing attrition and there is no potential for product failure, a state-dependent threshold policy on the level of pent-up demand determines whether upgrading in the current period is optimal. The threshold is monotonically non-increasing in the product's lag in leading-edge technology. As a result, we can construct a technology-based threshold policy similar to the structure of policies described in the literature (Balcer and Lippman 1984; Krankel et al. 2006; Liu and Özer 2009; Cho and McCardle 2009).

Incorporating TPRs into the decision process does not change the structure of the optimal upgrade policy. We provide two sufficient conditions pertaining to the optimality of offering TPRs. The first is based on the level of brand commitment for which there exists a threshold on pent-up demand for determining if the firm should offer a TPR. The second provides a bound on the launch cost for which the firm will always prefers to upgrade over the option of offering a TPR for any cost below the bound. Higher degrees of brand commitment raise the bound on cost supporting the premise that firms with high levels of brand commitment can rely less on TPRs when optimizing their upgrade strategies.

If there is a risk of a failed product launch, and TPRs do not generate additional sales, the optimal upgrade strategy maintains the form of a threshold policy based on pent-up demand. However, the threshold is non-monotonic in the technology lag implying that a technology based upgrade policy is not guaranteed to be optimal. This has important organizational implications, since forecasters may not work in technology departments. Furthermore, for firms where branding is an important determinant of consumer purchasing behavior, the result suggests that an upgrade policy constructed on critical values of pent-up demand is a more robust approach to managing product upgrades compared to one based solely on technology.

We establish a sufficient condition on the likelihood and degree of lost sales from a product failure to guarantee that the optimal upgrade strategy is characterized by a pent-up demand threshold policy. Therefore, when the risk of failure is sufficiently low, managers can continue to use an upgrade strategy based on pent-up demand. In addition, we provide a second sufficient state-dependent condition in terms of the impact of failure that guarantees that the firm's optimal strategy is to sell the incumbent product with a TPR for a sufficiently high level of pent-up demand. In these states, TPRs act as a mitigation strategy against the risk of a failed launch by incentivizing a portion of pent-up customers to buy the incumbent product in the current period. Subsequently, it may be optimal for the firm to release the upgrade once the risk associated with a product failure has been sufficiently reduced. This result further emphasizes the importance of having appropriate linkages between the decision processes governing the upgrade and pricing strategies.

We carry out a series of numerical experiments to examine how the model performs under different scenarios related to the pace and magnitude of technology advancements, where the pace refers to frequency of technology advancements and magnitude refers to the size of incremental demand due to an increase in technology. We find that the variability in upgrades and profits associated with brand commitment increases when market growth is driven by pace. Moreover, in fast paced markets, the rate of increase tends to increase as well in brand commitment. Although the variability in profits decreases when growth is driven by magnitude, profit levels are higher across all levels of brand commitment. The results also show that TPRs are primarily used by firms with low brand commitment. However, as pace and magnitude decrease, more firms make use of offering TPRs. We show that the pace and magnitude also impact a high brand commitment firm's optimal response to increased lost sales from a failed product. Finally, the results show that incorporating the potential for failure into the decision model helps to mitigate against tremendous profit loss, particularly for high brand commitment firms. The implications of these findings are discussed in the context of the smartphone industry.

2. Related Literature

Brand commitment plays a pivotal role in the timing and choice of purchasing decisions in markets for technology intensive consumer products (Johnson et al. 2006). Extensive research shows that many consumers remain committed to a particular brand, even when faced with sequential product releases with incremental improvements from different brands (Posavac et al. 2004; Lam et al. 2010). This holds true even though consumers can accurately differentiate between the relative value proposition of the various models in terms of price and performance (Liu and Liang 2014). There is a variety of inter-related factors that explain the enduring effect of brand commitment through multiple product cycles. If a consumer has purchased other products offered by the firm, then there is considerable disincentive for consumer to switch brands. The value of a brand to consumers increases with time due to learning, network effects, customization, and switching costs (Shugan 2005). Firms are focused on magnifying these effects through platform initiatives such as cloud services, support for online user groups and providing access to applications and proprietary content (Pon et al. 2014). While success of these initiatives varies among brands, they provide important motivation for customers to remain committed to a familiar brand.

Social networking has a powerful influence on brand commitment and consumer purchasing behavior. Research shows that social interaction often creates a process where adoption decisions tend to coincide among participants (Wang et al. 2012; Sun 2013). In addition, the pervasiveness of mobile technology products has made brand image an increasingly important aspect of social identity (Lam et al. 2010). The combined effect of social influence and identity helps to explain why consumers with brand commitment often postpone their purchase until the brand upgrades an aging product.

Consumer commitment to a dominant brand often results in a self-sustaining process. Prior to purchasing, committed consumers tend to place a higher premium on positive information about the brand's product while minimizing negative information (Ahluwalia et al. 2000; Posavac et al. 2004). Loss aversion also reinforces commitment as consumers usually see a dominant brand as the safest alternative (Hardie et al. 1993, Muthukrishnan et al. 2009). Likewise, after the purchase, consumers are much more likely to be satisfied with their choice and less likely to suffer buyer's remorse (Chu et al. 2013).

Research on technology markets demonstrates that branding has a major impact on consumer purchasing behavior. However, this factor has yet to be accounted for in decision models on the timing of product upgrades. Rather, the majority of this research focuses on the trade-off between performance and time-to-market. This trade-off was first explored by Cohen et al. (1996) for a fixed sales window based on market size potential, competitive offerings, and the pace of product improvements. Morgan et al. (2001) extend this research for a firm that introduces multiple generations of a product over a finite selling horizon in order to maintain or improve market share. Souza (2004) consider the problem in terms of an industry's clockspeed based on a firm's input costs related to product development, production, and inventory. Observing that many firms introduce product upgrades at fixed intervals, both Druehl et al. (2009) and Liao and Seifert (2015) analyze factors related to product development costs and diffusion in order to determine the optimal product introductory pace. In the aforementioned models, the advancement of technology is deterministic and market demand is independent of the level of technology. Thus, these models are not necessarily applicable to the upgrade problem for firms in industries where market potential increases due to stochastic improvements in technology driven by component suppliers.

Balcer and Lippman (1984), Krankel et al. (2006), Liu and Özer (2009), Cho and McCardle (2009), and Lobel et al. (2015) all study the timing of upgrades with exogenous stochastic technology improvements. Balcer and Lippman (1984) consider the timing of technology adoption over an infinite time horizon. Given a fixed cost of technology adoption, they demonstrate that a firm should adopt the state of the art technology once the lag in technology exceeds a threshold value. Cho and McCardle (2009) extend Balcer and Lippman (1984) to investigate the adoption of two complementary technologies over time. Their model analyzes the interaction between co-dependent technologies, and focuses on situations where an increase in the lag of one technology leads to an upgrade in the dependent technology. In both models, profit is a linear function of the technology levels, implying that profit is strictly improving in a firm's incumbent technology level. These models are most applicable to manufacturing firms that adopt new technology in order to reduce the cost of production.

Our formulation and assumptions are closely related to Krankel et al. (2006) and Liu and Özer (2009). Similar to Krankel et al. (2006), the firm's optimization problem is structured as a MDP where the firm makes a binary decision between launching and delaying the next-generation product release. Although the research by Krankel et al. (2006) has the added benefit of incorporating demand diffusion, their model pertians to a monopolistic environment where technology always increases the firm's potential market. Thus, delays in product upgrade result in discounted rather than lost sales. Liu and Özer (2009) consider a decision model from the perspective of a follower firm in a duopoly. The follower firm makes their upgrade decision in response to a market leader, given the assumption that the follower firm's demand rate depends on the performance gap between their own and the market leader's technology levels. The presence of the market leader is comparable to the outside option embedded in our model with respect to the impact of a lag in technology on demand. However, similar to Cohen et al. (1996), Morgan et al. (2001), and Souza (2004) market demand in Liu and Özer (2009) is independent of technology improvements and technology improvements only cause erosion in the firm's market share. Thus, beyond the inclusion of branding and product failure, our formulation of the problem differs from Krankel et al. (2006) and Liu and Özer (2009) because technological advancement can result in both incremental market growth as well as attrition in the firm's market potential.

Lobel et al. (2015) analyze the decision of whether to pre-announce launch dates for the next-generation product, or simply release upgrades on the go, in the presence of strategic customers. The results show that when facing strategic customers, firms should alternate between minor and major product upgrades. Furthermore, the firm should announce that there will be a long duration between the minor and major upgrade. The intuition behind the strategy is that the entire market will purchase after a major release. However, the firm must guarantee that there will be a sufficient duration between upgrades in order to incentivize the market to purchase the product with minor improvements. The stochastic advancement of technology encourages the entire market to re-purchase the product, but does not bring new consumers to the market. Thus, the model is applicable to industries where the entire consumer base would consider purchasing the first generation product. Indeed, Lobel et al. (2015) show that if technology improvement is slow and prices are high, the firm will release a single generation and exit the market after the entire consumer base purchases the inaugural product.

This paper complements the literature by studying the joint impact of brand commitment, technology pace, and magnitude, on the management of product upgrades and TPRs. An important contribution of this research is to explore how the risk of a failed product launch impacts product upgrades and price promotions. Although firms purchase most of their new technology from component suppliers, they are often faced with significant design, production and supply chain challenges related to the integration, manufacturing and distribution of a new product (Williamson 1971; Nellore and Balachandra 2001). This problem is particularly prevalent in consumer technology industries where the launch of each new product is extensively scrutinized by a plethora of online technical and consumer oriented review sites in terms of performance metrics and product flaws. To date, other than documenting the scope of the problem, no research has analyzed how the risk of a failed product launch impacts the frequency and profitability of product upgrades. Although price reductions have been utilized in the literature as a mechanism to manage inventory during product transitions (Li and Graves 2012), we show that pricing has implications as a mitigation strategy when there is a high risk of failure.

3. Model Description

Consider a firm selling a product comprised of component-based technology over an infinite planning horizon. At the start of each period the firm decides whether to continue selling the incumbent product or to release a product upgrade and whether or not to offer a TPR. We simplify the effects of the TPR to a reduced unit profit level accompanied by increasing demand. The firm's decision are denoted by the binary variables x and y, where the firm upgrades the product when $x = 1$ and offers a TPR when $y = 1$. We assume that the firm withdraws the previous generation from the market following an upgrade.

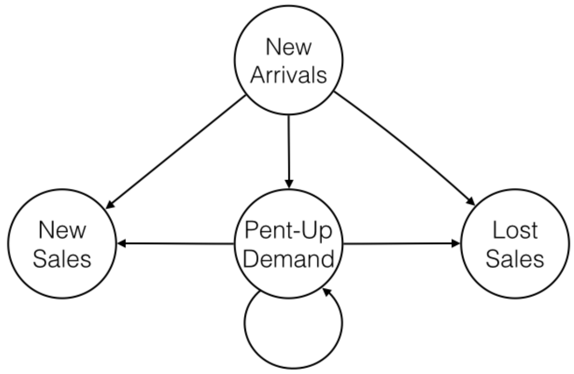
The firm faces two sources of uncertainty when planning upgrades. The first is the advancement of component technology in each period. The second source is that a newly released product may fail (due to design or manufacturing defects) resulting in lost sales. Although the increase in component capabilities is exogenous to the firm's actions, the probability that the product fails is partially controlled by the firm. For firms integrating multiple components into a product, a firm's risk of a failed product escalates with the size of the upgrade (Tatikonda and Rosenthal 2000; Meyer et al. 1997). Therefore, the level of uncertainty associated with releasing a failed product varies throughout the planning horizon and is dependent on the technology lag of the incumbent product.

A segment of the overall market has a preference for the firm's product based on factors related to branding such as marketing, reputation for quality, and network effects. This segment is commonly known as the firm's *serviceable market*. In each period, consumers from the firm's serviceable market look to purchase the firm's product. These consumers are referred to as *new arrivals*. If the firm's product lags leading-edge technology in a given period or the firm is selling a failed product, then only a portion of the new arrivals will result in sales. New arrivals who do not buy the product may delay their purchase to the next period rather than leave the market. Similar to the no purchase option in Li and Graves (2012), consumers leaving the firm's serviceable market can be interpreted as purchasing a product from a competitor. This portion of new arrivals along with previous arrivals who are still waiting for the release of an upgrade forms the firm's *pent-up demand*. The extent to which consumer continue waiting is dependent upon the firm's brand commitment, where greater degrees of brand commitment result in a higher proportion of consumers continuing to wait for the next-generation product. The firm can also experience erosion in pent-up demand, as waiting customers may exhaust their patience and leave the firm's market. The TPR serves as a mechanism to prevent some loss of sales from new arrivals and pent-up demand due to product failure or dated technology. The dynamics of consumers are illustrated in Figure 1. The exact movements of consumers in any period is dependent upon the state dynamics and firm decisions, which is described in the next section.

3.1 System State and Dynamics

After the firm makes the upgrade and pricing decisions, new arrivals look to purchase the product. The number of arrivals is dependent upon the number of potential customers in the serviceable market $n \in \Re^+$. The number of arrivals who purchase the product in a period is dependent upon the firm's incumbent level of technology relative to the leading-edge, $z \in \mathbb{Z}^+$, the degree of product failure $f \in \mathbb{Z}^+$ (where $f = 0$ signifies that the product is a complete success), and the firm's TPR decisions $y \in \{0, 1\}$. Given a technological lag z, failure level

Figure 1: The movements of consumers through the system. The number of new and lost sales is dependent upon the firm decisions and the state of the system.



f, and decision y, the proportion of arrivals that purchase a product in a period is $\rho_y(z, f)$. A portion of the customers who do not purchase the product (due to product failure or a lag in technology) may delay leaving the market. These customers become part of the pent-up demand $d \in \Re^+$, and wait to see if the firm releases a product upgrade in the next period. The proportion of consumers who delay their purchasing decision depends on the degree of the firm's brand commitment, which is represented by the brand commitment parameter $\theta \in [0, 1]$. We make the following assumptions on the consumer arrivals and sales.

Assumption 1 (New Arrivals)**.** *For serviceable market size n, the new arrivals in a period is $a(n)$. The proportion of new arrivals $a(n)$ is nondecreasing in the market size n with $da(n)/dn \leq 1$ and $a(0) = 0$. For technology lag z, failure level f, and TPR decision y, the proportion of new arrivals that purchase is $\rho_y(z, f)$, which is nonnegative, nonincreasing in z and f with $\rho_y(0, 0) = 1$ for $y \in \{0, 1\}$, and $\rho_0(z, f) \leq \rho_1(z, f)$ for all $z, f \in \mathbb{Z}^+$. The resulting number of total sales from new arrivals in the current period is $a(n)\rho_y(z, f)$. The proportion of new arrivals that do not purchase and wait at least one additional period for a new product release is $\theta\bar{\rho}_y(z, f)$, where $\bar{\rho}_y(z, f) = 1 - \rho_y(z, f)$ is the proportion of new arrivals that do not purchase a product. If the firm upgrades then all new arrivals look to buy the product in that period and will not wait any further periods.*

The first part of assumption 1 states that the per period arrivals do not change too sharply with an increase in n. In a period where the firm's product contains leading-edge technology (i.e., $z = 0$) and exhibits no degree of failure (i.e. $f = 0$), then all consumers that enter the market purchase the firm's product. If $f > 0$ and/or the firm did not upgrade ($x = 0$) and $z > 0$, then $a(n)\bar{\rho}_y(z, f)$ of the new arrivals do not buy in period t. For a fixed level of z and f, the firm will not have more sales when not offering a TPR ($y = 0$) compared to offering a TPR ($y = 1$). Finally, if consumers arrive in a period where the firm releases a new product but do not purchase due to a degree of product failure, they leave the firm's market. If the firm does not upgrade and has perfect (no) brand commitment, then $\theta = 1$ ($\theta = 0$) and consumers who did not purchase the incumbent product always (never) wait for an upgrade or price drop. If $\theta \in (0, 1)$, then the consumers have a certain willingness to wait. When consumers delay their purchasing decisions, they will only buy the product from the firm once the firm provides an upgrade or a TPR.

Assumption 2 (Pent-up Demand)**.** *The proportion of pent-up demand customers that purchase the product given decision* y *is* $\phi_y(z, f)$*. If the firm offers an upgrade and the level product failure is* f*, then the proportion of pent-up demand consumers who purchase the product is* $\phi_y(0, f)$*, where* $\phi_y(0, 0) = 1$ *and* $\phi_0(0, f) \leq \phi_1(0, f)$ *for all* $f \in \mathbb{Z}^+$*. If the firm does not upgrade nor offer a TPR, then none of the pent-up demand will purchase the product, i.e.* $\phi_0(z, f) = 0$ *for all* $z > 0$*. In addition,* $\phi_y(z, f)$ *is nonnegative and nonincreasing in* z *and* f*. If the firm does not upgrade, then the number of consumers from pent-up demand that continue to wait at least one additional period is* $\theta\bar{\phi}_y(z, f)$*, where* $\bar{\phi}_y(z, f) = 1 - \phi_y(z, f)$*. If the firm upgrades then all of the pent-up demand looks to buy the product and will not wait any further periods.*

Assumption 2 states that if the firm releases an upgraded product without any degrees of failure, then all pent-up demand purchases the product. If there is a degree of product failure, the magnitude of the product failure influences the degree of lost sales associated with the product launch. For example, a malfunctioning sensor in a multi-attribute product may have a modest impact on sales, whereas a defective battery could result in a much steeper loss of sales. The proportion of lost sales due to failure is less when the firm offers a TPR with the upgrade. Similar to sales, the proportion of consumers that purchase given a TPR and no upgrade decreases with lags in technology or higher degrees of product failure. If there is no upgrade, then part of the pent-up demand that did not purchase a product (in the event of a TPR) will continue to delay their purchase. If there is an upgrade, then all customers will look to buy the product. If there is a failure, then the firm will lose sales, and these customers will not continue to wait.

Assumption 3 (Stochastic Elements)**.** *The advancements of technology over time are statistically independent and identically distributed as a nonnegative integer random variable* ζ *with a finite expectation, i.e.* $E[\zeta] < \infty$*. An advancement of technology* ζ *results in an incremental market growth of* $g(\zeta)$*, where* $g(\zeta)$ *is nonnegative and nondecreasing in* ζ*. Given the technology lag, product failures are statistically independent across upgrades and do not depend on the path of technology realization. They are identically distributed as a nonnegative integer random variable* $\xi(z)$ *with a finite expectation, i.e.* $E[\xi(z)] < \infty$*.*

Assumption 3 discusses the stochastic nature of technology and product failure. The likelihood and extent of a product failure is stochastic and depends on the lag in technology. The probability of successfully launching the product is nonincreasing with the size of the upgrade in terms of technology. Although the upgrade is based on component technology, firms have to integrate the component technology into the existing product architecture. This is consistent with the literature, which supports that an "innovative" versus "incremental" product improvement has inherent risk in successfully launching the product (Bhaskaran and Ramachandran 2011). Once the firm realizes their sales, component vendors announce the technological progress made over the course of the period. The advancement of component technology is stochastic and increases the serviceable market of the firm.

3.2 Optimal Control Problem

The cost of launching the next-generation product is K. The fixed launch cost incorporates various activities related to withdrawing older generation products from the market, distributing and restocking the new product, marketing and re-branding, as well as costs related to sales, support, and administration. For a given lag z, if the firm upgrades the product and makes TPR decision y, then the level of product failure is a random variable $\xi(z)$. The firm incurs launch cost K and earns sales $a(n)\rho_y(0,\xi(z)) + d\phi_y(0,\xi(z))$ at profit margin of πr^y, where $r \in [0,1)$ corresponds to the size of the TPR discount. Since the firm upgraded there is no pent-up demand for the next period. After sales are realized, technology randomly increases by ζ, which grows the market by $g(\zeta)$. Therefore, the market potential for the firm in the next period is $n - a(n) + g(\zeta)$ and the state in the next period is $\big(0, \xi(z), n - a(n) + g(\zeta), \zeta\big)$. If $\xi(z) = 0$, then the product completely meets consumer expectation and the level of sales simplifies to $a(n) + d$.

If the firm does not upgrade, then the firm realizes $a(n)\rho_y(z,f) + d\phi_y(z,f)$ sales at the profit margin $r^y\pi$. The new arrivals that form pent-up demand is $\theta a(n)\bar{\rho}_y(z,f)$ and the remaining consumers from the pent-up demand at the start of the period is $\theta d\bar{\phi}_y(z,f)$. Since the failure level remains at f, given an increase in technology ζ, the state in the next period is $\big(\theta(a(n)\bar{\rho}_y(z,f) + d\bar{\phi}_y(z,f)), f, n - a(n) + g(\zeta), z + \zeta\big)$. Introducing the discount factor $\delta \in [0,1)$, the firm's optimal control problem is

$$V(d,f,n,z) = \max_{x,y\in\{0,1\}} E\big[\pi r^y S - xK + \delta V(D,F,N,Z)\big] \tag{3.1}$$

$$\text{s.t.} \quad S = a(n)\rho_y(\bar{x}z, F) + d\phi_y(\bar{x}z, F) \tag{3.2}$$

$$D = \bar{x}\theta\big(a(n)\bar{\rho}_y(\bar{x}z,F) + d\bar{\phi}_y(\bar{x}z,F)\big) \tag{3.3}$$

$$F = \bar{x}f + x\xi(z) \tag{3.4}$$

$$N = n - a(n) + g(\zeta) \tag{3.5}$$

$$Z = \bar{x}z + \zeta. \tag{3.6}$$

4. Analysis

Problem (3.1)-(3.6) is an infinite horizon continuous state MDP. The following proposition shows that there is indeed an optimal stationary policy.

Proposition 1. *There exists an optimal (Markovian) stationary policy that solves (3.1)-(3.6).*

To provide insight into the structure of the optimal policy we consider special cases using two conditions: (1) No risk of product failure and (2) TPRs that are ineffective in generating additional sales from pent-up demand or new arrivals. The first condition can occur when a firm has refined the design and manufacturing process to eliminate the risk of failure. The second condition approximate markets for highly innovative products, where consumer utility is driven by ownership of new technology rather than purchasing older products at discounted prices. Formally, the conditions are defined as follows:

Condition 1 (No risk of product failure). $Pr(\xi(z) = 0) = 1$ *for all* $z \in \mathbb{Z}^+$.

Condition 2 (Ineffective TPRs). $\rho_0(z,f) = \rho_1(z,f)$ *and* $\phi_0(z,f) = \phi_1(z,f)$.

These conditions are used to study the following four cases: (1) No risk of failure with ineffective TPRs; (2) no risk of failure with effective TPRs; (3) risk of product failure with ineffective TPRs; and (4) risk of product failure with effective TPRs. Using the four cases, we study the optimality of threshold policies based on pent-up demand. The policies have the form of either upgrading and/or offering a TPR once pent-up demand exceeds a state-based threshold value. This contrasts the existing literature where policy decisions are characterized by the firm's technology lag.

4.1 No Risk of Failure and Ineffective TPRs

If there is no risk of of product failure, then $f = 0$ in each stage of the MDP. For brevity we remove f from the state space. In addition, since TPRs do not stimulate additional sales, offering a TPR will only results in the firm earning less revenue. Thus, in the simplest case of no risk of failure and ineffective promotions, there is no incentive to ever offer a TPR. This implies that the firm's decision space is reduced to the binary decision of releasing an upgrade. The optimal policy states that for each pair (n, z), there exists a threshold level of pent-up demand $D^*(n, z)$, which dictates the firm's upgrade decision. It is optimal for the firm to introduce (delay) the next-generation product if and only if the pent-up demand is above (below) the value $D^*(n, z)$.

Proposition 2. *If Conditions 1 and 2 hold, then there exists a threshold $D^*(n, z)$ such that it is optimal to upgrade the product if and only if the pent-up demand $d \geq D^*(n, z)$.*

The threshold policy provided by Proposition 2 defines a set of switching curves based on pent-up demand over the state space $\{n, z\}$. Since the optimal policy is to upgrade when $d \geq D^*(n, z)$, if the level of pent-up demand is exactly at or above the threshold value, then it is optimal for the firm to invest in an upgrade. The next proposition demonstrates monotonicity of the upgrade threshold policy in z.

Proposition 3. *If Conditions 1 and 2 hold, then the optimal threshold value $D^*(n, z)$ is non-increasing in the technology lag z for a fixed value of n. An equivalent threshold policy can be constructed using $Z^*(d, n)$, where it is optimal to upgrade the product if and only if the technology lag $z \geq Z^*(d, n)$.*

Proposition 3 shows that there is greater pressure on the firm to upgrade their product as the technology lag grows, and that firms require a large amount of pent-up demand to warrant an upgrade at smaller lags. The monotonic behavior of threshold $D^*(n, z)$ implies that the optimal policy can alternatively be expressed as a threshold in the lag of technology $Z^*(d, n)$.

4.2 No Risk of Failure and Effective TPRs

If TPRs are effective in generating sales, then the firm needs to incorporate price promotions into their strategy. If there is no potential for product failure, then all new arrivals and pent-up demand will purchase the product following an upgrade independent of the TPR decision. Therefore, there is no incentive to offer a TPR when releasing an upgrade. This implies that the firm only has to consider three decisions in each period: waiting to upgrade without a TPR, waiting to upgrade with a TPR, and upgrading without a TPR.

Proposition 4. *If Condition 1 holds, then there exists a threshold $D^*(n, z)$ such that it is optimal to upgrade the product if and only if the pent-up demand $d \geq D^*(n, z)$. In addition, the* *optimal threshold value $D^*(n,z)$ is nonincreasing in the technology lag z for fixed n, implying an equivalent technology-based threshold $Z^*(n,z)$.*

Without the risk of product failure, the optimal policy is characterized by the same properties as Propositions 2 and 3 for the case of no risk and ineffective TPRs. Thus, a sufficient level of pent-up demand will result in the firm upgrading, even with TPRs. However, Proposition 4 does not provide insight into when a firm should offer a TPR. The next two results demonstrate that the decision on whether to offer a TPR is dependent on the firm's level of brand commitment as well as the cost of launching the product.

Corollary 1. *For states with component z such that $\theta \leq \frac{r\phi_1(z)}{\delta}$, if the firm delays upgrading, there exists a state-dependent threshold value in pent-up demand $\tilde{D}(n,z)$, such that it is optimal to offer a TPR if pent-up demand $d \geq \tilde{D}(n,z)$.*

The corollary provides a sufficient condition on the brand commitment parameter that guarantees a threshold value $\tilde{D}(n,z)$ separating the decision to offer a TPR. The upper bound on brand commitment that guarantees the threshold value is dependent and nonincreasing in the technology lag, suggesting that the firm will prefer to wait rather than offer a TPR as technology increases. A TPR is less effective in attracting consumer sales, as the lag of technology increases. The greater the degree of brand commitment, the less incentive a firm has to offer a TPR before an upgrade. The firm will maintain a sufficient level of pent-up demand and obtain more sales at the higher profit margin by not offering a TPR.

Firms have to upgrade frequently to prevent loss of sales from technology advancement. TPRs provide firms with an opportunity to release fewer upgrades by boosting sales of the incumbent product. The effectiveness of TPRs throughout the planning horizon is highly dependent upon the current state of the market, pent-up demand, and technology, as well as key parameters such as branding, the effectiveness of TPR, and the firm's discount factor (forward looking behavior). To obtain conditions under which a firm would prefer to upgrade rather than offer a TPR, we compare the respective profit for each decision. The difference in the number of sales between upgrading and offering a TPR is $\Delta s = a(n)\big(1-\rho_1(z)\big)+d\big(1-\phi_1(z)\big) = a(n)\bar{\rho}_1(z)+\bar{\phi}_1(z)d$. The corresponding difference in revenue is $\Delta\pi = \pi a(n)\big(1-r\rho_1(z)\big) + \pi d\big(1-r\phi_1(z)\big)$. With a slight abuse of notation we define $\tilde{K}(d,n,z) = \Delta\pi - \delta\theta\pi\Delta s$.

Corollary 2. *For states (d,n,z), if $K < \tilde{K}(d,n,z)$, upgrading is strictly more profitable than offering a TPR.*

The corollary provides a sufficient state dependent condition on the launch cost that guarantee that the firm will not offer a TPR. Moreover, analysis of the sufficient condition establishes that the threshold is increasing in each state-component, but decreasing in key model parameters. Observing that $\Delta\pi > \delta\theta\pi\Delta s$, greater differences in sales (between upgrading and offering a TPR) leads to a higher threshold value bounding K. Thus, if n and/or d increase, the impact on revenue from immediate sales due to upgrading increases, which raises the value of $\tilde{K}(d,n,z)$. If technology increases, then fewer people comprising the pent-up demand and new arrivals purchase the product given a TPR, leading to an increase in $\tilde{K}(d,n,z)$. This shows that TPRs are less effective compared to upgrading and that there is a greater range of launch costs where the firm will strictly prefer to upgrade when firms have high levels of pent-up demand, market size, and/or lags in technology. On the other hand, as branding (θ), forward looking behavior

(δ), and the revenue from TPRs increase, the value of $\tilde{K}$ decreases. In addition, as the launch cost increases, the inequality in Corollary 2 may no longer hold. If the inequality does not hold due to changes in the values of the states or parameters, then firms have to carefully consider if upgrading is the optimal strategy.

4.3 Risk of Failure and Ineffective TPRs

The potential for releasing a failed product implies that the threshold results established under Condition 1 are not guaranteed to hold. However, the following result demonstrates that the upgrade policy is still characterized as a state-dependent threshold policy based on pent-up demand, where it is optimal to upgrade if and only if $d \geq D^*(f, n, z)$.

Proposition 5. *If Condition 2 holds, then there exists a threshold value where it is optimal to upgrade if and only if $d \geq D^*(f, n, z)$. For fixed n and f the threshold $D^*(f, n, z)$ is generally non-monotonic in technology lag z.*

An interesting property of the optimal policy given the potential for a product failure is that the optimal threshold value $D^*(f, n, z)$, unlike the previous thresholds, is not guaranteed to be monotonically nonincreasing in z. Since the pent-up demand threshold value is not monotonically nonincreasing in z, a single technology lag threshold upgrade policy may not exist. The presence of multiple technology threshold values reveals that the structure of the profit-to-go function is more complex in the presence of product failures. When the technology lag is high, a firm may want to continue accumulating pent-up demand instead of releasing an upgrade. Although the lag in technology continues to grow, the firm may be able to increase the level of pent-up demand, making an upgrade profitable. This can provide incentive for firms to further delay product upgrades, trying to raise pent-up demand beyond the threshold value. On the other hand, if there is a substantial risk associated with long durations between upgrades, then the potential for product failures may force firms to release their upgrades more frequently.

4.4 Risk of Failure and Effective TPRs

In the previous sections where there was no risk of product failure and/or TPRs were ineffective, firms had no incentive to offer a price promotion when releasing an upgrade. However, in the most general case, TPRs may act as mitigation strategy to prevent excessive loss of sales due to product failures. Thus, in the general case, the firm has to consider all four strategies. Observing that TPRs do not impact the state dynamics when upgrading, the decision to offer a TPR with an upgrade is determined entirely by the expected sales in the current period.

Proposition 6. *A firm will not offer a TPR with an upgrade if and only if the following inequality holds:*

$$a(n)E\big[\rho_0(0, \xi(z))\big] + dE\big[\phi_0(0, \xi(z))\big] \geq r\Big(a(n)E\big[\rho_1(0, \xi(z))\big] + dE\big[\phi_1(0, \xi(z))\big]\Big). \tag{4.1}$$

The proposition shows that depending on the functional form of ρ and ϕ and the size of the discount r, either offering a TPR or not can be a strictly dominant strategy. Since the inequality (4.1) is linear in the level of pent-up demand d, there exist at most one point in pent-up where the optimal decision switches between offering and not offering a TPR for fixed (n, z). With additional structure on ϕ and the stochastic behavior of ξ, we can further characterize this behavior.

Corollary 3. *Consider fixed $(\tilde{d},\tilde{n},\tilde{z})$ where inequality (4.1) is satisfied. If $\phi_0(0, f) - r\phi_1(0, f)$ and $\rho_0(0, f) - r\rho_1(0, f)$ are increasing in f, and $\xi(z+1) \geq_{st} \xi(z)$, then inequality (4.1) will hold for all $z > \tilde{z}$. If $E[\rho_0(0, \xi(z)) - r\rho_1(0, \xi(z))] > 0$, then inequality (4.1) will hold for all $n \geq \tilde{n}$. If $E[\phi_0(0, \xi(z)) - r\phi_1(0, \xi(z))] > 0$, then inequality (4.1) will hold for all $d \geq \tilde{d}$.*

Remark 1. For fixed $(\tilde{d},\tilde{n},\tilde{z})$ where (4.1) does not hold, if $\phi_0(0, f) - r\phi_1(0, f)$ and $\rho_0(0, f) - r\rho_1(0, f)$ are decreasing in f and $\xi(z+1) \geq_{st} \xi(z)$, then inequality (4.1) will hold for all $z > \tilde{z}$. There are similar parallel sufficient conditions where inequality (4.1) will not hold for all $n \geq \tilde{n}$ and $d \geq \tilde{d}$.

The previous proposition and corollary describe when a firm should offer a TPR conditional on an upgrade. Thus far, the thresholds of the optimal decision have stated that when pent-up demand surpasses a certain level the optimal decision is to upgrade. However, if upgrades are risky (there is a high probability of excessive lost sales due to a product failure) and TPRs are effective, it is uncertain that this form of policy will hold in in general. Indeed,

Proposition 7. *For fixed (f, n, z) the optimal policy in general cannot be characterized by a threshold $D^*(f, n, z)$ where it is optimal to upgrade (with or without a TPR) for all $d \geq D^*(f, n, z)$.*

Due to the interaction of product failure and technology, a control policy where the firm upgrades given a adequately high level of pent-up demand remains valid for lower lags in technology. However, if the gap in technology grows to the point where the complexity of an upgrade is likely to lead to a failed product release, then offering a TPR may become an optimal strategy if the firm has a high level of pent-up demand.

Proposition 8. *If for all $z \in \mathbb{Z}^+$ the inequality*

$$\max_{y\in\{0,1\}} E\big[\phi_y(0, \xi(z))\big] \geq \max\big(r, \delta\theta\big) \tag{4.2}$$

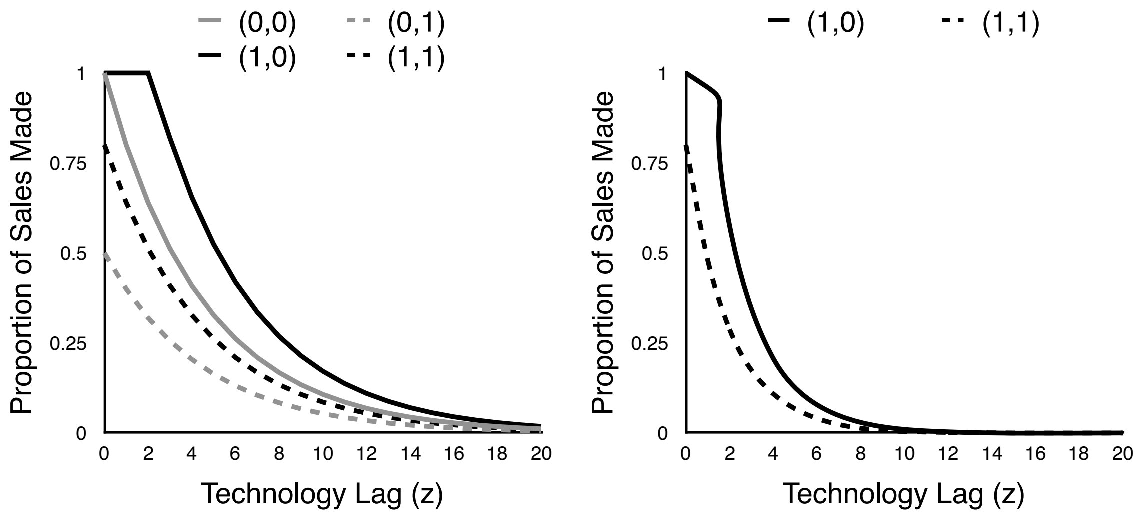
holds, then there exists a threshold value where it is optimal to upgrade with or without a TPR if and only if $d \geq D^*(f, n, z)$. For states (z, f), if the inequality

$$r\phi_1(z, f) \geq \max\left(\max_{y\in\{0,1\}} E\big[r^y\phi_y(0, \xi(z))\big], \delta\theta\right) \tag{4.3}$$

holds, then there exists a threshold value where it is optimal to offer a TPR without an upgrade if and only if $d \geq D^*(f, n, z)$.

The left-hand side (right-hand side) of condition (4.2) (condition (4.3)) is nonincreasing in the technology lag. As a result, if there is a sufficient risk of a product failure, condition (4.3) may be satisfied, and the firm is better off offering a price promotion and waiting to upgrade. The price promotion encourages a portion of the pent-up demand to purchase the incumbent

Figure 2: Level of sales for new arrivals and waiting consumers given technology lag z and failure level f.



(a) $\rho_y(z, f)$

(b) $\phi_1(z, f)$

product. The reduction in the level of pent-up demand in the subsequent period reduces the lost sales from a potential product failure. Thus, TPRs can be used as a strategy to mitigate against risky upgrades. Condition (4.3) is most likely to be realized when $f = 0$, showing that successful products can continue to provide firms with additional benefits in later stages of the upgrade release.

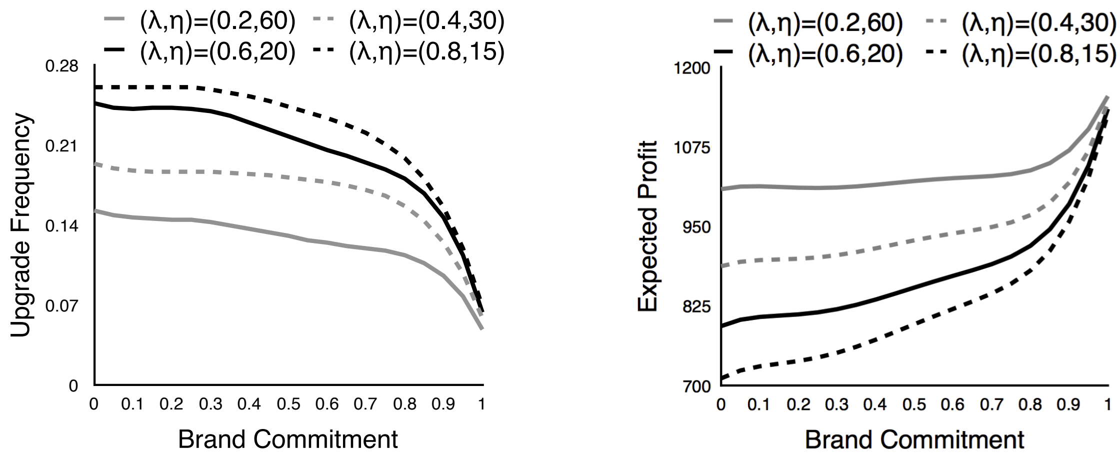
5. Numerical Experiments

We present results from four sets of numerical experiments that study the influence of technology advancement, failure, and brand commitment on the firm's upgrade strategy. The experiments simulate technology advancements and product failures to approximate the frequency of upgrades and TPRs. The experiments seek to answer the following questions:

1. What is the impact of brand commitment on the frequency of upgrades and profitability of firms?
2. What is the impact of brand commitment on the frequency of TPRs and what is the impact of TPRs on profitability?
3. How do high brand commitment firms respond to greater risks associated with product failure?
4. What is the impact on profits when firms neglect the potential for product failure?

For each experiment, we use value iteration to find the optimal stationary policy and perform 1000 simulations over 1000 time periods. For brevity, we only provide the functional forms and parameters that are directly relevant to the discussion. We refer the reader to Online Appendix B for complete details on the MDP, functional forms of the sales process, and parameter selection.

Figure 3: Upgrade frequency and the increase in profit due to brand commitment for various technological paces and magnitudes at a fixed expected market growth.



(a) Upgrade Frequency (b) Profit

The advancement of component technology in each period is binary, improving by an increment of 1 unit with probability $\lambda \in [0, 1]$ or remaining unchanged with probability $1 - \lambda$. An improvement in technology leads to η new consumers entering the firm's potential market. Therefore, λ represents the pace of technological advancement and η represents the magnitude of this advancement. The product $\lambda\eta$ is the firm's per period expected market growth. This functional form is the same as the one used to model technology in Krankel et al. (2006). We also model product failure as binary. If $f = 0$ ($f = 1$), then the product launch is a success (failure). If $f = 1$, the proportion of sales made decreases by a factor of μ, i.e. $\rho_y(z, 1) = \mu\rho_y(z, 0)$ and $\phi_y(z, 1) = \mu\phi_y(z, 0)$.

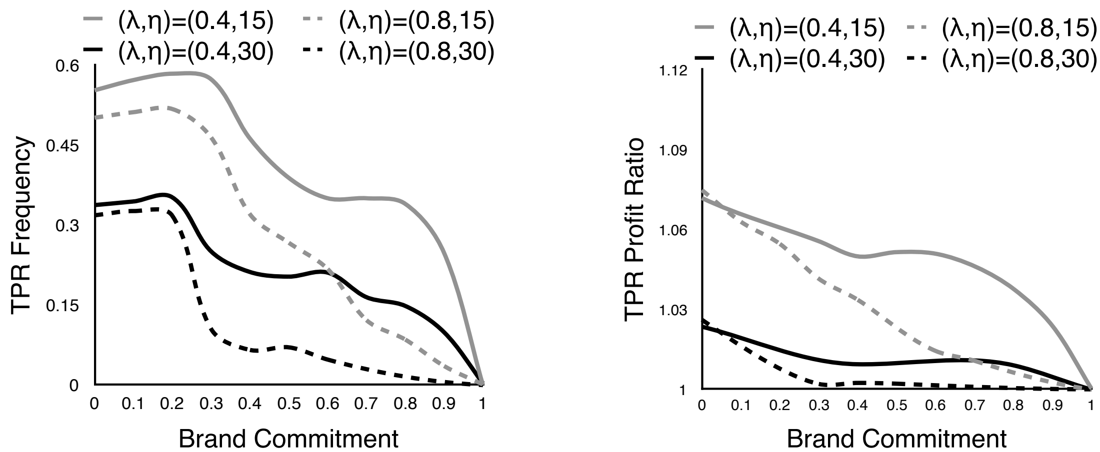
The volume of sales from the pool of new arrivals and pent-up demand is dependent upon whether the firm offers a TPR, if the firm's product meets consumer expectation, and the incumbent product's technology lag. Figure 2 (a) shows the proportion of sales from new arrivals for TPR decision y and failure level f for $\mu = 0.5$. Figure 2 (b) shows the proportion of sales from pent-up demand when the firm offers a TPR at failure level f.

5.1 Impact of Technology Pace and Magnitude

To study the impact of technology pace (λ) and technology magnitude (η) on the frequency of upgrades and profits, we vary each parameter fixing the expected per period market growth to $\lambda\eta = 12$ consumers. Figures 3 (a) plots the upgrade frequency and shows that the frequency of upgrades increases when market growth is driven by higher pace rather than higher magnitude across all levels of brand commitment. The figure also shows that the upgrade frequency decreases with brand commitment and that for high brand commitment firms this reduction in the number of upgrades is greater when technology is driven by faster technology pace.

Figure 3 (b), which plots the expected profit, shows that there is a corresponding trend between lower upgrade frequency and higher profits. As the pace of technology becomes faster, the relative increase in profit rises dramatically for firms with high levels of brand commitment. When market growth is driven by magnitude rather than pace, the overall level of profit increases

Figure 4: TPR frequency and TPR profit ratio at different levels of brand commitment, technology pace and magnitude.



(a) TPR Frequency

(b) TPR Profit Ratio

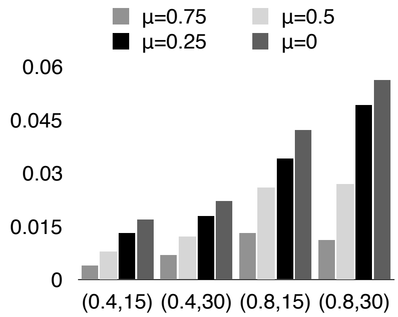
for all firms, while the variation in profit across brand commitment decreases. Therefore, given a particular value of per period market growth, the value of brand commitment diminishes with increase in magnitude matched by the corresponding decrease in pace. This shows that investments in increasing a firm's level of brand commitment have the potential to be more valuable in fast paced markets.

Fast paced technology improvements with low magnitude is becoming a common feature in mature markets, given the near ubiquitous penetration of CPUs, digital displays, memory and other related componentry and the fierce competition among component makers to supply combined markets. Therefore, the results suggest that brand commitment will have an important advantage for firms as the product category matures. Finally, the results show that when the formulation of the problem is restricted to full brand commitment (which has the interpretation of a monopolist), variation in the values of pace and magnitude for a fixed rate of market growth has little impact on upgrade frequency or profit.

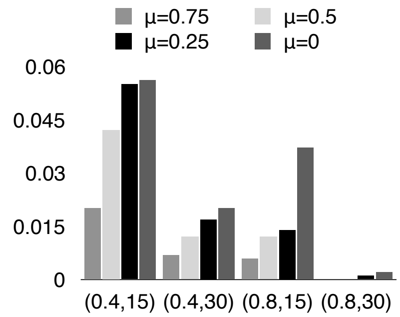
5.2 Impact of TPRs

We examine the variation in the impact of TPRs associated with brand commitment for four different combinations of pace and magnitude. Figure 4 (a) plots the frequency of TPR use. Figure 4 (b) plots the TPR profit ratio, which we define as the profit improvement using the optimal TPR policy relative to the case in which the firm does not use TPRs. Figure 4 (a) shows that while both slower pace and lower magnitude increase TPR frequency, this effect is much greater for the latter. When pace rises, the use of TPRs becomes increasingly concentrated to a narrow spectrum of firms with low brand commitment. As pace or magnitude decreases, the use of TPRs increase for all firms. Figure 4 (b) shows that the increase in profit associated with TPRs is almost exclusively due to magnitude rather than pace. Irrespective of pace, lower magnitude decreases the frequency of upgrades which provides more opportunities for firms to use TPRs profitably. This particularly holds for firms without the ability to aggregate meaningful pent-up demand. In situations, where slower pace is combined with lower magnitude, the TPR profit

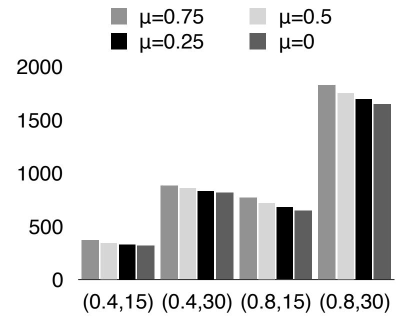
Figure 5: The impact of lost sales from failure for a high brand commitment firm at different levels of technology pace and magnitude.



(a) Difference in Upgrades



(b) Difference in TPRs



(c) Expected Profit

ratio for high commitment firms increases in tandem with the TPR frequency.

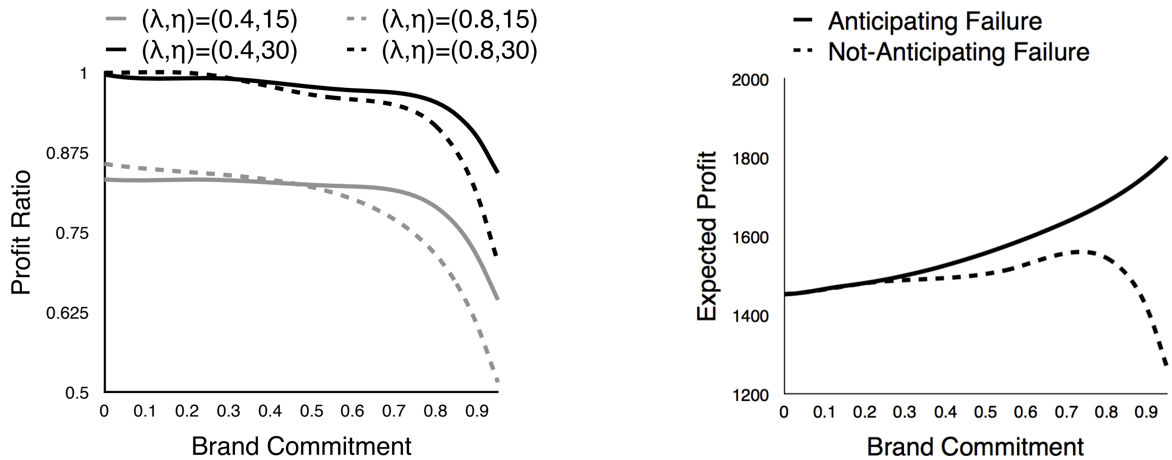
5.3 Response to Product Failure

To avoid excessive lost pent-up demand sales, high brand commitment firms respond to increased risks from failure by modifying their frequency of upgrades and/or TPRs. We examine how increasing lost sales from failed products impact firms with high brand commitment ($\theta = 0.9$) by comparing the difference in upgrades, TPRs, and profits for $\mu \in \{0.75, 0.5, 0.25, 0\}$ to the case of $\mu = 1$. Recall that a lower value of μ implies an increased loss of sales from releasing a failed product and that $\mu = 1$ is equivalent to the case of no product failure. For each case we consider four different market scenarios characterized by high or low magnitude and fast or slow pace.

Figure 5 (a) plots the upgrade frequency at μ minus the upgrade frequency when $\mu = 1$. The figure shows that the firm increases the frequency of upgrades as the level of lost sales increases and that the increase in frequency is greater at faster technology paces and higher technology magnitudes. Figure 5 (b), which plots the difference in TPR frequency compared to $\mu = 1$, shows that firms also respond to increased risk of product failure by increasing the frequency of offering TPRs. For fixed λ, the figure highlights that the increase in TPRs is greater for low technology magnitude. These results suggest that the optimal response to an increased risk in product failure is highly dependent upon the stochastic nature of technology. Figure 5 (c), which plots the expected profit at each level of μ, shows that the reaction strategies are largely effective in limiting the loss in profit due to product failure. The figure also shows that high brand commitment firms lose greater profit as the pace and magnitude of technology increase. This indicates that in fast pace and higher magnitude environments, firms need to rely on a more costly strategy of additional upgrades over TPRs in the presence of greater lost sales from failure.

These results support the conclusions drawn from the analysis of Proposition 8. We provide additional examples in Online Appendix B to illustrate the control policies when there is a high risk of failure. The examples show market scenarios where high levels of pent-up demand result in the use of TPRs rather than upgrading. The control plots demonstrate that a firm has greater incentive to use TPRs to lower pent-up demand at lower technology magnitudes. This occurs

Figure 6: Profits from unanticipated product failure at different levels of brand commitment.



(a) Profit Loss Ratio (b) Profits for $(\lambda, \eta) = (0.8, 30)$

because it is more difficult to recover from a disproportionate loss of sales in low magnitude environments.

5.4 Neglecting Product Failure

In this section we show the impact of a failed product launch when firms do not account for the risk of a product failure. Figure 6 (a) plots the ratio of profit loss from a firm that neglects the risk of failure relative to that when the firm accounts for this risk in its upgrade policy. Clearly, the decrease in the ratio of profit from neglecting failure is dependent upon the proportion of sales that the firm generates through pent-up demand. Therefore, neglecting failure has a greater impact for firms with higher levels of brand commitment. The results from Figure 6 (a) show that the degree of profit loss for high commitment firms ($\theta > 0.5$) is impacted more by magnitude than pace, but in both cases, increases in the two parameters will lower the profit ratio as well as increase the spectrum of firms that are impacted by neglecting the risk of failure. Indeed, in fast paced and high magnitude environments, firms with high brand commitment will build pent-up demand more quickly and consequently suffer more losses from the absence of a constraint to limit the size of this demand in proportion to the risk posed by a failed product launch. Figure 6 (b) plots the total profit across brand commitment for $(\lambda, \eta) = (0.8, 30)$ with and without accounting for failure. The figure shows that when profit loss potential is high, utilizing brand commitment to accumulate pent-up demand can lead to lower profits compared to a firm that does not accumulates any pent-up demand.

It is important to point out that these results show the ramifications of not accounting for failure under the best circumstances, since pent-up demand in the baseline model is still optimized based on the other factors in the upgrade policy. To the extent that the aggregation of pent-up demand extends beyond what is optimal due to poor planning or an unintended product delay, then the risk associated with product failure would increase.

5.5 Discussion on the Smartphone Industry

The smartphone industry provides a compelling illustration of how brand commitment impacts product upgrades. Apple has the highest level of brand commitment in the industry. Despite this, many times in the past, industry analysts have predicted the demise of the iPhone's market share as their product cycles have expanded relative to their competitors. Although Apple generally experiences sales erosion towards the latter stages of their product cycle, they always recapture these sales through the aggregation of pent-up demand (Laugesen and Yuan 2010). A recent example occurred in 2013 when Apple took 13 months to launch a single upgrade despite the release of numerous models with updated componentry from their competitors. Nevertheless, when Apple released its new model at the end of September of 2013, it sold close to 9 million phones globally in the opening weekend recapturing the lost market share from the previous three months. While iPhone may be the most prominent example of the impact of brand commitment on a product release cycle, these dynamics come into play in many other technology industries (Hui 2004; Sriram et al. 2006; Anderson et al. 2013). Although the impact on launch frequency and profits may be relatively modest in comparison to Apple, the difference of a few percentage points in profit may be a key determinant of a firm's survival in a highly competitive component based technology industry.

The smartphone industry also provides a vivid illustration of the consequences associated with the risks of aggregating pent-up demand. Blackberry had one of the highest measures of brand commitment in the industry which enabled the firm to build significant pent-up demand prior to each of its launches. However, due to factors related to poor product planning and unforeseen engineering problems, it launched a phone without a functional web browser after a sufficient duration between products where many committed customers were waiting for an upgrade. Solidifying Blackberry's downfall was their decision to release a touch screen model despite the fact that their remaining pool of committed customers were waiting for a phone with the firm's proprietary QWERTY keyboard (Silcoff et al. 2013). By comparison, LG, which often releases multiple smartphones throughout the year and does not count on the buildup of pent-up demand, experienced a number of disappointing product launches in 2012. This only impacted their sales for that calendar year but had little effect on their sales from product launches in successive years. To date Apple has managed to avoid failed product launches with the iPhone. However, the viral spread of the Bendgate rumor associated with the launch of the iPhone 6+ is a potent reminder of how quickly news on a potential product flaw can spread through social media sites. Fortunately for Apple, they were able to effectively counter this rumor through social media and third party reviews which demonstrated that the iPhone 6+ did not bend from regular use.

6. Future Research

There are several avenues for future research related to our formulation of the upgrade due to stochastic technology. Given the impact of pent-up demand on the firm's profit in fast paced technology environments, the question arises as to whether firms should invest in initiatives that incentivize customers to wait for an upgrade through loyalty programs. Although there is an extensive literature on the efficacy of these programs in a variety of settings, this issue has not been studied in the context of the product upgrades related to stochastic technology. Based on the experimental results, it appears that the potential benefits of these programs will be weighted more towards firms with high brand commitment. However, firms with lower levels of brand commitment may still benefit, if these programs reduce their reliance on TPRs.

Another avenue of research relates to whether the buildup of pent-up demand enables firms to achieve efficiencies in procurement and production. Not only does the firm's product launches meet greater demand, but there is also greater certainty in the level of sales during the launch period compared to the remaining portion of the product selling window due to stochastic technology. The inclusion of these efficiencies could help firms further optimize their upgrade policy based on the level of brand commitment. Reduced costs due to greater demand could help to further explain why firms with high brand commitment enjoy a major profit advantage over competitors.

Finally, this work could benefit from more precise data on how quickly firms in fast paced technology industries can recover from a failed product launch and how it impacts brand commitment going forward. Despite the fact that our model considers a best case scenario, we show that the inclusion of failure in the policy still provides considerable loss mitigation for high commitment firms in fast paced high magnitude markets. While there are some case studies of how a failed product launch impacts brand commitment, there is little empirical research investigating this issue. The importance of adjusting the policy to account for this risk will be even more compelling as the pace of technology and product complexity continues to accelerate.

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A. Technical Appendix

Proof of Proposition 1

Consider a general MDP that consists of

1. A Borel (or Polish) space $\{S, B(S)\}$, where S is a Borel subset of a complete separable metric space and $B(S)$ its family of Borel subsets.
2. A Borel Space $\{A, B(A)\}$, where again A is a Borel subset of a complete separable metric space and $B(A)$ its family of Borel subsets, and a collection of sets $A_s \in B(A)$ for each $s \in S$. Let $B(A_s)$ denote the induced collection of Borel subsets of A_s. Furthermore, we require existence of a measurable function δ mapping s into A with $\delta(s) \in A_s$ for each $s \in S$.
3. A family $\mathcal{P}(A_s)$ of probability measures on $B(A_s)$
4. Real-valued reward functions $r_t(s, a)$ which for each $t \in T$ satisfy
   a. $r_t(\cdot, \cdot)$ is measurable with respect to $B(S \times A_s)$,
   b. $r_t(s, \cdot)$ is integrable with respect to all $q \in \mathcal{P}(A_s)$ for all $s \in S$.
5. Conditional probabilities $p_t(\cdot|s, a)$ which for $t \in T$
   a. $p_t(G|\cdot, \cdot)$ is measurable with respect to $B(S \times A_s)$ for $G \in B(S)$,
   b. $p_t(G|s, \cdot)$ is integrable with respect to all $q \in \mathcal{P}(A_s)$ for all $s \in S$ and for $G \in B(S)$.

Theorem 6.2.12-c of Puterman (2014) states that for Polish space S, probability measure P on the Borel subset of S, there exists an optimal stationary policy if each action set of allowable action in state s, A_s is finite for a MDP as outlined above. Since the space A has 4 possible decisions and S, which is $\Re^{+2} \times \mathbb{Z}^{+2}$ is a Polish space, $r_t(s, \cdot)$ and $p_t(G|\cdot, \cdot)$ are measurable with respect to $B(S \times A_s)$ for $G \in B(S)$, and $r_t(s, \cdot)$ and $p_t(G|s, \cdot)$ are integrable with respect to all $q \in \mathcal{P}(A_s)$ for all $s \in S$ and for $G \in B(S)$, there is an optimal stationary policy.

Preliminary Results for Remaining Proofs

Before presenting the proofs, we present several technical results where Condition 1 holds. The first result pertains to the monotonicity properties of the state dynamics.

Lemma 1. *For two system states that evolve under the same sequence of technological advancements, if $n^1_{t_0} \leq n^2_{t_0}$ then $n^1_t \leq n^2_t$ for all $t > t_0$. If the two system states also have the same upgrade and pricing decisions, then*

- *for system states $(d^1_{t_0}, n^1_{t_0}, z_{t_0})$ and $(d^2_{t_0}, n^2_{t_0}, z_{t_0})$, if $d^1_{t_0} \leq d^2_{t_0}$ and $n^1_{t_0} \leq n^2_{t_0}$, then $d^1_t \leq d^2_t$ for all $t \geq t_0$, and*
- *for system states $(d_{t_0}, n_{t_0}, z^1_{t_0})$ and $(d_{t_0}, n_{t_0}, z^2_{t_0})$, if $z^1_{t_0} \leq z^2_{t_0}$, then $z^1_t \leq z^2_t$ for all $t \geq t_0$.*

Proof. To show monotonicity in the state component n, assume that $n^1_t \leq n^2_t$ for some $t \geq t_0$. Independent of the upgrade decision, $n^1_{t+1} = n^1_t - a(n^1_t) + g(\zeta) \leq n^2_t - a(n^2_t) + g(\zeta) = n^2_{t+1}$, where the inequality follows form the inductive assumption and the fact that $\frac{da(n_t)}{dn_t} \leq 1$.

To show monotonicity in the state component d_t for systems $(d^1_{t_0}, n^1_{t_0}, z_{t_0})$ and $(d^2_{t_0}, n^2_{t_0}, z_{t_0})$, consider policies that make product upgrade and pricing decisions in the same periods. In period t_0, the levels of pent-up demand for systems 1 and 2 are $d^1_{t_0}$ and $d^2_{t_0}$, respectively. Since $d^1_{t_0} \leq d^2_{t_0}$, assume that $d^1_t \leq d^2_t$ holds for some $t \geq t_0$. In period $t+1$, if the firm does not upgrade nor offer a price drop, then $d^1_{t+1} = \theta\big(d^1_t + a(n^1_t)\bar{\rho}_0(z_t)\big) \leq \theta\big(d^2_t + a(n^2_t)\bar{\rho}_0(z_t)\big) = d^2_{t+1}$, where the inequality holds due to the inductive assumption, the fact that the dynamics of n_t and z_t are independent of pent-up demand, and that $n^1_t \leq n^2_t$ for all time t. In period $t+1$, if the firm does not upgrade but offers a price drop, then $d^1_{t+1} = \theta\big(\phi_1(z_t)d^1_t + a(n^1_t)\bar{\rho}_1(z_t)\big) \leq \theta\big(\phi_1(z_t)d^2_t + a(n^2_t)\bar{\rho}_1(z_t)\big) = d^2_{t+1}$. If the firm upgrades in period t then $d^1_{t+1} = d^2_{t+1} = 0$ and the result $d^1_{t+1} \leq d^2_{t+1}$ trivially holds. This implies that $d^1_t \leq d^2_t$ for $t \geq t_0$.

A similar inductive reasoning is applicable to prove that $z^1_t \leq z^2_t$, for all $t \geq t_0$. In period t make the inductive assumption that $z^1_t \leq z^2_t$. If the firms do not upgrade in period $t+1$, then $z^1_{t+1} = z^1_t + \zeta_t \leq z^2_t + \zeta_t = z^2_{t+1}$. If the firm upgrades in period $t+1$, then $z^1_{t+1} = \zeta_t = z^2_{t+1}$ and the result trivially holds. □

The monotonicity property of the state dynamics implies that under the same realization of technology, if two firms have the same product introduction timing, then the firm with greater pent-up demand or better technology will maintain their advantage until both firms upgrade their products, whereas the firm with greater market potential will maintain the advantage until time T. In addition, the lemma establishes that the advantage resulting from greater pent-up demand and market potential is nonincreasing with time.

Proposition 9. *If for fixed n and z there are two systems with pent-up demands d^1 and d^2 respectively, where $d^1 \leq d^2$, then $V(d^2, n, z) \leq V(d^1, n, z) + \pi(d^2 - d^1)$.*

Proof. Consider two systems starting in stage t_0, system 1 with starting state $(d^1_{t_0}, n_{t_0}, z_{t_0})$ and system 2 with starting state $(d^2_{t_0}, n_{t_0}, z_{t_0})$, that experience the same sample path of technological advancement. Define the optimal upgrade and pricing policy for system 2 as $\tilde{\mu}_2$, the random variable τ as the time of the first product upgrade since t_0 under policy $\tilde{\mu}_2$, and $f(K)$ as is the total cost under policy $\tilde{\mu}_2$. Define the random set $\mathcal{T}_U$ as the time set between τ and ∞. In addition, define the random sets $\mathcal{T}_W$ and $\mathcal{T}_R$ as the time sets under policy $\tilde{\mu}_2$, where the firm waits for the first upgrade after t_0 without offering a price reduction and the firm waits for the first upgrade after t_0 while offering a temporary price reductions, respectively. Under $\tilde{\mu}_2$ and a sample path of technology, $\tilde{\Pi}^i_t$ is the revenue in period $t \geq t_0$ for system i. Let system 1's upgrade policy mimic $\tilde{\mu}_2$. i.e. system 1 mimics the upgrade decisions of system 2 irrespective of the system 1 state components. The resulting system 1 policy may not be Markovian in terms of system 1 state but it can be completely recovered from the history of the technology advancement process.

Denote $\Delta d_t = d^2_t - d^1_t$. Lemma 1 implies that if the firm does not upgrade nor drop the price, then $\Delta d_{t+1} = \theta\big(\Delta d_t\big)$ and if the firm does not upgrade but does offer a price drop, then $\Delta d_{t+1} = \theta\bar{\phi}_1(z_t)\Delta d_t$. Thus, $0 \leq \Delta d_{t+1} \leq \Delta d_t, \quad \forall t \geq t_0$. Since the systems have the same starting states in terms of n_t and z_t, it is clear that $0 \leq \Delta d_t \leq \Delta d_{t_0}$ for all $t \geq t_0$. The difference in the level of sales given a TPR in period t is $\Delta s_t = \phi_1(z_t)\Delta d_t$. Since $\theta \leq 1$, this implies that $\Delta d_t \geq \Delta s_t + \Delta d_{t+1}$ for all t. In addition, since $0 \leq \Delta d_t \leq \Delta d_{t_0}$ and $\Delta d_t \geq \Delta s_t + \Delta d_{t+1}$ for all t, it follows that $\sum_{t \in \mathcal{T}_R} \Delta s_t + \Delta d_\tau \leq \Delta d_{t_0}$.

Since the systems have the same starting states in terms of n and z, $\sum_{t\in\mathcal{T}_W}\delta^{t-t_0}\big(\tilde{\Pi}_t^2-\tilde{\Pi}_t^1\big)=0$. Since after the upgrade, both systems will have the same states, the only difference in revenue is the pent-up demand at time τ and so $\sum_{t\in\mathcal{T}_W}\delta^{t-t_0}\big(\tilde{\Pi}_t^2-\tilde{\Pi}_t^1\big)=\delta^{\tau-t_0}\pi\Delta d_\tau$. For the periods before τ where there are TPR, $\sum_{t\in\mathcal{T}_R}\delta^{t-t_0}\big(\tilde{\Pi}_t^2-\tilde{\Pi}_t^1\big)=\sum_{t\in\mathcal{T}_R}\delta^{t-t_0}r\pi\Delta s_t$. Letting $V^1_{t_0,\tilde{\mu}_2}$ and $V^2_{t_0,\tilde{\mu}_2}$ be random variables representing the present value of profit at t_0 using policy $\tilde{\mu}_2$,

$$\begin{aligned}
V^2_{\tilde{\mu}_2,t_0}-V^1_{\tilde{\mu}_2,t_0}&=\sum_{t\in\mathcal{T}_W}\delta^{t-t_0}\big(\tilde{\Pi}_t^2-\tilde{\Pi}_t^1\big)+\sum_{t\in\mathcal{T}_R}\delta^{t-t_0}\big(\tilde{\Pi}_t^2-\tilde{\Pi}_t^1\big)+\sum_{t\in\mathcal{T}_U}\delta^{t-t_0}\big(\tilde{\Pi}_t^2-\tilde{\Pi}_t^1\big)\\
&=\sum_{t\in\mathcal{T}_R}\delta^{t-t_0}r\pi\Delta s_t+\delta^{\tau-t_0}\pi\Delta d_\tau\\
&\le\pi\Delta d_{t_0}
\end{aligned}$$

Since the policy $\tilde{\mu}_2$ was optimal for System 2 and potentially suboptimal for System 1, the inequality $V(d^2_{t_0},n,z)=E[V^2_{\tilde{\mu}_2,t_0}]\le E[V^1_{\tilde{\mu}_1,t_0}+\pi\Delta d_{t_0}]\le V(d^1_{t_0},n,z)+\pi(d^2_{t_0}-d^1_{t_0})$ holds. Even though system 1 policy is not necessarily Markovian, it is history dependent. $V(d^1_{t_0},n,z)$ dominates the value of this policy, since there exist an optimal Markovian policy when optimizing over all history-dependent ones. □

Proposition 9 utilizes the results from Lemma 1 to bound the difference in the profit-to-go. The tightness of the bound depends on the upgrade strategy of the firm. Given that for both firms the loss of consumers in the serviceable market is the same, the firm with the demand advantage (either from a greater market size or more pent-up demand) will not be able to accumulate more demand going forward.

Proof of Propositions 2, 3, and 4

Since the case of no risk and ineffective TPRs is a special case of no risk and effective TPRs, for brevity we detail the proofs of of Propositions 2, 3 through the proof of Proposition 4. We divide proof of Proposition 4 into two sections. The first section shows the existence of the threshold D_2. We then show that when Condition 2 holds, the proof demonstrates the existence of the threshold condition D_1 proving Proposition 2. The second section shows the monotonicity property, which is similarly used to prove Proposition 3.

Proof of Proposition 4 Part I - Existence of the Optimal Threshold Value

Recall that we removed f from the state space and simplified the new arrival sales parameters to $\rho_y(z)$ and the sales from pent-up demand due to a TPR as $\phi_y(z)$. In addition, since there are only three decisions, we denote U as the value function given the decision to upgrade, W as the value function from the decision to wait to upgrade without offering a TPR, and R as the value function from the decision to wait to upgrade but offering a TPR. If $\frac{\partial}{\partial d}\big(U(d,n,z)-W(d,n,z)\big)>0$ and $\frac{\partial}{\partial d}\big(U(d,n,z)-R(d,n,z)\big)>0$ almost everywhere (a.e.) in d for all n and z, then the marginal benefit of upgrading would be increasing in pent-up demand and the proposition would hold true. Therefore, monotonicity in d would imply that if the firm would rather delay an upgrade than release an upgrade at $\tilde{d}_W$, then the firm would delay the upgrade rather than upgrade for any pent-up demand $d\le\tilde{d}_W$ and if the firm would rather offer a price reduction than release an upgrade at $\tilde{d}_R$, then the firm would rather offer a price reduction for any pent-up demand $d \le \tilde{d}_R$.

We employ value iteration method to establish the claim. The firm's profit-to-go for states (d, n, z) at value iteration index t for each decision is

$$\begin{aligned} U_t(d,n,z) &= \pi\big(d + a(n)\big) - K + \delta E\big[V_{t-1}(0, n - a(n) + g(\zeta), \zeta)\big], \\ W_t(d,n,z) &= \pi a(n)\rho_0(z) + \delta E\big[V_{t-1}(\theta(d + a(n)\bar{\rho}_0(z)), n - a(n) + g(\zeta), z + \zeta)\big], \\ R_t(d,n,z) &= r\pi\big(a(n)\rho_1(z) + \phi_1(z)d\big) + \delta E\big[V_{t-1}(\theta(\bar{\phi}_1(z)d + a(n)\bar{\rho}_1(z)), n - a(n) + g(\zeta), z + \zeta)\big]. \end{aligned}$$

Indeed, for any feasible tuple (d, n, z), if the firm upgrades in the current period, then the pent-up demand going forward is 0. Thus, d does not affect the future expectation of the profit-to-go given an upgrade, and $\frac{\partial}{\partial d}U_t(d, n, z) = \pi$ for any value of d. Consequently, the condition that $\frac{\partial}{\partial d}W_t(d, n, z)$ and $\frac{\partial}{\partial d}R_t(d, n, z)$ are less than π by a fraction strictly less than 1 is sufficient to prove monotonicity in d, which we demonstrate by induction. Using the starting condition $V_0(d, n, z) = 0$, the profit-to-go given the decision to wait is $W_1(d, n, z) = \pi a(n)\rho_0(z)$, which trivially implies that $\frac{\partial}{\partial d}W_1(d, n, z) \le \delta\theta\pi$. In addition, $R_1(d, n, z) = r\pi\big(a(n)\rho_1(z) + \phi_1(z)d\big)$, which implies that $\frac{\partial}{\partial d}R_1(d, n, z) \le \max\{r, \delta\theta\}\pi$. We make the inductive assumption that $W_t(d, n, z)$ and $R_t(d, n, z)$ are piecewise differentiable with at most countable number of points on non-differentiability and that $\frac{\partial}{\partial d}W_t(d, n, z) \le \delta\theta\pi$ and $\frac{\partial}{\partial d}R_t(d, n, z) \le \max\{r, \delta\theta\}\pi$.

Since $V_t(d, n, z) = \max\big\{U_t(d, n, z), W_t(d, n, z), R_t(d, n, z)\big\}$, based on the inductive assumptions regarding $W_t(d, n, z)$, the fact that $\frac{\partial}{\partial d}U_t(d, n, z) = \pi$, and the continuity of both functions, there exists at most one value of d (a set of measure zero), where $U_t(d, n, z) = W_t(d, n, z)$. Similarly, from the inductive assumption on $R_t(d, n, z)$, there exists at most one point (a set of measure zero), where $U_t(d, n, z) = R_t(d, n, z)$. Define the point where $U_t(d, n, z) = W_t(d, n, z)$ as $D^*_{t,W}(n, z)$ and the point where $U_t(d, n, z) = R_t(d, n, z)$ as $D^*_{t,R}(n, z)$. Between the decision of upgrading and waiting to upgrade, it is preferable for the firm to delay the upgrade for $d < D^*_{t,W}(n, z)$ and for the firm to upgrade at $d \ge D^*_{t,W}(n, z)$. Between the decision of upgrading and offering a price reduction, it is preferable for the firm to offer the price reduction for $d < D^*_{t,R}(n, z)$ and for the firm to upgrade at $d \ge D^*_{t,R}(n, z)$.

Defining $D^*_t(n, z) = \max\big\{D^*_{t,W}(n, z), D^*_{t,R}(n, z)\big\}$, the inductive assumptions imply that $\frac{\partial}{\partial d}V_t(d, n, z) \le \max\{r, \delta\theta\}\pi$ for $d < D^*_t(n, z)$ and $\frac{\partial}{\partial d}V_t(d, n, z) = \pi$ for $d \ge D^*_t(n, z)$, and thus $\frac{\partial}{\partial d}V_t(d, n, z) \le \pi$ a.e. for all (d, n, z). Let the probability of an increase of ζ in the performance of leading-edge technology occur with probability $\lambda(\zeta)$, where $\sum_\zeta \lambda(\zeta) = 1$. Since $W_{t+1}(d, n, z) = \pi a(n)\rho_0(z) + \delta\sum_\zeta \lambda(\zeta)V_t(d_{t,W}, n - a(n) + g(\zeta), z + \zeta)$, where $d_{t,W} = \theta\big(d + a(n)\bar{\rho}_0(z)\big)$, it follows that $W_{t+1}(d, n, z)$ is piecewise differentiable with a countable number of points of non-differentiability. By the chain rule,

$$\frac{\partial}{\partial d}W_{t+1}(d, n, z) = \delta\theta\sum_\zeta \lambda(\zeta)\frac{\partial}{\partial d_{t,W}}V_t(d_{t,W}, n - a(n) + g(\zeta), z + \zeta) \le \delta\theta\sum_\zeta \lambda(\zeta)\pi = \delta\theta\pi.$$

Similarly, since $R_{t+1}(d, n, z) = r\pi\big(a(n)\rho_1(z) + \phi_1(z)d\big) + \delta\sum_\zeta \lambda(\zeta)V_t(d_{t,R}, n - a(n) + g(\zeta), z + \zeta)$, where $d_{t,R} = \theta\big(\bar{\phi}_1(z)d + a(n)\bar{\rho}_1(z)\big)$, it follows that $R_t(d, n, z)$ is piecewise differentiable with a countable number of points of non-differentiability and that

$$\frac{\partial}{\partial d}R_{t+1}(d,n,z) = r\pi\phi_1(z) + \delta\theta\bar{\phi}_1(z)\sum_{\zeta}\lambda(\zeta)\frac{\partial}{\partial d_{t,R}}V_t(d_{t,R}, n-a(n)+g(\zeta), z+\zeta)$$
$$\leq r\pi\phi_1(z) + \delta\theta\bar{\phi}_1(z)\sum_{\zeta}\lambda(\zeta)\pi = \big(r\phi_1(z)+\delta\theta\bar{\phi}_1(z)\big)\pi \leq \max\{r,\delta\theta\}\pi.$$

This implies that $\frac{\partial}{\partial d}W_{t+1}(d,n,z) \leq \delta\theta\pi$ and that $\frac{\partial}{\partial d}R_{t+1}(d,n,z) \leq \max\{r,\delta\theta\}\pi$ and that the induction holds. Taking the limit in t produces , $\frac{\partial}{\partial d}W(d,n,z) \leq \delta\theta\pi$ and that $\frac{\partial}{\partial d}R(d,n,z) \leq \max\{r,\delta\theta\}\pi$. Thus, there exists a threshold value $D^*(n,z)$, which determines the optimal upgrade policy in any period. If both Conditions 1 and 2 hold, then $\rho_1(z) = \rho_0(z)$ and $\phi_1(z) = 0$ for all $z > 0$. This implies that $R_t(d,n,z) = rW_t(d,n,z) \leq W_t(d,n,z)$, and that $D^*(n,z) = D^*_{t,W}(n,z)$. Thus the proof of Proposition 2 is a special case of Proposition 4.

Proof of Proposition 4 Part II - Monotonicity of the Threshold

For fixed n, the optimal threshold value $D^*(n,z)$ is nonincreasing in the technology lag z if $\Delta_z D^*(n,z) = D^*(n,z+1) - D^*(n,z) \leq 0$. Since $\Delta_z U(d,n,z) = U(d,n,z+1) - U(d,n,z) = 0$, demonstrating that $\Delta_z D^*(n,t) \leq 0$ is equivalent to showing that $\Delta_z W(d,n,z) = W(d,n,z+1) - W(d,n,z) \leq 0$ and $\Delta_{z_t} R(d,n,z) = R(d,n,z+1) - R(d,n,z) \leq 0$. Using value iteration and a starting condition of $V_0(d,n,z) = 0$ for all (d,n,z), we have that $\Delta_z W_1(d,n,z) = \pi a(n)(\rho_0(z+1) - \rho_0(z)) \leq 0$, since sales are nonincreasing in the technological lag, and $\Delta_z R_1(d,n,z) = r\pi\big[a(n)\big(\rho_1(z+1)-\rho_1(z)\big) + d\big(\phi_1(z+1)-\phi_1(z)\big)\big] \leq 0$, since the pent-up demand sales from a TPR are also nonincreasing in the technological lag. These conditions imply that $\Delta_z V_1(d,n,z) \leq 0$. Assume for some iteration $t \geq 1$ that $\Delta_z W_t(d,n,z) \leq 0$ and $\Delta_z R_t(d,n,z) \leq 0$, which in turn implies that $\Delta_z V_t(d,n,z) \leq 0$. Define $\Delta_z\rho_i(z) = \rho(z+1) - \rho(z)$, for $i \in \{0,1\}$, $\tilde{d}_W(z) = \theta\big(d + \bar{\rho}_0(z)a(n)\big)$, and $\tilde{n} = n - a(n) + g(\zeta)$. Since $W_{t+1}(d,n,z) = \pi a(n)\rho(z) + \delta\sum_{\zeta}\lambda(\zeta)V_t\big(\tilde{d}_W(z),\tilde{n},z+\zeta\big)$, it follows that

$$\begin{aligned}
\Delta_z W^*_{t+1}(d,n,z) &= \pi a(n)\Delta_z\rho_0(z) + \delta\sum_{\zeta}\lambda(\zeta)\bigg(V_t(\tilde{d}_W(z+1),\tilde{n},z_t+1+\zeta) - V_t(\tilde{d}_W(z),\tilde{n},z_t+\zeta)\bigg)\\
&\leq \pi a(n)\Delta_z\rho_0(z) + \delta\sum_{\zeta}\lambda(\zeta)\bigg(V_t(\tilde{d}_W(z+1),\tilde{n},z_t+\zeta) - V_t(\tilde{d}_W(z),\tilde{n},z_t+\zeta)\bigg)\\
&\leq \pi a(n)\Delta_{z_t}\rho_0(z)\big(1-\delta\theta\big)\\
&\leq 0.
\end{aligned}$$

The first inequality follows from the fact that the value function is nonincreasing in the technology lag. The second inequality follows from the fact the difference in the future value functions are bounded by $-\pi\delta\delta\theta_z\rho_0(z)a(n)$. The final inequality results from the fact $\Delta_z\rho_0(z) < 0$, while π and $a(n)$ are nonnegative and $\delta\theta < 1$. Thus, the inductive assumption on $\Delta_z W^*_t(d,n,z)$ holds.

We now analyze the decision to offer a TPR. Define $\Delta_z\phi_1(z) = \phi_1(z+1) - \phi_1(z)$ and $\tilde{d}_R(z) = \theta\big(\bar{\phi}_1(z)d + \bar{\rho}_1(z)a(n)\big)$. Since $R_{t+1}(d,n,z) = r\pi\big(a(n)\rho_1(z) + \phi_1(z)d\big) + \delta\sum_{\zeta}\lambda(\zeta)V_t(\tilde{d}_R(z),\tilde{n},z+\zeta)$, it follows that

$$\begin{aligned}
\Delta_t R^*_{t+1}(d,n,z) &= r\pi\big(a(n)\Delta_z\rho_1(z) + d\Delta_z\phi_1(z)\big) \\
&\qquad + \delta\sum_\zeta \lambda(\zeta)\Big(V_t\big(\tilde{d}_R(z+1),\tilde{n},z+1+\zeta\big) - V_t\big(\tilde{d}_R(z),\tilde{n},z+\zeta\big)\Big) \\
&\le r\pi\big(a(n)\Delta_z\rho_1(z) + d\Delta_z\phi_1(z)\big) \\
&\qquad + \delta\sum_\zeta \lambda(\zeta)\Big(V_t\big(\tilde{d}_R(z+1),\tilde{n},z+\zeta\big) - V_t\big(\tilde{d}_R(z),\tilde{n},z+\zeta\big)\Big) \\
&\le r\pi\big(a(n)\Delta_z\rho_1(z) + d\Delta_z\phi_1(z)\big)\big(1-\delta\theta\big) \\
&\le 0.
\end{aligned}$$

The inequalities arise for similar reasons as with the case of $\Delta_z W^*_{t+1}$ with the additional aspect that $\Delta_z\phi_1(z) < 0$. Therefore, the induction also holds on $\Delta_z R^*_{t+1}(d,n,z)$. Taking the limit in t, $\Delta_z W^*(d,n,z) \le 0$ and $\Delta_z R^*(d,n,z) \le 0$. Thus, the optimal threshold value $D^*(n,z)$ is nonincreasing in the technology lag z. The proof demonstrating that $\Delta_z W(d,n,z) = W(d,n,z+1) - W(d,n,z) \le 0$ is equivalent to proving Proposition 3.

Proof for Corollary 1

The claim is true if $\frac{\partial}{\partial d}\big(R(d,n,z) - W(d,n,z)\big) \ge 0$. From the proof of Proposition 4, we know that $\frac{\partial}{\partial d}W(d,n,z) \le \delta\theta\pi$ and that $\frac{\partial}{\partial d}R_{t+1}(d,n,z) = r\pi\phi_1(z) + \delta\theta\bar{\phi}_1(z)\sum_\zeta \lambda(\zeta)\frac{\partial}{\partial d_{t,R}}V_t(d_{t,R}, n - a(n) + g(\zeta), z+\zeta)$, where $d_{t,R} = \theta\big(\bar{\phi}_1(z)d + a(n)\bar{\rho}_1(z)\big)$. We also know that $\frac{\partial}{\partial d}R(d,n,z) \ge r\pi\phi_1(z)$, since each component of $\delta\theta\bar{\phi}_1(z)\sum_\zeta \lambda(\zeta)\frac{\partial}{\partial d_{t,R}}V_t(d_{t,R}, n-a(n)+g(\zeta), z+\zeta)$ is positive. If $\frac{\partial}{\partial d}R(d,n,z) \ge r\pi\phi_1(z) \ge \delta\theta\pi \ge \frac{\partial}{\partial d}W(d,n,z)$, then the claim holds. Thus, the condition $\theta \le \frac{r\phi_1(z)}{\delta}$ is sufficient to guarantee a threshold $\tilde{D}(n,z)$ exists between waiting to upgrade and offering a TPR.

Proof for Corollary 2

Independent of the decision to upgrade or offer a TPR the expected state component for market potential in the next period is $\tilde{n} = n - a(n) + g(\zeta)$. The difference in the number of sales between offering an upgrade and a TPR is $\Delta s = a(n)\big(1-\rho_1(z)\big) + d\big(1-\phi_1(z)\big) = a(n)\bar{\rho}_1(z) + \bar{\phi}_1(z)d$. The corresponding difference in revenue is $\Delta\pi = \pi a(n)\big(1 - r\rho_1(z)\big) + \pi d\big(1 - r\phi_1(z)\big)$. If the firm offers a TPR, then the level of pent-up demand in the next period is $\theta(\bar{\phi}_1(z)d + a(n)\bar{\rho}_1(z)) = \delta\theta s$. Introducing the $\tilde{R}(d,n,z)$

$$\begin{aligned}
\tilde{R}(d,n,z) &= r\pi\big(a(n)\rho_1(z) + \phi_1(z)d\big) + \delta E\big[V(\delta\theta s,\tilde{n},\zeta)\big] \\
&\ge r\pi\big(a(n)\rho_1(z) + \phi_1(z)d\big) + \delta E\big[V(\delta\theta s,\tilde{n},z+\zeta)\big] \\
&= R(d,n,z),
\end{aligned}$$

where the inequality follows from Lemma 1. From Proposition 9, $-\pi\delta\theta s \le V(0,\tilde{n},\zeta) - V(\delta\theta s,\tilde{n},\zeta)$. If $U(d,n,z) \ge \tilde{R}(d,n,z)$, then $U(d,n,z) \ge R(d,n,z)$ and the decision to upgrade dominates the decision to offer a TPR. Since $U(d,n,z) = \pi\big(a(n)+d\big) - K + \delta E\big[V(0, n -$

$a(n) + g(\zeta), \zeta)\big]$, it follows that

$$\begin{aligned} U(d,n,z) - \tilde{R}(d,n,z) &= \Delta\pi - K + \delta\big(V(0,\tilde{n},\zeta) - V(\delta\theta s,\tilde{n},\zeta)\big) \\ &\geq \Delta\pi - K - \delta\theta\pi\Delta s. \end{aligned}$$

Since $\tilde{K}(d,n,z) = \Delta\pi - \delta\theta\pi\Delta s$, if $\tilde{K}(d,n,z) > K$ then $U(d,n,z) > R(d,n,z)$.

Proof of Proposition 5

If Condition 1 does not holds and 2 does, then we only consider the decision between upgrading and waiting to upgrade because the firm has no incentive to offer a TPR. Let $q_j(z)$ be the probability of encountering a product failure of degree j given a technology lag z. Thus, $\sum_j q_j(z) = 1$ for each $z \in \mathbb{Z}^+$. The marginal value of pent-up demand for the upgrade decision is $\frac{\partial}{\partial d}U(d,f,n,z) = \pi \sum_j q_j(z)\phi_0(0,j)$ for any value of d, so the condition that $\frac{\partial}{\partial d}W(d,f,n,z) \leq \delta\theta\pi \sum_j q_j(z)\phi_0(0,j)$ is sufficient to prove monotonicity in d. Using the starting condition $\frac{\partial}{\partial d}W_0(d,f,n,z) = 0$ provides the inductive assumption that $W_t(d,f,n,z)$ is piecewise differentiable with at most countable number of points on non-differentiability and that $\frac{\partial}{\partial d}W_t(d,f,n,z) \leq \delta\theta\pi \sum_j q_j(z)\phi_0(0,j)$. Using the argument that there exists at most one point (a set of measure zero) where $U_t(d,f,n,z) = W_t(d,f,n,z)$ implies that $\frac{\partial}{\partial d}V_t(d,f,n,z) \leq \pi \sum_j q_j(z)\phi_0(0,j)$. Since $\tilde{d} = \theta\big(d + a(n)\bar{\phi}_0(z,f)\big)$, by the chain rule,

$$\begin{aligned} \frac{\partial}{\partial d}W_{t+1}(d,f,n,z) &= \delta\theta \sum_{\zeta} \lambda(\zeta) \frac{\partial}{\partial \tilde{d}} V_t(\tilde{d}, n - a(n) + g(\zeta), f, z + \zeta) \\ &\leq \delta\theta\pi \sum_j q_j(z)\phi_0(0,j) \end{aligned}$$

which completes the induction. Taking the limit in t, $\frac{\partial}{\partial d}W^*(d,f,n,z) \leq \delta\theta\pi \sum_j q_j(z)\phi_0(0,j)$.

We construct the following example to show that the threshold is non-monotonic.

Example 1. *Consider a market where the probability of releasing a new product with $f = 0$ at a technology level z is $q_0(z) = 2/(2 + z_t)$ and the probability of releasing a product with $f = 1$ is $q_1(z) = 1 - q_0(z)$. In addition, consider the parameters and functional forms $K = 0.75$, $\pi = 1$, $\rho(0,1) = \phi_0(0,1) = 0.1$, $\rho(z,0) = 1/(1+z)$, $\delta = 0$, and $a(n) = n$. The parameters imply that a product is either a hit ($f = 0$) or a miss ($f = 1$). Figure 7 plots the state dependent threshold policy $D^*(0,n,z)$ for various levels of n and z, and clearly demonstrates that the threshold values are not monotonically nonincreasing in z. Figure 7 plots the threshold with respect to z for the parameters given by Example 1 clearly demonstrating that the threshold is non-monotonic in z.*

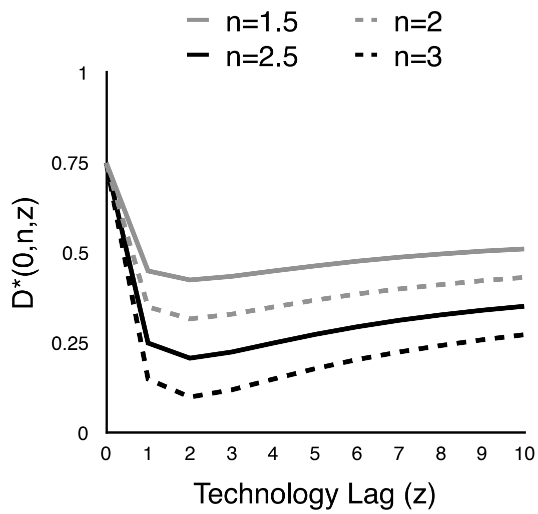
Proof of Proposition 6

Let $U(d,f,n,z)$ be the value function from the decision to upgrade without a TPR and $B(d,f,n,z)$ the value function from upgrading with a TPR. This implies that

$$U(d,f,n,z) = E\big[\pi\big(a(n)\rho_0(0,\xi(z)) + d\phi_0(0,\xi(z))\big) + \delta V(0,\xi(z),\tilde{n},\zeta)\big] \tag{A.1}$$

$$B(d,f,n,z) = E\big[r\pi\big(a(n)\rho_1(0,\xi(z)) + d\phi_1(0,\xi(z))\big) + \delta V(0,\xi(z),\tilde{n},\zeta)\big]. \tag{A.2}$$

Figure 7: State-dependent threshold policy with potential for a product failure



Observing that the future component of both value functions are the same, then if (4.1) (does not) hold, it is (not) optimal to offer a TPR with an upgrade.

Proof of Propositions 7

We construct a simple example to prove that there does not exist a threshold where it is optimal to upgrade for $d \geq D(f, n, z)$. Consider the states $n = 0$ and $f = 0$ where $\delta = 0$, the probability that $f = 0$ following an upgrade is $q_0(z)$, and the firm loses all sales if it releases a product failure. Since $n = 0$ and $\delta = 0$, $W(d, 0, 0, z) = 0$ for all d and z, the value from upgrading is $U(d, 0, 0, z) = q_0(z)\phi_0(0, 0)d$, the value from upgrading with a TPR is $B(d, 0, 0, z) = rq_0(z)\phi_1(0, 0)d$, and the value from offering a TPR is $R(d, 0, 0, z) = r\phi_1(z, 0)d$. Therefore, for any z where $r\phi_1(z, 0) \geq q_0(z)\phi_0(0, 0)$, the optimal decision for any $d > 0$ will be to offer a TPR.

Proof of Propositions 8

From the value-to-go equations of U and B given by (A.1) and (A.2), respectively, it follows that $\frac{\partial}{\partial d}U(d, f, n, z) = \pi E\big[\phi_0(0, \xi(z))\big]$ and $\frac{\partial}{\partial d}B(d, f, n, z) = r\pi E\big[\phi_1(0, \xi(z))\big]$. In the situation where neither Conditions 1 and 2 hold, defining $d_{t,W} = \theta(d + a(n)\bar{\rho}_0(z, f))$, and $d_{t,R} = \theta(d\bar{\phi}_1(z, f) + a(n)\bar{\rho}_1(z, f))$, the value of waiting and not offering a TPR and offering a TPR without upgrading is

$$W(d, f, n, z) = \pi\big(a(n)\rho_0(z, f)\big) + \delta E\big[V(d_{t,W}, f, n - a(n) + g(\zeta), z + \zeta)\big]$$
$$R(d, f, n, z) = r\pi\big(a(n)\rho_1(z, f) + d\phi_1(z, f)\big) + \delta E\big[V(d_{t,R}, f, n - a(n) + g(\zeta), z + \zeta)\big]$$

respectively. Following the value iteration approach from before, we make the inductive assumption that $W_t(d, f, n, z)$ and $R_t(d, f, n, z)$ are piecewise differentiable with at most countable number of points on non-differentiability, $\frac{\partial}{\partial d}W_t(d, f, n, z) \leq \delta\theta\pi$, and $\frac{\partial}{\partial d}R_t(d, f, n, z) \leq \max\{r, \delta\theta\}\pi$. Since $V_t(d, n, z) = \max\big\{U_t(d, f, n, z), B_t(d, f, n, z), W_t(d, f, n, z), R_t(d, f, n, z)\big\}$, the inductive assumptions imply that the bound $\frac{\partial}{\partial d}V_t(d, n, z) \leq \pi$ continues to hold in the most general setting. This is fairly intuitive, since the presence of failure or TPRs lower the value of pent-up demand. Since $W_{t+1}(d,n,z)$ and $R_{t+1}(d,n,z)$ are piecewise differentiable with a countable number of points of non-differentiability and

$$\begin{aligned}\frac{\partial}{\partial d}W_{t+1}(d,f,n,z) &= \delta\theta E\left[\frac{\partial}{\partial d_{t,W}}V_t(d_{t,W},f,n-a(n)+g(\zeta),z+\zeta)\right] \leq \delta\theta\pi,\\ \frac{\partial}{\partial d}R_{t+1}(d,f,n,z) &= r\pi\phi_1(z,f)+\delta\theta\bar{\phi}_1(z,f)E\left[\frac{\partial}{\partial d_{t,R}}V_t(d_{t,R},f,n-a(n)+g(\zeta),z+\zeta)\right]\\ &\leq r\pi\phi_1(z,f)+\delta\theta\bar{\phi}_1(z,f)\pi \leq \max\{r,\delta\theta\}\pi,\end{aligned}$$

the induction holds. If $\max_{y\in\{0,1\}} E\big[\phi_y(0,\xi(z))\big] \geq \max(r,\delta\theta)$, then (4.2) is satisfied, and either (1) $\frac{\partial}{\partial d}U(d,f,n,z) \geq \frac{\partial}{\partial d}W(d,f,n,z)$ and $\frac{\partial}{\partial d}U(d,f,n,z) \geq \frac{\partial}{\partial d}R(d,f,n,z)$ or (2) $\frac{\partial}{\partial d}B(d,f,n,z) \geq \frac{\partial}{\partial d}W(d,f,n,z)$ and $\frac{\partial}{\partial d}B(d,f,n,z) \geq \frac{\partial}{\partial d}R(d,f,n,z)$. Thus, inequality (4.2) is sufficient for guaranteeing that the optimal threshold has the form of upgrading (with or without a TPR) if $d \geq D^*(f,n,z)$.

If (4.3) is satisfied, then $\frac{\partial}{\partial d}R(d,f,n,z)/\pi \geq r\phi_1(z,f) \geq \max_{y\in\{0,1\}} E\big[r^y\phi_y(0,\xi(z))\big] = \max(\frac{\partial}{\partial d}U(d,f,n,z), \frac{\partial}{\partial d}B(d,f,n,z))$. Expanding Corollary 1 to incorporate the state space f shows that if $\theta \leq \frac{r\phi_1(z,f)}{\delta}$, then $\frac{\partial}{\partial d}R(d,f,n,z) \geq \frac{\partial}{\partial d}W(d,f,n,z)$. Thus, by transitivity, there exists a threshold in pent-up demand where the optimal policy has a form that it is optimal to offer a TPR without upgrading for $d \geq D^*(f,n,z)$, when inequality (4.3) is satisfied.

B. Numerical Experiment Appendix

In section 5 we introduce the numerical experiments and summarize the main findings. In this appendix, we provide details on the functional forms, approximate dynamic programming formulation, and parameter values for each experiment. For clarity, we detail the functional form's in order of the assumptions in section 3.

B.1 Functional Forms and Baseline Parameters

New Arrivals

The market arrival process for a serviceable market size of n is $a(n) = \alpha n$, where $\alpha \in (0,1]$. The parameter α captures the speed at which consumers from the serviceable market look to purchase a product. If technology advances at the end of the period, then the market in the following period is $N = n(1-\alpha)+\eta$. If technology does not advance, then the market potential in the following period is $N = n(1-\alpha)$. The initial market size at the start of the infinite planning horizon is n_0. For all experiments $\alpha = 0.4$ and $n_0 = 100$.

Sales from New Arrivals

The sales is dependent upon if the firm offers a TPR, if the firm has releases a failed product, and the technology lag. The parameters β and μ represent the boost and loss of sales for the firm when offers a TPR and selling a failed product. Thus, $\beta \geq 1$ and $\mu \in [0,1]$. The level of sales with technology lag z, failure level f, and TPR decision y is $\rho_y(z,f) = 1 \wedge \big(\beta^y\mu^f\gamma^z\big)$,

Table 1: Values for the sales from new arrivals - $\rho_y(z, f)$

	$f = 0$		$f = 1$	
	$y = 0$	$y = 1$	$y = 0$	$y = 1$
$x = 1$ $(z = 0)$	1	1	μ	$\beta\mu$
$x = 0$ $(z \geq 0)$	γ^z	$\beta\gamma^z$	$\mu\gamma^z$	$\beta\mu\gamma^z$

Table 2: Values for the sales from pent-up demand - $\phi_y(z, f)$

	$f = 0$		$f = 1$	
	$y = 0$	$y = 1$	$y = 0$	$y = 1$
$x = 1$ $(z = 0)$	1	1	μ	$\beta\mu$
$x = 0$ $(z \geq 0)$	0	$\beta\omega^z$	0	$\beta\mu\omega^z$

and $\gamma \in [0, 1]$. The parameter γ is consumer sensitivity to a lag in technology. The values are displayed for the different states based on the firm decision and level of f in Table 1. For all experiments and examples $\gamma = 0.8$.

Sales from Pent-up Demand

For consistency with new arrivals, the parameters β and μ represent the boost and loss of pent-up demand sales for the firm when offers a TPR and selling a failed product. The proportion of pent-up demand customers that purchase the product in each period is given by $\phi_y(z, f) = 1 \wedge \left(\beta^y \mu^f \omega^z (1 - \mathbb{I}_{y=0,z\neq 0}\right)$, where $\omega \in [0, 1]$ and $\mathbb{I}_{y=0,z\neq 0}$ is an indicator variable that is 1 when the firm does not upgrade nor offer a TPR $(y = 0, z \neq 0)$ and 0 otherwise. The parameter ω represents how influential the lag in technology is in determining sales when there is a TPR and no upgrade. The values are displayed for the different states based on the firm decision and level of f in Table 2. For all experiments and examples $\omega = 0.6$.

Approximate Markov Decision Process

There are two challenges in using value iteration to solve the MDP. First the state space is countable, but can be infinite. Second, the level of pent-up demand and the cumulative arrivals are continuous. To make the state space finite, we set bounds on the state spaces for z, n, and d, such that $z \in \{0, \cdots, \bar{Z}\}$, $n \in \{0, \cdots, \bar{N}\}$, and $d \in \{0, \cdots, \bar{D}\}$ and adjust the problem accordingly.

To find an appropriate value of $\bar{Z}$, we plot the level of sales from new sales and pent-up demand based on the parameters γ and ϕ. For $\gamma = 0.8$ (which is the maximum value used in each experiment for both γ and ϕ), the marginal changes in sales is inconsequential after z exceeds 20, and thus $\bar{Z} = 20$. The market size increases and decreases stochastically, but in expectation the market size in period $t+1$ is given by $n_t(1 - \alpha) + \eta\lambda$. The expected market size increases over time approaching the limiting value of $\eta\lambda/\alpha$, and thus we set $\bar{N} = \eta\lambda/\alpha$. In the case where there are no sales due to excessive lag in technology, no TPRs to lower the level of pent-up demand, and arrivals have reached its steady state $(\bar{N}\alpha = \eta\lambda)$, the pent-up demand in period $t + 1$ is $\theta(d_t + \eta\lambda)$. This has a limiting value of $\theta\eta\lambda/(1 - \theta)$ and we set $\bar{D} = \theta\eta\lambda/(1 - \theta)$. Note that in this case we limit our experiments to $\theta \in [0, 1)$.

Since the level of pent-up demand and the cumulative arrivals are continuous, interpolation is used to discretize the values for the future state dynamics. Define the D^0 and N^0 as the floor function of pent-up demand D given by equation (3.3) and market potential $\bar{\alpha}N$, and define

$$D^1 = \begin{cases} \lceil D \rceil & D > D^0 \\ \lceil D \rceil + 1 & D = D^0, \text{ and} \end{cases}$$

$$N^1 = \begin{cases} \lceil N \rceil & N > N^0 \\ \lceil N \rceil + 1 & N = N^0. \end{cases}$$

For non-integer values of D and N, if the probabilities of being in the states D^i and N^j for $i, j \in \{0, 1\}$ are given respectively by $q_D^0(d_t, n_t, z_t) = D^1(d_t, n_t, z_t) - D(d_t, n_t, z_t)$, $q_D^1(d_t, n_t, z_t) = D(d_t, n_t, z_t) - D^0(d_t, n_t, z_t)$, $q_N^0(n_t) = N^1(n_t) - N(n_t)$, $q_N^1(n_t) = N(n_t) - N^0(n_t)$, then the approximate dp of (3.1)-(3.6) is

$$\begin{aligned} V(d, f, n, z) = \max_{x,y \in \{0,1\}} \bar{x} \Big(\pi r^y S_0 + \delta \sum_{i,j \in \{0,1\}} q_D^i q_N^j \big(\lambda V(D^i, f, N^j + 1, z + 1) + \bar{\lambda} V(D^i, f, N^j, z) \big) \Big) \\ + x \sum_{i,j,f \in \{0,1\}} q_D^i q_N^j p_f(z) \Big(\pi r^y S_1 - K + \delta \big(\lambda V(D^i, f, N^j + 1, z + 1) + \bar{\lambda} V(D^i, f, N^j, z) \big) \Big) \end{aligned} \tag{B.1}$$

$$\text{s.t.} \quad S_0 = a(n)\rho_y(z, f) + d\phi_y(z, f) \tag{B.2}$$

$$S_1 = a(n)\rho_y(0, f) + d\phi_y(0, f) \tag{B.3}$$

$$D = \bar{x}\theta\big(a(n)\bar{\rho}_y(z, f) + d\bar{\phi}_y(z, f)\big). \tag{B.4}$$

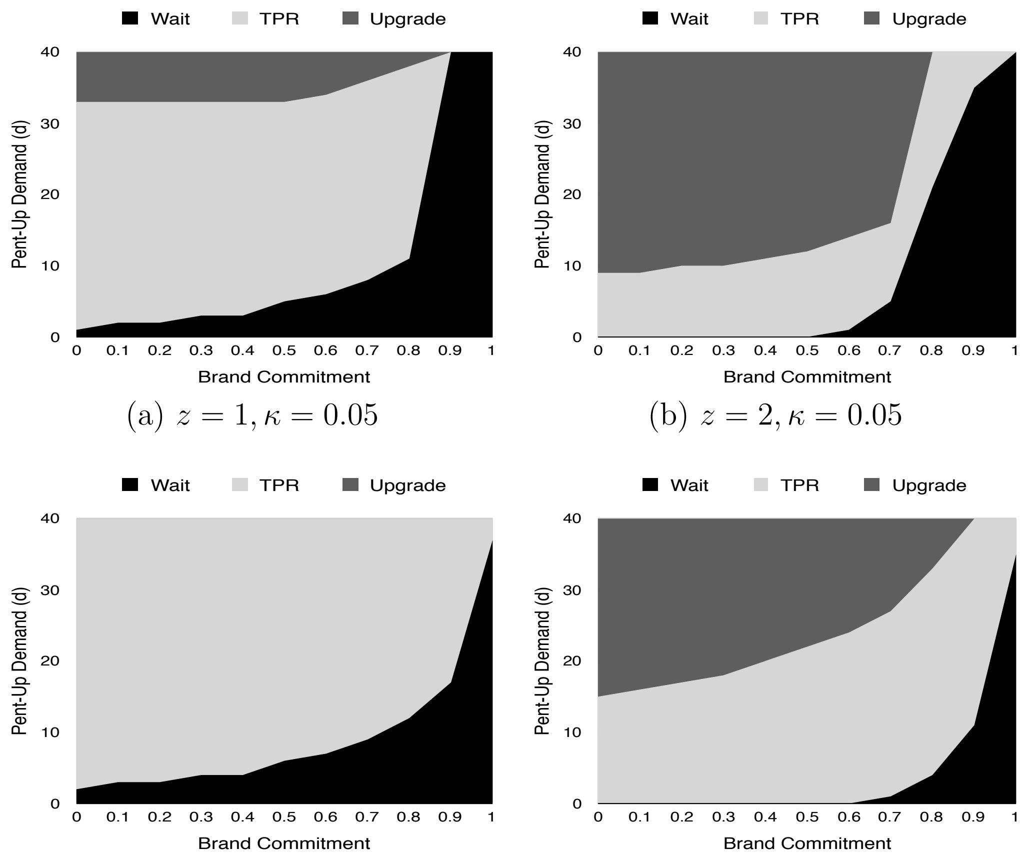
B.2 Experiment Details

In the first set of experiments, the value of pace and magnitude were fixed such that the expected per period market growth $\lambda\eta = 12$. For technology paces $\lambda \in \{0.2, 0.4, 0.6, 0.8\}$ the corresponding size of market growth due to an increase in technology $\eta \in \{60, 30, 20, 15\}$. The experiments considered a cost of $K = 15$, $\beta = 1$ and $\mu = 1$, which implies that TPRs are ineffective at increasing demand and there is no potential for product failure. The profit earned for a full product sale in all experiments was $\pi = 1$.

In the second and third set of experiments, the value of pace and magnitude were taken from the set $\lambda \in \{0.4, 0.8\}$ and $\eta \in \{15, 30\}$. The results were taken at $K = 20$, with TPR revenue $r = 0.7$, a sales increase factor of $\beta = 1.6$, and a lost sales factor of $\mu = 0.5$. The probability that a product is a hit ($f = 0$) is $p_0(z) = e^{-\kappa z}$ and the probability that a product is a miss ($f = 1$) is $p_1(z) = 1 - e^{-\kappa z}$. Recall that the probability that technology increases is λ and that the magnitude of market growth given an advancement is η. The value of κ is set to 0.05. The only difference between the second and third set of experiments was that the level of lost sales was varied between $\mu \in \{0.75, 0.5, 025, 0\}$.

The fourth set of experiments, the value of pace and magnitude were again taken from the set $\lambda \in \{0.4, 0.8\}$ and $\eta \in \{15, 30\}$. The launch cost was set to $K = 15$ and $\beta = 1$ implying that firms did not use TPRs. To demonstrate the extent to which product failure can be damaging to high brand commitment firms, the lost sales factor associated with a launch failures was $\mu = 0$ and the probability of a failure was quasi-convex, where $p_1(z) = \sum_{i=0}^{z} i/15$.

Figure 8: Optimal control plots across brand commitment and states $(d, 0, 100, z)$ at different values of κ.



(a) $z = 1, \kappa = 0.05$ (b) $z = 2, \kappa = 0.05$

(c) $z = 1, \kappa = 0.15$ (d) $z = 2, \kappa = 0.15$

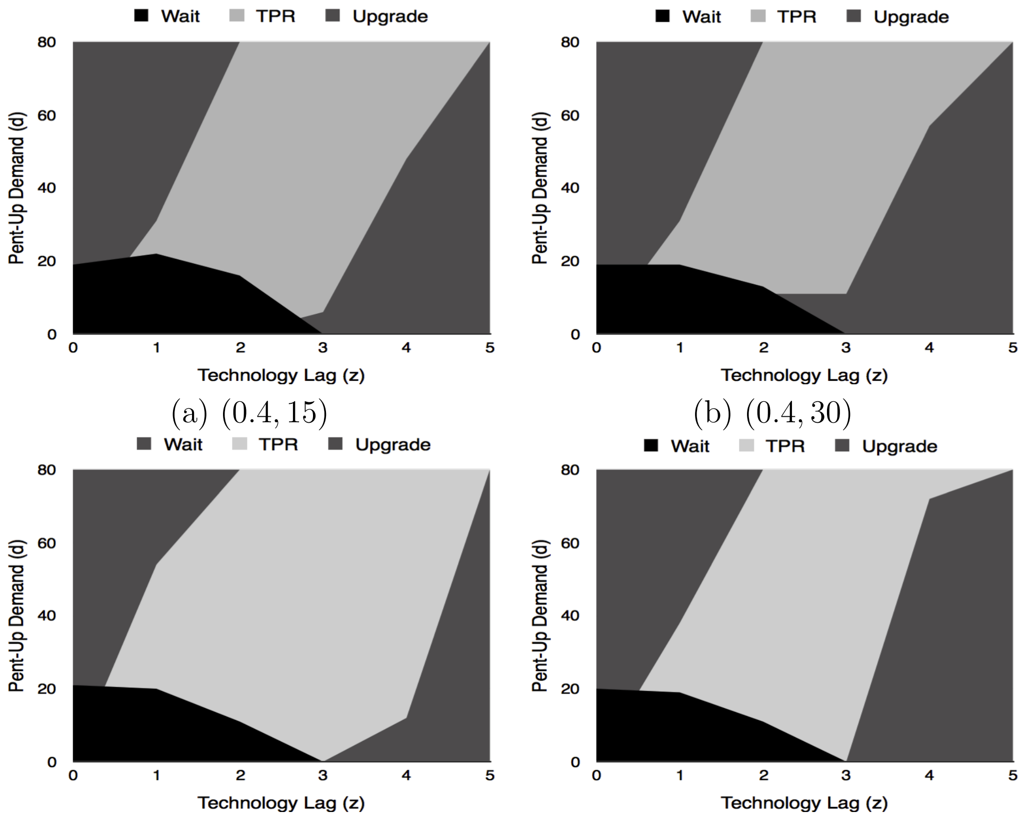
B.3 Additional Numerical Examples

In this section we present a number of control plots using the same functional forms as in the body of the paper. Consider a fast paced $\lambda = 0.8$, high magnitude $\eta = 30$, with parameters $K = 20$, $r = 0.75$, $\alpha = 0.25$, $\beta = 1.6$, and $\mu = 0.1$. The probability that a product is a hit $(f = 0)$ is $p_0(z) = e^{-\kappa z}$ and the probability that a product is a miss $(f = 1)$ is $p_1(z) = 1 - e^{-\kappa z}$. We vary the value of κ (where a higher value of κ implies that there is a greater chance of failure) to show the impact of the potential of product failure. The control plots were taken at states $n = 60$ and $z = 2$.

Figure 8 shows the optimal control policy across brand commitment for the states $(d, 0, 100, z)$ where $d \in \{0, 1, \ldots, 40\}$ and $z \in \{1, 2\}$ and for $\kappa \in \{0.05, 0.15\}$. For a fixed value of z, we observe that an increase in κ raises the amount of pent-up demand necessary to optimally release an upgrade. Comparing the figures at $z = 2$, we can also see that the threshold separating waiting and offering a TPR decreases. As a result, the increase in the probability of a product failure, even for a fixed value of TPR, provides incentive for offering a TPR to lower the level of pent-up demand going forward.

To demonstrate a situation where increases in pent-up demand cause a switch in the optimal upgrade strategy from upgrading to offering a TPR, we set $K = 10$, $r = 0.75$, $\alpha = 0.25$, $\beta = 2$,

Figure 9: Optimal control plots for $\theta = 0.9$ and states $(d, 0, 100, z)$ at different values of (λ, η).



(a) $(0.4, 15)$ (b) $(0.4, 30)$ (c) $(0.8, 15)$ (d) $(0.8, 30)$

and $\mu = 0.1$. The probability that a product is a hit ($f = 0$) is $p_0(z) = e^{-\kappa z}$ and the probability that a product is a miss ($f = 1$) is $p_1(z) = 1 - e^{-\kappa z}$, where κ is set to 0.25. Offering a TPR to lower the potential loss in sales from pent-up demand is only meaningful for firms with high levels of brand commitment. As a result, we consider a firm with $\theta = 0.9$.

Figure 9 illustrates the optimal control policy for states $(d, 0, 100, z)$ where $d \in \{0, 1, \ldots, 80\}$ and $z \in \{0, 1, \ldots, 5\}$ and for $\lambda \in \{0.4, 0.8\}$ and $\eta \in \{15, 30\}$. The threshold plots show that at higher values of z, there is a distinct threshold separating the decision between upgrading and TPRs, where pent-up demand above the threshold corresponds to the decision to offer a TPR. We observe that, the threshold separating upgrading and TPRs is increasing with higher magnitude for a fixed pace. This implies that at a lower magnitude a firm has greater incentive to use a TPR to lower the level of pent-up demand before a risky upgrade.