PRICE-MATCHING COMPETITION IN THE PRESENCE OF STRATEGIC CUSTOMERS

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Abstract. Competition between retailers leads to an increase in the total inventory on the market while strategic customer behavior works in the opposite direction. Price matching (PM) alters the balance between these forces by working in the same direction as strategic customers and serves as a signal that dramatically changes the equilibrium. We show that PM deserves the close attention of all market participants because it can not only mitigate the loss from strategic customer behavior but may even lead to gains from increases in the level of this behavior under competition. At the same time, combined effects of PM and strategic behavior may cause the equilibrium retailer profit with PM to be less than the worst equilibrium profit without PM. Manufacturer never benefits from PM except for branded products when the sales at reduced prices are undesirable. On the other hand, policymakers may encourage PM since it can improve the aggregate welfare.

1. Introduction

In recent years, wide adoption of dynamic pricing strategies prompted concerns that customers might try to outsmart retail firms by timing their purchases in anticipation of price markdowns over the course of a sales season. The study of demand management systems facing such phenomenon, known as *strategic customer behavior*, has been a subject of extensive research in the Management Science community; see, e.g., Shen and Su (2007), Aviv and Vulcano (2010), and Aviv et al. (2009).

One of the early key papers to explain the negative impact of strategic customer behavior, Coase (1972), appears in the economics literature. Coase describes a situation in which a monopolist sells a durable good to a large set of customers with different valuations. Ideally, the monopolist could employ perfect segmentation by charging each customer his own valuation. The monopolist would initially charge a high price from the high-valuation customers, followed by a sequence of price reductions to capture more and more customers. As a result, the monopolist would extract all of the customer surplus. In contrast to this ideal scenario, Coase examines what happens if the customers are strategic. Here, if high-valuation customers anticipate a price decline, they would rationally expect and wait for a price reduction, instead of buying at premium prices. Coase argues that as a consequence of such behavior, in equilibrium, the monopolist effectively sells the product at marginal cost. Indeed, this relatively simple model demonstrates that strategic behavior could dramatically decrease the monopolist's revenues. The following intuitive explanation of Coase's result is key to understanding many of the ideas and results of the relevant literature in Management Science. The monopolist in Coase's paper, also referred to as a "durapolist" in the literature, sells products to the customers over *multiple* periods of time. Hence, the monopolist operates in a competitive situation in which it is engaged in a pricing game against its future "replicas." As a result, this self-competition could be highly detrimental to the monopolist's bottom-line revenue performance.

Coase (1972) suggests a number of ways for a seller to avoid the adverse impact of strategic customer behavior. For example, the seller can make a contractual arrangement with the customers in which he agrees not to sell more than a given quantity of the product. This *capacity rationing* proposition has been studied in papers such as Liu and van Ryzin (2008). In the latter paper,

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the authors consider a seller that can deliberately understock a product, hence creating a shortage risk for the customers and discouraging them from waiting for markdowns. The authors find that when the market consists of a large number of high-valuation customers, capacity rationing is useful; otherwise, the firm should serve the entire market at a low price. Moreover, under competition, the effectiveness of capacity rationing is reduced, and there exists a critical number of firms beyond which rationing never occurs in equilibrium. Levin et al. (2010) and Cachon and Swinney (2009) demonstrate the effectiveness of capacity rationing policies, and both provide a sharper understanding of the intricate relationship between the pricing and quantity decisions; see also Su (2007) and Su and Zhang (2008).

Another strategy suggested by Coase (1972) is for the monopolist to offer customers a buy-back agreement. Specifically, if the product is offered at any time in the future at a lower price, the monopolist agrees to accept product returns and to issue the customers full refund. In fact, in Coase's model, this is equivalent to paying the customers the difference between their purchase prices and the offered discounted price at any time. The rationale behind this strategy is that it ties the hands of the monopolist's future "replicas." Since the customers know that future price discounts require the retailer to pay back early purchasers, they do not anticipate the monopolist to offer significant discounts. Consequently, such rational perception drives the strategic customers to purchase at premium prices. This type of strategy, to which we shall refer as *price matching* (PM), is a central subject of our current research.

Companies have long used PM as a promotional tool since, from a customer's point of view, it makes company offerings more attractive. To the best of our knowledge, policies considered in the literature involve either matching a competitor price at the time of purchase (*external PM*) or matching the firm's own price in the future (*inter-temporal PM*). Lai et al. (2010) call these two types *concurrent* and *posterior*, respectively. The authors argue that both types of PM are prevalent in various industries, but also point out that, in practice, these types are often offered together. Indeed, a check of the BestBuy.com website on May 18, 2014 revealed the following policy statement:

For previous purchases, should you find a lower price in-store, in print or online from an authorized Canadian dealer we will beat it by 10% of the difference. Present us with your original receipt within 30 days of purchase. Tell us which competitor is offering the lower price; we will verify the price and that the item is in stock and available for immediate sale and delivery. If our own price is reduced present us with your original receipt within 30 days of purchase and we will refund the difference.

Thus, in practice, matching a competitor's price can occur during the same period of time as matching the firm's own price. In this paper, we consider this comprehensive type of PM.

There is an extensive literature on PM surveyed by Lai et al. (2010) and Nalca et al. (2013). For example, Salop (1986) shows that concurrent PM ("meeting-competition clause") can serve a tool that facilitates *collusion* among firms, and hence views it as an anticompetitive device. Holt and Scheffman (1987) is another early article highlighting PM ("best-price provision") as a tool enabling tacit collusion. The above research papers do not consider dynamic pricing, strategic customer behavior, or capacity constraints. Interestingly, we also find that PM may facilitate collusive capacity decisions resulting in the elimination of clearance sales. In fact, this may happen even in highly competitive markets. However, such collusion may be advantageous for the local economy, compared to a market scenario where PM is considered illegal.

Another stream of research on concurrent PM, pioneered by Png and Hirshleifer (1987), explores PM as a tool for *price discrimination* between customers that are differentiated by their search costs, levels of sophistication, and information. This body of work does not consider capacity constraints, except for Nalca et al. (2013) who explored PM, inventory, and pricing duopoly in an uncertain demand environment. A particular emphasis of that paper is the role of the availability verification clause in PM as a discriminating tool. In contrast, our paper considers settings where

PM cannot be used for price discrimination but rather provides the conditions under which PM is better than intertemporal price discrimination (without PM). A third group of papers, starting with Jain and Srivastava (2000), studies concurrent PM as a *signal* to the customers that the firm is low-priced (the reader is referred to Winter (2008) for a comprehensive review that include additional works on the signaling aspect of concurrent PM). In our paper, we discuss the way in which the signaling role of PM in dynamic environments depends on customer expectations.

Cooper (1986) is the first to show for a duopoly with myopic customers that *posterior* PM can be a tool facilitating tacit collusion and the performance of this tool can be close to that of a perfect cartel. Our paper confirms anticompetitive properties of PM for any arbitrary number of retailers and shows that, for some market situations, collusion would be impossible in the absence of strategic customers. Png (1991) considers a monopolist who sells a fixed stock of perishable items to strategic customers over two periods, and may offer posterior PM. The total number of customers is fixed, and each of them has a fixed valuation level that can be either low or high. However, the proportion of high valuation customers is uncertain. The author finds that the seller favors PM when capacity is large, but leans toward price discrimination as the uncertainty about the mixture of low/high valuations customers increases. Finally, Lai et al. (2010) studies a two-period model with strategic customer behavior. Similarly to Png (1991), customers are segmented into high and low (fixed) valuation segments. The high-valuation segment is additionally split into strategic and myopic. Myopic customers are further split into those who do and those who do not claim the PM reimbursement. The paper uses a framework of rational expectations and determines that PM is beneficial if the difference in valuation levels is neither too low nor too high. The paper also shows that customer surplus may increase as a result of PM, and that Pareto improvements in the customers' surplus and seller's profit are possible when the variance in the size of the high-valuation segment is high. When consumers are sufficiently strategic, we also obtain settings where surplus increases as a result of PM while profits remain constant. Moreover, we find that PM is welfare-improving in the majority of market situations.

The market setting we consider in this paper is one in which the initial (first period) price is exogenously set. This is often the case when the manufacturer of a product mandates a *manufacturer suggested retail price* (MSRP); e.g., see discussions in Cachon and Swinney (2009) and Liu and van Ryzin (2008), §4.4. In fact, retailers may follow the MSRP under repeated interactions with the manufacturer, even if this suggested price is not binding (see, e.g., Buehler and Gärtner (2013)). MSRP is a special case of a wider phenomenon of "manufacturer's resale price maintenance" studied by Orbach (2008) who cites long-standing examples of this practice in the pharmaceutical and other industries. In the presence of resale price maintenance, the main retailer decision is the quantity of the product. Thus, quantity competition among retailers is another important characteristic of the market we consider. But then, as products, such as medicines, lose their freshness or go out of fashion/season, they commonly appear on clearance sales. Butz (1996) cites many sources indicating that the practice of retail price maintenance is ubiquitous. The author shows "that manufacturers have many, many instruments to exert control [over resale price] and to some extent will do so whether or not the law permits it." Butz presents an argument that the concurrent PM (the actual term used is "meet-the-competition provisions") becomes one of such tools when the manufacturer finance rebates for the retailers who charge the suggested retail price.

Intuitively, the efficacy of posterior PM depends on the degree to which the customers exhibit strategic behavior. Such behavior exerts a downward pressure on the equilibrium product quantity in the market effectively becoming an opposing force to competition which tends to increase the supply. We show that whenever PM changes the equilibrium structure, the aggregate procurement quantity (total inventory) decreases, leading to a higher second-period price. The paper determines the conditions under which the retailers can mitigate the loss from strategic customer behavior by using PM, and examines how the level of competition affects this mitigation. Additionally, we show that when PM results in sales in both periods, this effectively voids the MSRP; thus, signaling to the manufacturer that the first-period price is too high. Finally, we augment the above theoretical insights with some illustrations in §5, showing that different types of PM equilibria can result in gains or losses exceeding the direct losses from strategic customer behavior.

2. Model description

We consider a two-period model of a competitive market in which n retailers, indexed by the set $I = \{1, \dots, n\}$, sell a limited-lifetime product. For clarity in our analyses and discussions, we assume that they are identical. As mentioned in the introduction, the first-period price, p_1, is fixed (e.g., MSRP). The retailers select their profit-maximizing inventory levels in anticipation of the market outcome. The per-unit cost of inventory is c, which obviously includes the cost charged by the manufacturer, but also embeds the retailer's effort to increase its *"market attraction."* We assume that the unit cost is equal for all retailers, driven in part by our market structure in which the manufacturer is common, the product is undifferentiated, and the retailers operate under similar conditions; see, e.g., §4.4 in Liu and van Ryzin (2008). Since the retailers know the market and consider PM as a strategic-behavior mitigating tool, we assume that they set their PM policies at the same time they select their inventory levels. Let y^i denote the inventory (capacity) of retailer i at the beginning of the season, and $m^i \in \{0, 1\}$ be retailer i's decision on utilizing a PM policy (where 0 and 1 mean "no" and "yes," respectively); define the vectors $y = \left(y^1, \dots, y^n\right)$ and $m = \left(m^1, \dots, m^n\right)$ accordingly. In the second period, the retailers are "free" to select their own prices, but we assume that under the competitive market structure, they converge to a price that clears the market (see, e.g., Dixon (2001)). To this end, we utilize a Cournot model to predict the second (clearance) period price as a function of the remaining inventory at the end of the first period. For example, Flath (2012) shows that products such as bicycles, records, and thermos bottles are appropriately described by the Cournot model.

In the first period, the market consists of *regular* customers with a mass normalized to 1, without loss of generality (change of scale). The first-period valuations of these regular customers are drawn from a uniform distribution on the interval $[0, 1]$. Two essential parameters in our model affect the customers' behavior. First, we use a parameter, $\beta \in [0, 1]$, to capture a typical decrease in valuations for seasonal and limited lifetime products. For example, a fashion product may lose 25% of its value (i.e., $\beta = 0.75$) – from a customer's standpoint – if that customer purchases the product at the end (second period) rather than at the beginning (first period) of the season. We refer to this parameter as *product durability*. We confine our analysis to the interesting case in which $\beta > c$, which means that some regular customers may be willing to purchase at a price that is above cost in the second period. If that is not the case, it is easy to show that any PM-equilibrium would have to result in sales in the first period only. Second, recall that a strategic customer is one that considers the possibility of postponing the time of purchase to the second period, by taking into account the possible price reduction and product availability in that period, as well as the PM payback (that would become *irrelevant* if the customer postpones the purchase). We use the parameter $\rho \in [0, 1)$, to which we refer as the *level of strategic behavior*, as a factor (weight) that the customers apply to the expected second-period surplus and to any reimbursements from PM. In particular, a value of ρ close to 1 means that the market consists of customers that are "fully" strategic, whereas $\rho = 0$ means that the customers are *myopic* (i.e., they always purchase in the first period if the price p_1 yields a non-negative surplus). The customers are homogeneous in their level of strategic behavior.

Similarly to Lai et al. (2010) and Cachon and Swinney (2009), we assume that in addition to the regular customers, there is an infinite number of bargain-hunting customers who can buy any number of units during the second period, at a unit salvage value $s < c$. Alternatively, one can think of this situation as a market setting in which remaining inventory can be returned to the supplier for a reimbursement of s per unit (e.g., through buyback agreements, or the ability of the supplier to divert the product to a secondary market channel).

Obviously, a key challenge in the theoretical study of markets with strategic customers, is the identification and characterization of market equilibria. To this end, one must pay careful attention to the assumptions regarding the information available to the decision makers: the customers and the retailers. For instance, in mature markets, where manufacturers regularly launch new versions of similar products, the retailers are typically able to conduct comprehensive customer behavior studies. In this vein, we assume that the retailers can determine how PM decisions affect the first-period demand. Additionally, we assume that the customers can form stable expectations regarding the price changes and product availability in the second period. Yet, similar to Lai et al. (2010), the customers observe only the retailers' PM-policies but not the inventory levels. Thus, we analyze the market using a game theoretical framework that follows the sequence of events listed below.

First, the customers form rational expectations about the second period market for all possible combinations of retailers' PM offerings. Second, given the customers' expectations, the retailers determine their PM policies and inventories. Third, sales are realized for the first period (see discussion in §2.2 for details). Finally, in the second period, if units of the product remain on shelf, the retailers engage in clearance sales, and, if they use PM, they reimburse the difference in prices between two periods.

2.1. **Customer Behavior.** Contingent on the PM offering information m, the customers form rational expectations about the product availability and the price in the second period, and make a decision about purchasing at the price p_1 or waiting for the second period. We model their behavior according to the following lines. Customers, who do not observe inventories, form expectations via two key parameters: first is the *expected* availability, $\bar{\alpha}(m) \in \{0,1\}$, which indicates whether inventory will be left at the end of the first period, and hence will be cleared. Second, in case that inventory is left ($\bar{\alpha}(m)=1$), is the expected clearance price $\bar{p}_2(m)$. For brevity of exposition, when there is no risk of confusion, we may avoid the explicit functional notation, using $\bar{\alpha}$ and $\bar{p}_2$ in short.

Confined to the pair $(\bar{\alpha},\bar{p}_2)$ for given m, the customers make their buy-or-wait decisions using a hierarchical procedure, as follows. At first, each customer would compare the price p_1 to its valuation v. If $v < p_1$, the customer will *wait* for the second period. Otherwise, the customer, who can gain an immediate surplus of $(v-p_1)$, will bring into consideration the second period – the essence of strategic behavior. Specifically, a customer who considers buying from a PM retailer will calculate the *net gain* that can be achieved in the *second period* by postponing the purchase; i.e., in addition to the loss of the immediate surplus $(v-p_1)$. That net gain consists of two values: (i) the expected PM payback that will be *forgone* due to the wait; and (ii) the expected surplus that would be *gained* in the second period. Altogether, we have

$$\Delta(v) \triangleq -\bar{\alpha}(p_1-\bar{p}_2)^+ + \bar{\alpha}(\beta v-\bar{p}_2)^+ .$$

Recall that we use the parameter ρ to express the degree of strategic behavior in the market. Following this approach, a customer with valuation $v \geq p_1$ will attempt to purchase a unit from a PM retailer in the first period if $\rho\Delta(v)-(v-p_1) \leq 0$. But observe that this functional expression is always decreasing in v, and hence, because $\rho\Delta(p_1)-(p_1-p_1) \leq 0$, we conclude that any customer with valuation larger than the threshold

$$v_1^{\min}(\bar{\alpha},\bar{p}_2) = \text{constant} = p_1 \tag{1}$$

will attempt to buy a unit from a PM retailer. Since $v_1^{\min}$ does not depend on expectations, we drop its functional dependence on $(\bar{\alpha},\bar{p}_2)$ in the rest of the paper. When a unit is not available at a PM retailer, the customer considers purchasing from a non-PM retailer. In such case, the decision would be based on whether $(v-p_1) \geq \bar{\alpha}\rho(\beta v-\bar{p}_2)^+$. Here, it is easy to verify that there are three cases of interest: (i) $p_1 \leq \bar{p}_2/\beta$, for which the valuation threshold p_1 would be adopted; (ii) $\frac{\bar{p}_2}{\beta} \leq p_1 \leq 1-\bar{\alpha}\rho(\beta-\bar{p}_2)$, for which the threshold $(p_1-\bar{\alpha}\rho\bar{p}_2)/(1-\bar{\alpha}\rho\beta)$ would be adopted; and

(iii) $p_1 \geq 1-\bar{\alpha}\rho\left(\beta-\bar{p}_2\right)$, for which no customer would buy the product in the first period, effectively using the threshold 1. In summary, any customer with valuation larger than the threshold

$$v_0^{\min}\left(\bar{\alpha},\bar{p}_2\right)=\max\left\{p_1,\min\left\{\frac{p_1-\bar{\alpha}\rho\bar{p}_2}{1-\bar{\alpha}\rho\beta},1\right\}\right\} \tag{2}$$

will attempt to buy a unit from a non-PM retailer in the first period. The above analysis demonstrates that an opportunity to buy from a PM retailer completely eliminates strategic customer behavior, regardless of its level ρ, the durability of the product β, or the expectations $(\bar{\alpha},\bar{p}_2)$. However, it is not clear when such elimination of strategic behavior is beneficial for the participants in the market.

2.2. **First-Period Sales Distribution Among Retailers.** In this section, we present a sales allocation mechanism, which allows for the calculation of sales during the first period, denoted by the vector $q=\left(q^1,\ldots,q^n\right)$. First, we assume that the first-period demand is allocated among the retailers with PM; then, the unsatisfied demand (due to stockouts) is split among the retailers that do not offer PM. Inside each group of retailers, customers buy in the order of their valuations. Second, we assume that the ability of a retailer to attract sales is proportional to its level of inventory.

To a certain degree, this model specification is in congruence with our uniform per-unit cost assumption, discussed in the beginning of the section: retailers that bring larger capacity could possibly enjoy economies of scale in procurement costs, but on the other hand may want to spend more on sales efforts in order to attract reasonable demand. The proportional allocation mechanism is not always the standard assumption. Admittedly, we utilize this assumption in order to enable us to obtain clear and relatively-elegant theoretical results. For similar reasons of gaining analytical tractability, papers such as Zhao and Atkins (2008), Liu and van Ryzin (2008), and Bazhanov et al. (2015) have considered alternative allocation schemes.

Let n_1 be the number of PM retailers, and let Y_1 be the aggregate inventory for those retailers; similarly, define n_0 and Y_0 for the non-PM retailers. Moreover, consider specific expectations $(\bar{\alpha},\bar{p}_2)$ for the corresponding PM policies m. The total demand that the PM retailers experience in the first period is $(1-p_1)$ as driven by the threshold value $v_1^{\min}=p_1$. Therefore, there are three special cases of interest, that depend on the aggregate inventory Y_1. (i) In case that $Y_1 \geq 1-p_1$, the PM retailers satisfy all of the demand, each selling a quantity $q^i=(1-p_1)\cdot y^i/Y_1$, whereas the non-PM retailers do not make any sales. Additionally, the regular customers remaining for the second period would have valuations uniformly distributed in the range $[0,\beta p_1]$ at that time. (ii) In case that $1-v_0^{\min}\left(\bar{\alpha},\bar{p}_2\right) \leq Y_1 < 1-p_1$, the PM retailers cannot satisfy all of the demand. Thus, the sales for the PM retailers are given by $q^i=y^i$, serving the valuation segment $[1-Y_1,1]$. Next, since the latter segment turns its demand to the PM retailers, and since $v_0^{\min}\left(\bar{\alpha},\bar{p}_2\right) \geq 1-Y_1$, it is easy to see that the non-PM retailers will experience no demand. Consequently, the regular customers remaining for the second period would have valuations uniformly distributed in the range $[0,\beta\left(1-Y_1\right)]$ at that time. (iii) In case that $Y_1 \leq 1-v_0^{\min}\left(\bar{\alpha},\bar{p}_2\right)$, the situation with the PM retailers remains the same as in case (ii). However, here it is easy to verify that the non-PM retailers would sell the quantities $q^i=\min\left(\left(1-Y_1-v_0^{\min}\left(\bar{\alpha},\bar{p}_2\right)\right)\frac{y^i}{Y_0},\ y^i\right)$. The regular customers remaining for the second period would have valuations uniformly distributed in the range $\left[0,\beta\cdot\max\left\{1-Y_1-Y_0,v_0^{\min}\left(\bar{\alpha},\bar{p}_2\right)\right\}\right]$ at that time.

2.3. **Second-Period Clearance Sales.** Since the product offerings are undifferentiated, the retailers lower their prices until all remaining inventory is cleared; i.e., the second period price p_2 (identical for all retailers) would be set to a sufficiently low level that would make demand equal to the total remaining inventory. It is noteworthy that since PM and inventory decisions are made at the same time and the demand is deterministic, a retailer would never have to withhold previously acquired inventory from clearance because of the PM. Instead, a rational retailer simply avoids stocking any inventory that is not eventually sold.

Let $Y \triangleq Y_1 + Y_0$ and $Q \triangleq Q_1 + Q_0$. Following the analysis in the previous section, we anticipate that inventory will be left only when $v_0^{\min}(\bar{\alpha}, \bar{p}_2) > 1 - Y$. In such case, clearance of the inventory (i.e., *completing* the sales of all of the original inventory Y) can be made either by targeting the customer with the original valuation of $(1 - Y)$, by setting $p_2 = \beta(1 - Y)$. Or, turning to the stream of bargain-hunters, by setting $p_2 = s$. Obviously, the second-period price that would maximize revenue is

$$p_2 = \max\{s, \beta(1 - Y)\}, \tag{3}$$

which is independent of the PM offers present in the market.

In the rest of the paper, we limit our attention to situations in which the second period valuations are sufficiently high so that $\beta > s/p_1$ (a condition similar to the logical restriction $\beta > c$). If this condition does not hold, it is possible to show that, in a two-period equilibrium, the second period price cannot exceed s, $v_0^{\min} = p_1$ under rational expectations, and strategic customer behavior has no effect on any of the possible equilibria.

2.4. **The First Period Capacity and Price-Matching Decisions.** We continue our analysis by looking at the retailers' profit optimization problems in the first period. Recall that since the second-period market is cleared, each retailer's second period inventory (equals to its sales) is given by $y^i - q^i$. Obviously, because of the interactions among the retailers, we must describe any given retailer's profit as a function of the other retailers' decisions as well as the customers' expectations $(\bar{\alpha}(m), \bar{p}_2(m))$. To this end, define y^{-i}, m^{-i} as the vectors of inventories and PM decisions of all retailers except i. We can now present the objective functions for the retailers:

$$\begin{aligned} &r^i(y^i, m^i, y^{-i}, m^{-i}, \bar{\alpha}(m^i, m^{-i}), \bar{p}_2(m^i, m^{-i})) \\ &\qquad = -cy^i + p_1 q^i + p_2(y^i - q^i) - q^i(p_1 - p_2)^+ \cdot \mathbb{1}\left\{\{Y > Q\} \cap \{m^i = 1\}\right\} \end{aligned} \tag{4}$$

where the q^i-values depend on the values $(y^i, m^i, y^{-i}, m^{-i}, \bar{\alpha}(m^i, m^{-i}), \bar{p}_2(m^i, m^{-i}))$, as explicitly described in §2.2. We conclude that the best response of retailer i belongs to a set of (y^i, m^i) pairs:

$$BR^i\left(y^{-i}, m^{-i}, \bar{\alpha}(\cdot, m^{-i}), \bar{p}_2(\cdot, m^{-i})\right) \triangleq \operatorname*{Argmax}_{y^i, m^i}\left\{r^i(y^i, m^i, y^{-i}, m^{-i}, \bar{\alpha}(m^i, m^{-i}), \bar{p}_2(m^i, m^{-i}))\right\},$$

where notation $\bar{\alpha}(\cdot, m^{-i}), \bar{p}_2(\cdot, m^{-i})$ emphasizes the dependence of the best response set on the expectations corresponding to either value of $m^i \in \{0, 1\}$ but only the given value of m^{-i}.

Using the set of best responses, one can proceed to characterize general Nash equilibria in the retailer game. However, our primarily focus is on two levels: the level of competition and the level of strategic behavior. The retailers are identical and it is natural to consider cases when they behave in the same way. As we show in §3 below, the resulting symmetric equilibria cover almost 100% of all inputs. One cannot rule out the existence of asymmetric equilibria and they may provide some additional insights, but, given a rich collection of results obtained for the symmetric case, the asymmetry would merely distract from the main effects considered in this paper.

In a *symmetric* pure-strategy Nash equilibrium each retailer makes the same PM decision $\hat{m}$ and procures the same fraction $\frac{1}{n}\hat{Y}$ of the total inventory $\hat{Y}$. Additionally, we consider *symmetric* expectations characterized by only two pairs of values $(\bar{\alpha}(\hat{m}, \hat{m}, \ldots, \hat{m}), \bar{p}_2(\hat{m}, \hat{m}, \ldots, \hat{m}))$ and $(\bar{\alpha}(1 - \hat{m}, \hat{m}, \ldots, \hat{m}), \bar{p}_2(1 - \hat{m}, \hat{m}, \ldots, \hat{m}))$ corresponding to, respectively, the equilibrium PM profile $(\hat{m}, \hat{m}, \ldots, \hat{m})$ and any possible one-retailer deviation. Formally, since expectations depend only on the first argument, we drop the remaining arguments in the rest of the paper. For duopoly, symmetry of expectations requires an assumption that the customers can identify the PM decision that constitutes a deviation. For given symmetric expectations $\bar{\alpha}(\cdot), \bar{p}_2(\cdot)$, a symmetric equilibrium is a pair $(\hat{m}, \hat{Y})[\bar{\alpha}(\cdot), \bar{p}_2(\cdot)]$ (a pair $(\hat{m}, \hat{Y})$ as a function of $\bar{\alpha}(\cdot), \bar{p}_2(\cdot)$) such that $(\hat{m}, \frac{1}{n}\hat{Y})$ provides a best response to a symmetric strategy profile of other retailers, i.e.,

$(\hat{m}, \frac{1}{n}\hat{Y}) \in BR^i\left((\frac{1}{n}\hat{Y}, \ldots, \frac{1}{n}\hat{Y}), (\hat{m}, \ldots, \hat{m}), \bar{\alpha}(\cdot), \bar{p}_2(\cdot)\right)$, where $(\frac{1}{n}\hat{Y}, \ldots, \frac{1}{n}\hat{Y})$ and $(\hat{m}, \ldots, \hat{m})$ are $n-1$ dimensional vectors, which stand in for y^{-i} and m^{-i} respectively.

2.5. **Rational Expectations Equilibrium.** In order to gauge the effect of PM, a participant of the market (a retailer, a manufacturer, or a local regulator) must first be able to understand which equilibria are possible for that particular market scenario. In order to predict market outcome, we utilize a rational expectations equilibrium framework. Specifically, we identify the set of decisions, made by the retailers and the consumers, such that they are optimal in the sense described earlier, but are also consistent with the customers expectations $(\bar{\alpha}(\cdot), \bar{p}_2(\cdot))$. That is, the equilibrium inventory levels and PM decisions of the retailers must lead to precisely the same observed product availability and clearance prices as expected by the customers. Recall that, according to (3), the observed second period price corresponding to the total inventory $\hat{Y}$ is equal to $\max\{s, \beta(1-\hat{Y})\}$. Moreover, if the total first-period sales corresponding to $(\hat{m}, \hat{Y})$ are $\hat{Q}$, then the observed second-period availability is $\mathbb{1}\{\hat{Y} > \hat{Q}\}$. Thus, we define *rational expectations symmetric equilibrium* (RESE) in pure strategies as follows:

Definition 1. *The tuple $(m^*, Y^*, \alpha^*(\cdot), p_2^*(\cdot))$ is a RESE if*

- *m^* and Y^* are a symmetric equilibrium PM decision and a total inventory level corresponding to symmetric expectations $\alpha^*(\cdot)$ and $p_2^*(\cdot)$, i.e., $(m^*, Y^*) = (\hat{m}, \hat{Y})[\alpha^*(\cdot), p_2^*(\cdot)]$;*
- *the expected and the observed equilibrium second-period availabilities and prices coincide, i.e., for the corresponding first-period sales Q^*, $\alpha^*(m^*) = \mathbb{1}\{Y^* > Q^*\}$ and $p_2^*(m^*) = \max\{s, \beta(1-Y^*)\}$;*
- *and, for a single retailer deviating from m^* into a different PM strategy $1-m^*$ and this retailer's optimal inventory decision y', we have, for the corresponding first-period sales Q' under the deviation, $\alpha^*(1-m^*) = \mathbb{1}\left\{\frac{n-1}{n}Y^* + y' > Q'\right\}$ and $p_2^*(1-m^*) = \max\left\{s, \beta(1 - \frac{n-1}{n}Y^* - y')\right\}$.*

The last requirement of the above definition clarifies why expectations have to depend on the PM profile. In the absence of such dependence, expectations may not match the availability and clearance price observed under the deviations. Thus, a deviating retailer may be able to take advantage of these irrational expectations breaking the equilibrium as the result. On the other hand, if customers adjust expectations when they see a PM deviation, the deviator no longer has this unfair advantage.

For retailers, it is important to know which outcomes can emerge depending on the market situation. From the model perspective, the market situation is described by particular model inputs and potential outcomes correspond to the equilibria that exist in the retailer game. In the next section, we characterize all possible equilibria in closed form starting with those using PM. This characterization facilitates analysis of the impact of PM on retailers, consumers, and the local economy. Moreover, switches between equilibrium types due to changes in the inputs (such as the levels of strategic behavior or competition) inform market participants about potential jumps in profits, consumer surplus, and welfare.

3. Characterization of RESE

There are two fundamental types of RESE that can potentially arise in the proposed model: with PM and without PM. We will refer to them, respectively, as PM and either N if no-PM is the retailer's decision or NA if PM is not available for other reasons. Each of these principal types are further classified into subtypes based on the structure of the market outcome. In particular, whether sales occur in both or only in one of the periods, and in which period they occur. We discuss PM first, and then contrast it with no-PM equilibria.

3.1. **Price-matching RESE.** When PM is used by all retailers, there are two types of equilibria which differ in how customers interpret the PM offers: whether or not the clearance sales should be expected. As we show, the equilibrium with (without) the second-period sales is characterized by relatively high (low) MSRP. The reader will also see that, in competitive markets ($n \geq 2$), for sufficiently high level of strategic behavior and cost-to-durability ratio there is even an interval of MSRP where the equilibria of both types exist. This indicates that consumer expectations is the only determinant of PM equilibrium structure in such markets. The persistence of equilibria is ensured, per standard game theory reasoning, because it is not rational for a profit-maximizing retailer to deviate unilaterally. Overall, the characterization drives the point that strategic customer behavior critically affects the equilibrium type and the resulting profit.

Following the general logic of Nash equilibrium in the retailer game, we consider two types of one-retailer deviations: into a no-PM strategy with its corresponding best-possible inventory decision and a PM and inventory strategy that also changes the availability of the product. The second type of deviation is possible because the profit function is discontinuous at the point where $Y = Q$. For example, in the first part of the theorem, customers rationally expect that the product is available in the second-period under the equilibrium PM and inventory strategies, i.e., $\alpha^*(1) = 1$ and $Y^* > Q^* = 1 - p_1$. The PM-deviation by retailer i in that case would result in a smaller total inventory level $Y' = y^i + \frac{n-1}{n}Y^* = 1 - p_1 = Q'$ and no availability in the second period: $\alpha = \mathbb{1}\{Y' > Q'\} = 0$. The comparison of the the associated profits leads to a quadratic inequality in p_1 (keeping all other inputs fixed) resulting in condition (1.2). Similarly, the comparison with a no-PM deviation leads to (1.1) under the additional condition of rationality of clearance price expectations $p^*(0)$ in a no-PM deviation. We provide a point-by-point discussion of the conditions immediately following the theorem. In the rest of the paper, v^* is the equilibrium value of $v_1^{\min}$, which, along with other equilibrium values, may be explicitly identified with the type of RESE, e.g., $v^{*,PM1}$ or $Y^{*,PM2}$ if necessary.

Theorem 1. *If PM is possible, the PM-equilibria with the following structure exist if and only if the respective conditions hold:*

PM1 (Clearance sales, $\alpha^* = 1$): $v^* = p_1, p_2^* = c + \frac{\beta-c}{n+1}, Y^* = \frac{n}{n+1}\left(1 - \frac{c}{\beta}\right)$, *and* $r^* = \frac{(\beta-c)^2}{(n+1)^2\beta}$ *under conditions*

$$\text{(1.1)}\ \frac{c}{\beta} < CB_1(\rho,\beta,n) \triangleq \frac{1-2\rho+\rho^2\beta}{(1-\rho\beta)^2 + (1-\beta)\rho[n-(1-\rho\beta)]},\ p_1 \geq P_{11} \triangleq 1 - \frac{n-1+\rho\beta}{n+1}\left(1-\frac{c}{\beta}\right)\ \textit{or}$$

$$\text{(1.2)}\ \frac{c}{\beta} \geq CB_1(\rho,\beta,n)\ \textit{and}\ p_1 \geq P_{12}(c,\beta,n), \textit{where } P_{12} \textit{ is the larger root of a quadratic equation;}$$

PM2 (No clearance sales, $\alpha^* = 0$): $v^* = p_1, Y^* = 1 - p_1$, *and* $r^* = \frac{1}{n}(p_1 - c)(1 - p_1)$ *under conditions*

$$\text{(2.1)}\ \frac{c}{\beta} < CB_2(\rho,\beta,n) \triangleq \frac{1-2\rho+\rho^2\beta}{(1-\rho\beta)^2 + (1-\beta)n\rho^2\beta},\ p_1 \leq P_{21} \triangleq \frac{c}{\beta}\frac{(1-\rho\beta)^2}{1-2\rho+\beta\rho^2};\ \textit{or}$$

$$\text{(2.2)}\ \frac{c}{\beta} \geq CB_2(\rho,\beta,n)\ \textit{and}\ p_1 \leq P_{22}(c,\beta,n), \textit{where } P_{22} \textit{ is the larger root of a quadratic equation.}$$

All bounds P_{11}, P_{12}, P_{21}, and P_{22} are greater than $\frac{c}{\beta}$ if $n < \infty, \rho > 0$, and $\beta < 1$; $P_{11}, P_{12}, P_{22} \to \frac{c}{\beta}$ as $n \to \infty$, and $P_{21} = \frac{c}{\beta}$ if either $\rho = 0$ or $\beta = 1$; $P_{11}, P_{12}, P_{22} \to 1$ as $\frac{c}{\beta} \to 1$.

Condition $p_1 \geq P_{11}$ in part (1.1) guarantees that a possible deviator into no-PM has sales only in the second period *under rational expectations in a deviation*, i.e., $\alpha^*(0) = 1$ and $v_0^{\min}(\alpha^*(0), p_2^*(0)) > 1 - \frac{n-1}{n}Y^*$. As a result, the effective price in this case is the same for both PM and no-PM retailers implying that the best deviator profit and inventory level remain the same as before the deviation. Since valuation threshold $v_0^{\min}(\alpha^*(0), p_2^*(0)) = \frac{p_1 - \rho p_2^*(0)}{1-\rho\beta}$ (associated with the demand of a deviating no-PM retailer) depends on ρ, the resulting rational expectation of clearance price $p_2^*(0) = \beta(1-Y^*)$ changes with the level of strategic behavior. As shown in the proof, no-PM deviations under other conditions would dominate.

Condition $p_1 \geq P_{12}$ in part (1.2) results from a quadratic inequality stating that PM deviator profit with sales only in the first period does not exceed the equilibrium one: $(p_1 - c)y^i = (p_1 - c)(1 - p_1 - \frac{n-1}{n}Y^*) \leq r^*$. The threshold value P_{12} is the larger root of the corresponding quadratic equation. Intuitively, an increase in competition reduces the ability of a single retailer to control the availability of the stock in the second period. Therefore, $p_1 \leq P_{12}$ becomes less restrictive with an increase in n as shown in Corollary 2 below and illustrated in Figure 2 (the area of inputs where PM1 exists increases in n).

In part (2), customers rationally expect no sales in the second period when all retailers use PM. Condition $p_1 \leq P_{21}$ in (2.1) guarantees that the retailer's equilibrium profit is not dominated by the profit of a deviator into no-PM with sales in both periods. Similarly to (1.1), the level of strategic behavior enters this condition through the dependence of $v_0^{\min}$ on ρ which affects the resulting rational $p_2^*(0)$. Condition $p_1 \leq P_{22}$ in (2.2) guarantees that the deviator's profit into PM with sales in both periods does not exceed the equilibrium one (similarly to (1.2), this profit comparison leads to a quadratic inequality). Any other forms of deviations do not dominate equilibrium profits.

Theorem 1 also points to a special role played by the cost-to-durability ratio $\frac{c}{\beta}$. Indeed, low values of this ratio in combination with a relatively high first-period price lead to high profitability of the second-period sales, which is one of the key determinants of the equilibrium structure. Cost-to-durability thresholds CB_1 and CB_2 provided in the statement are the intersection points of the pairs of p_1-boundaries P_{11}, P_{12} and P_{21}, P_{22}, respectively. In particular, when $\frac{c}{\beta} < CB_1$, the condition $p_1 \geq P_{11}$ (comparison with a no-PM deviation) is more restrictive than $p_1 \geq P_{12}$ (comparison with PM deviation). We illustrate the areas of existence of PM1 and PM2 equilibria in the $(p_1, c/\beta)$ cross-section of the parameter space for $\rho = 0.3$, $n = 4$, and $\beta = 0.5$ in Figure 1. Both CB_1 and CB_2 are simultaneously positive, zero, or negative depending on the level of strategic behavior (since both denominators are positive, and the numerator is positive if and only if $\rho < \left(1 - \sqrt{1-\beta}\right)/\beta$). When CB_1 and CB_2 are positive, $CB_1 \leq CB_2 \leq 1$ where the first inequality is strict unless $\beta = 1$, $\rho = 0$, or $n = 1$. Moreover, when $\beta = 1$ or $\rho = 0$, both CB_1 and CB_2 equal one. As a result, positive CB_1 and CB_2 split cost-to-durability ratio values into relatively low, intermediate, and high ranges $(0, CB_1), [CB_1, CB_2)$, and $[CB_2, 1)$ that determine the functional forms of the equilibrium boundaries. For a specific cost-to-durability ratio, the classification depends on other inputs because, as follows from above, a given $\frac{c}{\beta}$ can be less than CB_1 only for small levels of competition (if $\beta < 1$ and $\rho > 0$) and strategic customer behavior.

Equilibrium PM1 includes the cases with low and intermediate cost-to-durability ratio and relatively high first-period price leading to attractive sales in the second period. All customers with $v \geq p_1$ buy in the first period and obtain reimbursement $p_1 - p_2^*$ in the second one. The customers with $v \in [p_2^*, p_1)$ wait for clearance sales. Since the effective price for all customers is p_2^*, we call this a "clearance" PM equilibrium.

In the case of PM2, the intermediate and high cost-to-durability ratio, as well as low p_1, make two-period sales with reimbursement less attractive than first-period sales only. All customers with valuations p_1 or higher buy in the first period. Retailers divide the profit associated with the total inventory that is just enough to cover the first-period market. Since the inventory is determined by externally set MSRP, retailer competition is effectively eliminated and this case can be interpreted as an MSRP-facilitated collusion. Since there are no second-period sales we refer to PM2 as a "no-clearance" equilibrium. PM2 cannot exist if customer valuations remain constant ($\beta = 1$ implies $P_{21} = c$ and $CB_2 = 1$). This outcome is intuitive because the less the decrease in customer valuations between two periods, the more profitable the second-period sales. In either of

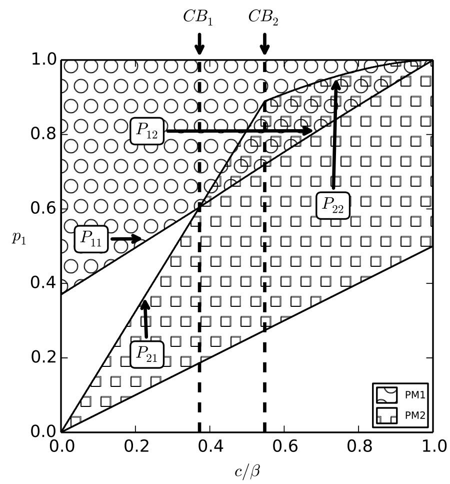


FIGURE 1. PM1 and PM2 regions in the $(p_1, c/\beta)$ slice of the parameter space for fixed $\rho = 0.3$, $n = 4$, and $\beta = 0.5$

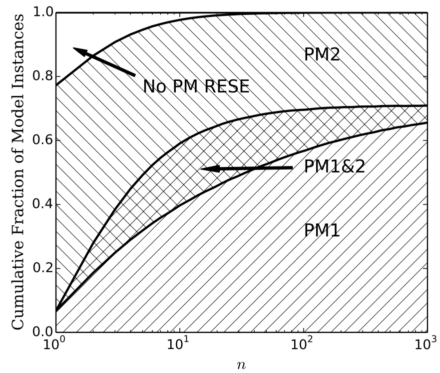


FIGURE 2. Fractions of model inputs where a particular PM RESE structure exists for given n

these two PM-equilibria, customers behave as if they are myopic and, consequently, inventory level Y^* and profit r^* do not depend on the level of strategic behavior.

The fraction of model inputs where PM-equilibria exist is illustrated in Figure 2 as a function of $1 \leq n \leq 1,000$. The fraction is computed by volume in the region of all inputs (ρ, β, c, s, p_1) satisfying the feasibility constraints $0 \leq \rho < 1$, $0 \leq s < c < \beta \leq 1$, and $\max\{\frac{s}{\beta}, c\} < p_1 \leq 1$. The figure is an area plot that shows the fractions of inputs resulting in a particular equilibrium structure (PM1 only, both PM1 and PM2, PM2 only, and neither PM1 nor PM2) as the heights of the respective shaded areas for each n. As n increases, the fraction of inputs where PM1 exists increases. However, the fractions of inputs with PM2 only, both PM1 and PM2, as well as neither equilibrium decrease.

Figure 2 only provides information with respect to the level of competition. The corollary below augments it by establishing a full set of monotonic properties of the PM1 and PM2 regions. In particular, it characterizes the overlap of PM1 and PM2 as well as the area where neither PM equilibrium exists.

Corollary 2. (1) *If $CB_1 > 0$, for low cost-to-durability ratio $\frac{c}{\beta} < CB_1$, we have $P_{21} < P_{11}$ and there are no PM-equilibria for $P_{21} < p_1 < P_{11}$. Moreover, $P_{11} = P_{12} = P_{21}$ if $\frac{c}{\beta} = CB_1$.*

(2) *For any* $\frac{c}{\beta}$, *we have* $P_{12} < P_{22}$ *if* $n > 1$ *and* $P_{12} = P_{22} = P_2 \triangleq \frac{1}{2}\left[1 + c + \sqrt{(1-\beta)\left(1 - c^2/\beta\right)}\right]$ *if* $n = 1$. *Therefore, for high cost-to-durability ratio* $\frac{c}{\beta} \geq CB_2$ *(possible only if* $\beta < 1, \rho > 0$*) there is an overlap* $P_{12} \leq p_1 \leq P_{22}$ *in the MSRP-range of PM1 and PM2 existence. In this overlap,* $r^{*,PM1} < r^{*,PM2}$ *if* $n > 1$ *and* $r^{*,PM1} = r^{*,PM2}$ *if* $n = 1$.

(3) *If* $\frac{c}{\beta} > CB_1 > 0$, *then* $P_{12} < P_{21}$. *Thus, for* $n > 1$ *and intermediate cost-to-durability ratio* $CB_1 \leq \frac{c}{\beta} < CB_2$ *(possible only if* $\beta < 1, 0 < \rho < (1 - \sqrt{1-\beta})/\beta$*), there is an overlap* $P_{12} \leq p_1 \leq P_{21}$ *in the MSRP-range of PM1 and PM2 existence. In this overlap,* $r^{*,PM1} < r^{*,PM2}$.

(4) *Inequality* $\frac{c}{\beta} \geq CB_1$ *is equivalent to a lower bound on* ρ.

(5) *The lower* p_1*-bounds* P_{11}, P_{12} *and upper* p_1*-bounds* P_{21}, P_{22} *depend on inputs as follows:*

	c	β	ρ	n
P_{11}	$\nearrow$	$\searrow$	$\searrow$	$\searrow$
P_{12}	$\nearrow$	$\searrow$	$\equiv$	$\searrow$
P_{21}	$\nearrow$	$\searrow$	$\nearrow$	$\equiv$
P_{22}	$\nearrow$	$\searrow$	$\equiv$	$\searrow$

The overlap in MSRP ranges of PM1 and PM2 equilibria established in Corollary 2 (Parts 2 and 3) is exclusive for oligopolistic PM-retailers and intermediate values of MSRP. That is natural because the monopolist optimally chooses whether to supply the product in one or both periods. For competitive settings, very low MSRP means that the second-period regular-customer market is small and has an extremely low margin. In contrast, a very high MSRP means that the first-period market is very small. Thus, the possibility of either type of equilibrium arises only for intermediate MSRP. Moreover, by Part 4, since $\frac{c}{\beta} \geq CB_1$ in the overlap, it can take place only if customers are sufficiently strategic. Thus, it is natural that customer expectations start to affect the equilibrium outcome. While PM2 is always better for competing retailers in the overlap, the magnitude of its difference with PM1 deserves a further study and we return to it in subsequent sections. The overlap does not exist for a monopoly, the highest-possible level of durability, or myopic customers.

While Theorem 1 provides a complete characterization of PM equilibria for a given market situation, market participants may want to forecast adjustments to equilibrium structure when the market situation changes. By Part 5 of Corollary 2, the areas where PM1 or PM2 exist *expand* when customers become *more* strategic. The changes in other parameters affect the areas of PM1 and PM2 existence in the opposite way. For example, when the level of competition *increases*, the area of PM1 *expands* while the area of PM2 *shrinks*.

The knowledge of possible shifts in the equilibrium structure is of particular importance when a market situation is close to the boundary between equilibria with notably different profits. In such situations, equilibrium can be unstable with respect to parameter changes or misestimations. Part 1 of Corollary 2 implies that for small levels of strategic behavior, i.e., $\rho < \left(1 - \sqrt{1-\beta}\right)/\beta$, which yields $CB_1 > 0$, PM equilibria may not exist. This observation stimulates an interest in the properties of equilibria when the PM-option is not available or when PM is available but remains unused. These equilibrium structures are considered below.

3.2. **RESE when PM is not available (NA).** For our study of PM we use the benchmark game where PM is not available. As shown by Bazhanov et al. (2015), there are four types of symmetric rational expectations NA equilibria identified in Theorems 3 and 5 cited below. Here and in other no-PM equilibria, we use v^* to denote the equilibrium value of $v_0^{\min}$. Similarly, these theorems provide the equilibrium expectations $\alpha^*(0)$ and $p_2^*(0)$ in the absence of PM. When PM is not available, the expectations $\alpha^*(1)$ and $p_2^*(1)$ corresponding to a one-retailer deviation into PM are undefined.

Theorem 3. *A unique NA with the stated structure exists if and only if the respective conditions hold:*

NA1 (No sales in the first period): *$v^* = 1, \alpha^*(0) = 1, p_2^*(0) = c + \frac{\beta - c}{n+1}, Y^* = \frac{n}{n+1}(1 - c/\beta)$, and $r^* = \frac{(\beta-c)^2}{(n+1)^2\beta}$ under condition $p_1 \geq 1 - \frac{n}{n+1}\rho(\beta - c) \triangleq P_1^N$.*

NA2 (No sales in the second period): *$v^* = p_1, \alpha^*(0) = 0, Y^* = 1 - p_1$, and $r^* = \frac{1}{n}(p_1 - c)(1 - p_1)$ under condition $p_1 \leq \frac{nc}{n-1+\beta} \triangleq P_2^N$.*

NA3 (Sales in both periods, $p_2^* > s$): *$v^* = \frac{p_1 - \rho\beta(1-Y^*)}{1-\rho\beta}, \alpha^*(0) = 1, p_2^*(0) = \beta(1 - Y^*)$, where Y^* is the larger root of a quadratic equation, and $r^* = \frac{1}{n}\left[(p_1 - c)(1 - v^*) + (p_2^* - c)(Y^* - 1 + v^*)\right]$ under condition $P_2^N < p_1 < P_1^N$ and one of the following:*
(a) $\frac{n-1}{n}(p_1 - s)(1 - v^*)Y^* \leq (c - s)(1 - s/\beta)^2$, or (b) condition (a) does not hold, $Y^* < 1 - s/\beta$, and $r^* \geq \tilde{r}^i$, where $\tilde{r}^i$ is the maximum profit of a firm deviating from this equilibrium in such a way that $p_2 = s$ (the total inventory is greater than $1 - \frac{s}{\beta}$).

The equilibrium characteristics Y^*, v^*, and r^* are continuous on the boundaries between these forms of NA. Moreover, under NA3, $Y^* > \max\left\{\frac{n}{n+1}\left(1 - \frac{c}{\beta}\right), (1 - p_1)\right\}$.

The following proposition shows the relationships between p_1-bounds in NA and PM-equilibria.

Proposition 4. (1) *The area of NA2 existence is always inside the area of PM2 existence, i.e., $P_2^N \leq \min\{P_{21}, P_{22}\}$.*
(2) *For $n > 1$, the area of NA1 existence is always inside the area of PM1 existence, i.e., $P_1^N > P_{11}$ and $P_1^N \geq P_{12}$ (strict for $\beta < 1$). For $n = 1$, the area of PM1 existence is always inside the area of NA1 existence, i.e., $P_1^N = P_{11}$ and, for $\frac{c}{\beta} > CB_1$, $P_1^N < P_{12}$.*

Notably, there are inputs for which either PM equilibrium can be realized if retailers use PM, or a price-discriminating equilibrium NA3 is realized if PM is not available. Due to differences in equilibrium structures, the change in profit can be discontinuous when PM becomes available.

For a monopoly ($n = 1$), Theorem 3 exhaustively covers all feasible parameter values. Starting from a duopoly, there is an area of inputs where none of the equilibria described in Theorem 3 may exist. At the same time, for oligopoly retailers with strategic customers, by the theorem below, there exists one more form of NA with sales in both periods and $p_2^* = s$ (NA4). This form exists only inside the p_1-range of NA3, i.e., there exists a non-empty set of input parameters where both NA3 and NA4 may exist and, by Proposition 4, either PM1 or PM2 may exist if PM is available.

Theorem 5 ("Salvaging" NA4: $p_2^* = s$)**.** *NA with $v^* = \frac{p_1 - \rho s}{1 - \rho\beta}, \alpha^*(0) = 1, p_2^*(0) = s, Y^* = \frac{n-1}{n}\frac{p_1 - s}{c - s}(1 - v^*)$, and $r^* = \frac{p_1 - s}{n^2}(1 - v^*)$ exists if and only if one of the following mutually exclusive conditions hold:*

(a) *salvaging is forced on retailers, i.e., $\frac{n-1}{n}Y^* \geq 1 - \frac{s}{\beta}$;*
(b) *condition (a) does not hold and the deviator profit is strictly increasing in the interval corresponding to $p_2 > s$, which is equivalent to $1 - \frac{s}{\beta} > \frac{n-1}{n}Y^* \geq \left(1 - \frac{s}{\beta}\right)^2 \frac{c + \beta v^* - 2s}{\beta(1-s/\beta)^2 + (p_1 - \beta)(1 - v^*)}$;*
(c) *conditions (a) and (b) do not hold, $Y^* > 1 - \frac{s}{\beta}$, and either the deviator profit is strictly decreasing in the interval corresponding to $p_2 > s$ (in this case the deviator profit never exceeds r^*), or $r^* \geq \tilde{r}^i$, where $\tilde{r}^i$ is the maximum deviator profit in this interval.*

Conditions of NA3 and NA4 existence indicate proximity of the market situation to a boundary of the area of existence. Namely, if the equilibrium exists only because the equilibrium profit r^* exceeds the profit of a potential deviator $\tilde{r}^i$ (condition (b) for NA3 and (c) for NA4), the equilibrium can be very sensitive to parameter changes.

Proposition 6. *(1) NA4 exists only if $c - s < \frac{n-1}{n}\frac{\beta(1-s)^2}{4(\beta - s)}$ (otherwise, there are no p_1 and ρ leading to $p_2^* = s$) and $p_1 < P_4^N \triangleq 1 - \rho(\beta - s)$ (otherwise, NA4 form of v^* does not permit sales in the first period). Moreover, $P_4^N < P_1^N$ and p_1-bounds are equivalent to upper bounds on ρ, with $\rho_4^N \triangleq \frac{1-p_1}{\beta - s}$*

and $\rho_1^N \triangleq \frac{n+1}{n}\frac{1-p_1}{\beta-c} > \rho_4^N$. *(2) For any inputs where both NA3 and NA4 exist,* $r^{*,NA4} < r^{*,NA3}$. *Moreover,* $r^{*,NA4} < \frac{1}{n}(p_1 - c)(1 - p_1)$.

Proposition 6 implies, first, that NA4 exists only when the unit salvage value is relatively close to the cost and when the first-period price is relatively low, resulting in first-period sales that are enough to compensate for the second-period loss. Since the p_1-upper bound in NA4 is strictly below P_1^N, the p_1-upper bound in NA3, NA4 never coexists with NA3 if $p_1 \in [P_4^N, P_1^N)$. If NA4 exists for $\rho = 0$, keeping other inputs fixed, it may also exist for $\rho < \rho_4^N$. Thus, part (2) of Proposition 6 in conjunction with a nonempty range $[\rho_4^N, \rho_1^N)$ may lead to a substantial "discontinuous" gain from increasing strategic behavior. Indeed, for $\rho \in [\rho_4^N, \rho_1^N)$, NA4 does not exist and no other NA-equilibria may exist except NA3, whose profit is higher than that of NA4.

3.3. **RESE without PM when PM is available.** An introduction of a PM decision into the retailer game increases the set of possible strategies. Thus, in the PM-game, no-PM equilibria may still exist but under more restrictive conditions than in the no-PM game since a retailer has an additional dimension to deviate. We denote by N an equilibrium where the PM option is available but not used. A formal statement (Proposition 16, Appendix), which is illustrated in Figure 3, shows that the additional flexibility for retailers in the form of PM-option indeed restricts the areas of existence of N-equilibria (except for N2 and N1 for $n > 1$) in comparison with the corresponding areas of NA-equilibria. These additional restrictions can be interpreted as conditions of "stability" of NA-equilibria with respect to PM option.

The information about PM-policy gives an additional signal for customer expectations. For example, if p_1 is relatively high, implying sales in the second period, the declaration of PM by a profit-maximizing retailer may lead to a higher p_2 than without PM since, under PM, p_2 cannot be below unit cost. On the other hand, if p_1 and β are relatively low, any second-period sales may result in $p_2 < c$. In this case, the declaration of PM implies the absence of second-period sales.

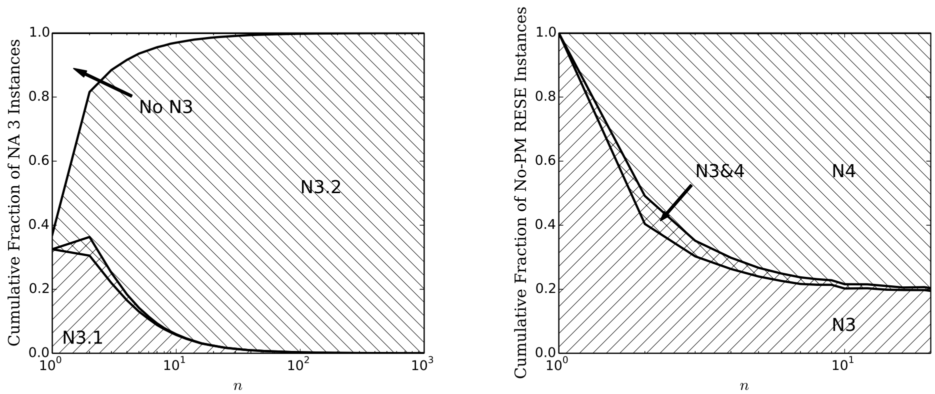
According to the definition of RESE, customer expectations need to be specified both for a symmetric PM decision profile and for all one-retailer deviations into no-PM. In this section, $\alpha^*(0)$ and $p_2^*(0)$ specify equilibrium expectations for a symmetric no-PM strategy profile, while $\alpha^*(1)$ and $p_2^*(1)$ — for a one-retailer deviation into PM. Rational customer expectations associated with a deviation determine two different subtypes of N3, which we call N3.1 (for $\alpha^*(1) = 0$) and N3.2 (for $\alpha^*(1) = 1$). Both these subtypes correspond to an otherwise identical NA3 structure. A summary of these outcomes is presented visually in Figure 3(a) as fractions of NA3 instances. There is a very small area of inputs where both N3.1 and N3.2 can exist. The incidence of N3.1 and no-N3 quickly diminishes and tends to zero as the market approaches perfect competition ($n \to \infty$). On the other hand, N3.2 type becomes dominant and absorbs the entire NA3 area as $n \to \infty$.

The behavior of fractions of NA4 instances is similar but the decreases in N4.1, an overlap of N4.1 and 4.2, and no-N4 are more rapid. The highest values of these fractions occur in a duopoly and are, respectively, 7.0%, 0.17%, and 0.26%.

N1 and N2 equilibrium types cannot exist for the inputs where PM-equilibria do not exist. On the other hand, N3 and N4 can exist. We examine the fractions of no-PM model instances, where N3 and/or N4 exist, visually in the area plot of Figure 3(b). The overwhelming majority of no-PM instances corresponds to N-equilibria. The remaining fraction of no-PM instances where neither N3 nor N4 exist is too small to be seen on the figure (its maximum over n is just 0.044%).

For brevity, we use N(A) to refer to either N-equilibrium if PM is available but retailers do not use it or NA if PM is not available due to other reasons. Visualizations of possible equilibrium types across model inputs are provided in Figures 5 and 6 (illustrating Examples 1 and 3 in §5).

FIGURE 3. For given n, fractions of inputs resulting in different types of
(a) N3 within NA3 (b) N within no-PM



4. When is PM beneficial for participants of the market?

By affecting the equilibrium, PM impacts all participants of the market. Thus, in this section, we consider PM effects on retailers in terms of their profit, on the manufacturer in terms of the total inventory, and on the customers in terms of the surplus, as well as on the local economy in terms of the aggregate welfare.

4.1. **PM effect on retailer profits.** As shown above, the availability of the PM option does not always lead to the existence of RESE with PM. But even if PM1 or PM2 exists, there are areas of inputs where PM-equilibria coexist with various forms of N(A), and it is not obvious that PM-profits are always greater in these areas. Indeed, it turns out that PM leads sometimes to a lower total profit than N(A)3 and/or N(A)4.

Assuming that, for given inputs, equilibria X and Y exist (possibly in different games), we say that X is *beneficial (equivalent, detrimental) for retailers* compared to Y if benefit $B^{X,Y} \triangleq r^{*,X} - r^{*,Y} > 0$ ($B^{X,Y} = 0, < 0$). Equilibrium X is beneficial (equivalent, detrimental) in an area of inputs if it is beneficial (equivalent, detrimental) for any inputs in this area.

Figure 4 (a) displays the area plot of fractions of PM1 inputs where N3 and/or N4 may also exist (implying the existence of NA3 and/or NA4 for these inputs). The overlap is quite large, and Figure 4 (b) shows that PM1 is detrimental compared to N3 and/or N4 in approximately 30% of the model inputs where PM1 coexists with either PM2 or N3 and/or N4. Recall that, depending on n, as illustrated in Figure 2, PM1 exists in approximately 10% to 70% of the volume of model inputs. Hence, up to 20% of possible model inputs may lead to a PM equilibrium that is detrimental compared to a RESE without PM.

The plot for the overlap of PM2 with N3 and/or N4 is similar to Figure 4 (a) with the only difference that the cumulative fraction of the overlap is around 80% for $n = 2$ and approaches 100% for n closer to 100. However, unlike PM1, PM2 is either beneficial or equivalent to *all* other RESE in the areas of coexistence. For a monopolist, PM1 coexists only with N1, therefore the n-axes in the plots of Figure 4 start from $n = 2$.

The proposition below provides conditions for the dominance of an equilibrium profit either under PM, or N(A)3 and N(A)4. For the convenience of exposition, we use $r^{*,N3}, r^{*,N4}$ instead of $r^{*,N(A)3}, r^{*,N(A)4}$, and we let w^2 denote the ratio of profits $r^{*,PM1}$ over $r^{*,N4}$, normalized by $n^2/(n+1)^2$, i.e., $w^2 \triangleq (\beta - c)^2(1 - \rho\beta)/\{\beta(p_1 - s)[1 - p_1 - \rho(\beta - s)]\}$.

Proposition 7. (1) *For any inputs in the overlap of PM1 and the corresponding N(A),*

(1.1) $r^{*,PM1} < r^{*,N3}$ *if* $p_1 > 1 - \frac{n}{n+1}(\beta - c)$ *and* $c \geq 3\beta - 2\left(1 + \frac{1-\beta}{n}\right)$*;*

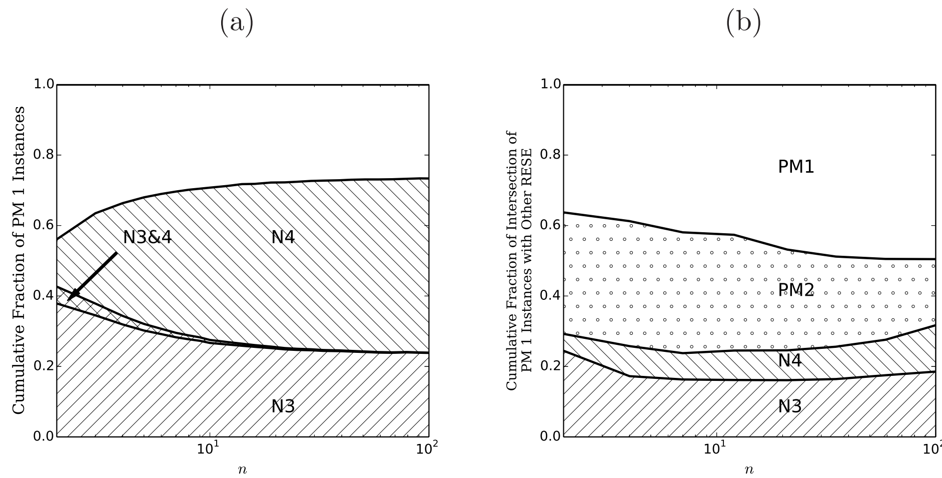


FIGURE 4. For given n, (a) fractions of PM1 instances where PM1 coexists with N3 or N4; (b) fractions of the intersection of PM1 instances with PM2 and N3 or N4 where the profit of the corresponding RESE is the greatest

(1.2) $r^{*,PM1} > r^{*,N4}$ *if and only if either* $w > \frac{3}{2}$, *or* $1 < w \leq \frac{3}{2}$, *and* $n > \frac{1}{w-1}$ *(*w *increases in* ρ*).*

(2) *For any inputs in the overlap of PM2 and the corresponding N(A),*

(2.1) $r^{*,PM2} \geq r^{*,N3}$ *with strict inequality if* $n > 1$ *or* $n = 1$ *and* $p_1 < P_{21}$;

(2.2) $r^{*,PM2} > r^{*,N4}$.

Part (1.1) implies that PM1 can be less profitable than N(A)3 when the product is not durable because, as we mentioned above, low durability decreases the second-period profits and, consequently, the attractiveness of PM1. Indeed, the lower bound on c in part (1.1) holds for any c and n if $\beta \leq \frac{2}{3}$ and never holds for $\beta > \frac{4+c}{5}$.

By part (1.2), since w increases in ρ, the more strategic customers are, the lower the minimum level of competition n when PM1 is beneficial compared to N(A)4. The necessary condition $w > 1$ for PM1 to be beneficial is equivalent to a lower bound on PM1 profit, i.e. $\frac{(\beta-c)^2}{\beta} > \frac{(p_1-s)(1-p_1-\rho(\beta-s))}{1-\rho\beta}$.

4.2. **PM effect on the total inventory.** An important part of this investigation is the PM effects on retailer inventory policies with the associated impact on all participants of the market. The total inventory, in turn, affects the existence of the second-period sales and, when these sales exist, the second period price. The results are summarized in the following proposition.

Proposition 8. *For the same inputs except PM-policy, in the areas where a PM-equilibrium and N(A) coexist, the total inventory under PM is not greater than under N(A), namely,*

(1) *PM total inventory and prices are the same as under N(A) if PM1 coexists with N(A)1 or PM2 coexists with N(A)2;*

(2) *PM total inventory is less than under N(A) if PM1 or PM2 coexists with N(A)3 or N(A)4.*

Hence, when the introduction of PM changes the realized RESE structure, the total inventory decreases. This result is consistent with the literature that shows that PM, by encouraging early purchases, allows retailers to increase prices (Png (1991), Lai et al. (2010)). In contrast, Nalca et al. (2013) showed that concurrent PM with availability checks may increase total inventory when retailers, facing uncertain demand, have stockouts.

Given that the wholesale price is fixed, the smaller inventory reduces the manufacturer's profit. Therefore, a current-profit-maximizing manufacturer, that is able to set the first-period price, may want to prevent the use of PM by retailers. On the other hand, a branded product manufacturer may prefer retailers to sell only at MSRP to maintain product reputation (e.g., Orbach (2008)), supporting PM2 as a result. The manufacturer's benefits from this support depend on the particular conditions of PM2 because the first-period price of branded products is usually high whereas PM2 exists for relatively low p_1.

Compared to N(A)3 or N(A)4, PM1 does not bring any benefits even for a branded product manufacturer. If PM1 is realized in the areas of equilibria coexistence, it means that retailers, using PM, avoid too high an MSRP. This situation is a signal for the manufacturer to target a lower first-period price. Alternatively, the manufacturer may negotiate a restriction against using PM. This no-PM restriction may benefit retailers, because, as shown in the previous subsection, retailer profits under PM1 may be even lower than under "salvaging" N(A)4, which is the worst RESE for retailers in a no-PM game (Figure 4 (b) and part (1.2) of Proposition 7).

4.3. **PM effects on customers and the local economy.** The above results partially support the findings in the literature that PM may be used as an anti-competitive practice. Indeed, recall that the price in PM2 is regulated by MSRP and, when another RESE with a lower second-period price is also possible, PM2-profit is always higher. If PM is not available and NA2 (which is equivalent to PM2 in profit) exists, then, as n increases, the outcome changes to NA3 or NA4 with $p_2 < p_1$. On the other hand, PM2 is guaranteed to exist for such inputs. Thus, a declaration of PM under PM2 merely serves as a tool to avoid competitive pricing. Customers do not receive any reimbursements.

The interpretation of PM as anti-competitive is not that obvious for another PM-equilibrium PM1, where PM is not just a declaration, and customers *do* obtain reimbursements in the second period. This RESE is the most beneficial for retailers in approximately 40% to 50% of inputs for which the other RESE may also exist. On the other hand, there is a significant share of inputs (Figure 4 (b)) where PM1 is detrimental compared to no-PM equilibria N(A)3 and even "salvaging" N(A)4. When PM1 is indeed detrimental for retailers, it is not clear whether it is beneficial for customers compared to N(A)3 or N(A)4. PM1 is indeed better than N(A)3 or N(A)4 for high-valuation customers who buy in the first period because their surplus is larger under PM1 due to reimbursements. In contrast, the low-valuation customers, who would buy in the second period under N(A)3 or N(A)4, are worse off under PM1 because, by Proposition 8, the PM-price is always higher than the second-period price under N(A)3 or N(A)4. Such mixed effects of PM raise a non-trivial question: is it possible that a PM-equilibrium is beneficial for the total customer surplus and/or aggregate welfare?

The total equilibrium customer surplus is $\Sigma = \Sigma_1 + \Sigma_2$, where Σ_1 and Σ_2 are the first-period and second-period surpluses respectively. We consider the actual or realized surplus, which is greater than the expected surplus. In the latter one, the second-period surplus would be discounted by ρ similarly to the individual second-period surplus used to determine the customer choice of buying or waiting. In contrast, the actual surplus measures the realized customer benefits depending on a type of RESE. The present value of the surplus is not an adequate measure for this purpose because it ignores or underestimates the realized second-period surplus of customers if they are myopic or, respectively, have low ρ.

The result below shows that PM1 is better for customers than N(A)1 and PM2.

Proposition 9. *For the same inputs, the change of equilibrium structure from N(A)1 to PM1 increases the total customer surplus except for a durable product ($\beta = 1$) and $p_1 = 1$ when the surplus remains the same; the change from any structure to PM2 decreases the surplus.*

Moreover, for the local economy (excluding the manufacturer), PM is socially beneficial in terms of the aggregate welfare $W \triangleq \Sigma + nr$ for the inputs where N(A)1 and PM1 coexist. Indeed, the total profit nr is the same in both equilibria, whereas, under PM, part of sales occur in the first period where customers enjoy fresh product effectively paying the reduced second-period price due to the reimbursements. For $n \geq 2$, PM1 can exist for the same inputs as N(A)3 or N(A)4. For $n = 2$, the fraction of inputs with a welfare-increasing switch from N(A)3 to PM1 is 81.5% (of the inputs where both NA3 and PM1 exist), and this fraction increases in n. For N(A)4, the fraction of inputs where PM1 improves W is even higher.

This result complements the finding of Lai et al. (2010) for a single retailer, who show, contrary to the literature, that PM can increase customer surplus when the uncertainty of the high-end market volume is high. In our setting, PM can be surplus- and welfare-improving even without uncertainty, e.g., when p_1 and ρ are sufficiently high leading to N(A)1 and PM1 existence.

Unlike PM1, PM2 is always disadvantageous for customers. However, by Proposition 7, PM2 is profitable for retailers, which raises a non-trivial question about the welfare-improving ability of PM2. PM2 improves W for $n = 1$ in 78% of inputs where PM2 and NA3 exist: intermediate p_1, relatively high difference $c - s$, *high* ρ and small β. Similarly to PM1, this share increases in n. In the area where PM2 and NA4 exist: s close to c, *high* ρ, n, and low β, p_1. This share starts from 99.9987% for $n = 2$ and increases in n. Thus, the local policymakers may help the retailers to escape from the "salvaging" N(A)4 by encouraging the use of PM.

Hence, when retailers operate under anti-competitive MSRP, another "collusive" tool – PM – improves social welfare for the local economy in most of the cases when customers are strategic. This effect results, first, from the fixed first-period price and, second, from an increase in the first-period sales. The latter effect increases the first-period surplus and the total profit — always under PM2 and, in some cases, under PM1 despite reimbursements since the second-period price is higher under PM1 than under no-PM equilibria. When PM is welfare improving, these increases exceed the loss of the second-period surplus under PM2 or a decrease in it under PM1.

5. Effectiveness of PM in counteracting strategic customer behavior

While previous sections provided qualitative description of PM-effects, the results below show that possible benefits or losses from PM can be essentially higher than losses from strategic customers. We contrast the cases of monopoly and oligopoly because the effects of PM are more pronounced under competition and can be qualitatively different in these two cases.

5.1. **PM performance.** This subsection introduces a suitable measure of PM performance as a profit-increasing tool relative to the effect on profit from an increase in the level of strategic behavior. Assume that all inputs except ρ are fixed and customers are more strategic for ρ^H than for $\rho^L < \rho^H$. Moreover, in the no-PM game, one of NA equilibria (denoted as NAL) is realized for ρ^L and, possibly, another NA equilibrium (denoted as NAH) is realized for ρ^H. Finally, a PM equilibrium is realized when customers are more strategic (Figures 5(b)), while the theoretical existence of a no-PM equilibrium with the same structure as NAH in the PM-game is not excluded. The corresponding no-PM profits are $r^{*,NAH}$ at ρ^H and $r^{*,NAL}$ at ρ^L. The PM-profit at ρ^H is $r^{*,PM}$.

Suppose the increase in ρ leads to a loss in the no-PM game, namely, $r^{*,NAH} - r^{*,NAL} < 0$. The *performance of PM as a tool for mitigating the loss from customer strategic behavior* is the ratio of the benefit from PM at ρ^H to the absolute value of the loss, i.e., $\eta(\text{NAL,NAH,PM}) \triangleq \frac{r^{*,PM} - r^{*,NAH}}{r^{*,NAL} - r^{*,NAH}}$. For brevity, we omit the arguments of the measure when it does not lead to confusion. This measure is negative when PM is detrimental, $\eta \in (0, 1]$ when PM leads to a *mitigation*, and $\eta > 1$ when PM results in a *gain*. For example, $\eta = 1$ means that PM mitigates 100% of the loss from increase in ρ. Theoretically, η can go to infinity when the change in ρ is close to zero, the profit in the no-PM game is continuous in ρ, and $r^{*,PM} - r^{*,NAH}$ is separated from zero due to discontinuous changes in the equilibrium structure resulting from the introduction of PM.

Similarly, suppose the increase in ρ leads to a gain in the no-PM game, e.g., as a result of the switch from NA4 to NA3 or under NA3 for large ρ and β. The *performance of PM as a tool for enhancing the gain from increasing strategic behavior* is the ratio of the benefit from PM at ρ^H to the absolute value of the gain, i.e., $\eta(\text{NAL,NAH,PM}) \triangleq \frac{r^{*,PM}-r^{*,NAH}}{r^{*,NAH}-r^{*,NAL}}$.

We keep to the following refinement of the notion "the gain from increasing strategic behavior." If equilibria A and B exist for both ρ^H and $\rho^L < \rho^H$ with $r^{*,A}|_{\rho=\rho^H} < r^{*,A}|_{\rho=\rho^L} < r^{*,B}|_{\rho=\rho^H}$, A is realized only for ρ^L, and B is realized only for ρ^H, the difference $r^{*,B}|_{\rho=\rho^H} - r^{*,A}|_{\rho=\rho^L} > 0$ *cannot be conclusively considered a gain* from increased ρ because the reason for the switch to B is not necessarily related to the increase in ρ. This difference may be a gain from another undetermined factor causing the switch.

Since equilibria can be multiple for both PM and no-PM, and for both ρ^H and ρ^L, the analysis below is concentrated on the cases when the benefits from PM are maximal as well as on the cases when PM is detrimental with the description of the corresponding areas of inputs.

5.2. **Monopoly.** We first consider the case of monopoly because of its analytical simplicity and qualitative differences from oligopoly. In particular, PM benefits neither the retailer nor customers if customers are myopic. Indeed, by Theorems 1 and 3 with myopic customers, a two-period PM-equilibrium (PM1) and a no-PM equilibrium without first-period sales (N(A)1) exist only in a degenerate case with $p_1 = 1$. A PM-equilibrium without second-period sales (PM2) exists only for a non-durable product ($\beta < 1$) and overlaps only with a no-PM equilibrium that has the same structure (N(A)2). Thus, when customers are myopic and the drop in valuations is relatively low, the major area of inputs belongs to a no-PM price-discriminating equilibrium N(A)3 ("salvaging" equilibrium N(A)4 does not exist for a monopolist).

The situation changes when customers are strategic and the product is not durable. Findings below, illustrated in Figure 5, specify the dependence of PM-benefits on the market parameters. In particular, there is an area leading to a PM-benefit only for the monopolist. This area, the overlap of PM2 and NA1, is not covered by Proposition 7 and exists only for high levels of strategic behavior. Indeed, the following lemma (illustrated in Figure 5(b)) shows that PM-equilibria exist only for sufficiently high ρ.

Lemma 10. *For $n = 1$, the conditions of PM-equilibria existence $p_1 \geq P_{11}$ and $p_1 \leq P_{21}$ are equivalent to lower bounds on ρ : $\rho \geq \frac{2(1-p_1)}{\beta-c} \triangleq \rho^{PM1}$, and $\rho \geq \frac{1}{\beta}\left[1-\sqrt{p_1(1-\beta)/(p_1-c)}\right] \triangleq \rho^{PM2}$ respectively, where $\rho^{PM2} \in (0,1]$, $\rho^{PM2} \to 0$ if $p_1\beta \to c+0$, and $\rho^{PM2} = 1$ if $\beta = 1$.*

Recall that the boundaries of PM-equilibria (Theorems 1 and Corollary 2) and their intersection points are such that $P_{12}|_{n=1} = P_{22}|_{n=1} = P_2$ and $CB_1|_{n=1} = CB_2|_{n=1} = CB$.

Proposition 11. (1) *A PM-equilibrium is beneficial for a monopolist compared to a no-PM equilibrium in and only in the union of (1.1) the overlap of PM2 and NA1 leading to benefit $B^{PM2,NA1} = (p_1-c)(1-p_1) - \frac{(\beta-c)^2}{4\beta} > 0$ that is constant in ρ, decreasing in β and increasing in c; and (1.2) the overlap of PM2 and NA3 leading to benefit $B^{PM2,NA3} = \frac{\beta p_1-c}{2-\rho\beta}\left[p_1(1-\beta)-(1-\rho\beta)^2(p_1-c)\right] > 0$ that is increasing in ρ.*

(2) *Retailer is indifferent between PM and no-PM equilibria in and only in the union of the overlaps of PM1 and N(A)1, PM2 and N(A)2, and the boundary between N3 and PM2.*

(3) *PM is less profitable than price discrimination (and, consequently, PM is not used) if and only if $\frac{c}{\beta} < CB, p_1 > \frac{c}{\beta}$ and $\rho < \min\{\rho^{PM1}, \rho^{PM2}\}$.*

Moreover, PM-equilibria never lead to a gain from an increase in ρ.

The proposition illustrates the nature of the relations between p_1-bounds in PM and N(A)-equilibria. When the second-period sales are relatively attractive, i.e., cost-to-durability ratio is low ($\frac{c}{\beta} < CB$), PM can be preferred to price discrimination (N3) only if the level of strategic

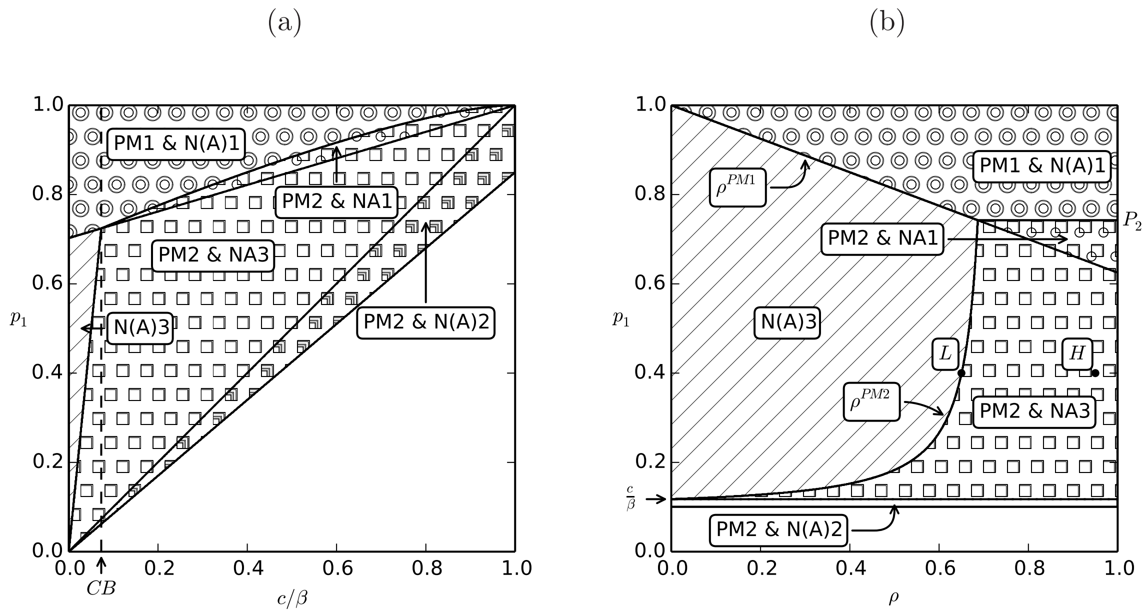


FIGURE 5. RESE for $n = 1$, $\beta = 0.85$ in (a) $(c/\beta, p_1)$ cross-section of inputs for fixed $\rho = 0.7$; and (b) (ρ, p_1) cross-section for fixed $c = 0.1$

behavior is high. When the second-period sales are less attractive, i.e., $\frac{c}{\beta} \geq CB$, PM is always no worse than price-discrimination. In this case, bound P_2, which does not depend on ρ, separates two forms of PM-equilibria — with sales in both periods (PM1) and only in the first one (PM2).

For a monopolist, PM-equilibria, when they exist, are never detrimental. However, PM never leads to a gain from increased strategic behavior. The following example illustrates Proposition 11.

Example 1. $p_1 = 0.4, \beta = 0.85, c = 0.1, \rho^L = 0.65, \rho^H = 0.95.$

Price-discriminating no-PM equilibrium NA3 exists for both ρ (Figure 5(b)) and, if PM is not available, the loss from increased ρ is $r^{*,NA3}|_{\rho=0.95} - r^{*,NA3}|_{\rho=0.65} = 0.170294 - 0.180010 = -0.009716$. If PM is available at ρ^H, PM2 is realized ($\rho^H > \rho^{PM2} = \frac{1-\sqrt{0.2}}{0.85} = 0.6503$) with $r^{*,PM2} = 0.18$, which mitigates almost all the loss. The performance of PM is η(NA3,NA3,PM2) $= 0.9989$. The performance decreases with the difference $\rho^{PM2} - \rho^L$ (ρ^L moves to the left from ρ^{PM2}).

5.3. **Oligopoly.** This subsection shows that in the oligopoly, unlike monopoly, PM can lead to substantial gains from strategic behavior as well as amplify losses depending on the market situation.

The competitive case has the following major differences from monopoly: "salvaging" equilibrium N(A)4, which may coexist with N(A)3, and the area of coexistence of PM1 and PM2. These differences lead to a much richer pattern of overlaps of PM and no-PM equilibria than under monopoly. The analysis of PM in the overlaps is simplified by the following results obtained above: (i) PM2-profit always exceeds the profit of PM1 (Corollary 2); (ii) N(A)3-profit always exceeds the profit of N(A)4 (Proposition 6); (iii) PM2 is always beneficial compared to N(A)3 and 4 (Proposition 7). By (ii) and (iii), the maximum benefit from PM2 belongs to the area where PM2 overlaps with N(A)4, which is specified in the following proposition.

Proposition 12. *For given inputs with $\rho^H > 0$, let PM2 and N(A)4 exist and, additionally, N(A)4 exists for the same inputs except $\rho^L < \rho^H$. Then PM2 at ρ^H leads to a gain from increased strategic behavior, bounded from below by $\eta(NA4,NA4,PM2)|_{\rho^L=0}$ as follows: $\eta(NA4,NA4,PM2) \geq \eta(NA4,NA4,PM2)|_{\rho^L=0} = 1 + \frac{(1-\rho^H\beta)(1-p_1)[n(p_1-c)-(p_1-s)]}{(p_1-s)\rho^H(p_1\beta-s)} > 1.$*

The following example illustrates the minimum value of $\eta(\text{NA4,NA4,PM2})|_{\rho^L=0}$ in n (attained at $n = 2$) for moderate values of other parameters (the existence of the equilibria in the examples below is shown in the appendix).

Example 2. $n = 2, \rho^L = 0, p_1 = \beta = \rho^H = 0.5, c = 0.1, s = 0.05.$

The performance of PM is $\eta(\text{NA4,NA4,PM2})|_{\rho^L=0} = 1 + \frac{3}{4} \cdot \frac{0.5\cdot(0.8-0.45)}{0.45\cdot 0.5\cdot 0.2} = 1 + \frac{3\cdot 0.35}{0.36}$, i.e., the increase in profit due to the introduction of PM2 at $\rho = 0.5$ is almost four times greater than the loss of profit under NA4 due to increased strategic behavior from $\rho = 0$ to $\rho = 0.5$. This gain is impossible without strategic customers because, for these data and small ρ, PM2 does not exist.

The case $p_1 = \beta$ used in Example 2 also provides a simple characterization of inputs where PM1 is beneficial compared to NA3:

Proposition 13. *Under the conditions of PM1 (Theorem 1) and NA3 (Theorem 3) with $p_1 = \beta$, $r^{*,PM1} > r^{*,NA3}$ if and only if $\beta > \frac{1+c}{2}$ and either $\rho > 2\frac{1-\beta}{\beta-c}$, or $\rho \in \left(\frac{1-\beta}{\beta-c}, 2\frac{1-\beta}{\beta-c}\right]$ and $n > \frac{1-\beta}{(\beta-c)\rho-1+\beta}$.*

This proposition shows that PM1 is never beneficial compared to NA3 for β close to c, namely, for $\beta \leq \frac{1+c}{2}$. If the necessary conditions $\beta > \frac{1+c}{2}$ and $\rho > \frac{1-\beta}{\beta-c}$ hold, PM1 *may be* better for retailers than NA3. PM1 *is* better for any level of competition n if the level of strategic behavior is quite high, i.e., $\rho > 2\frac{1-\beta}{\beta-c}$, and for sufficiently high $n > \frac{1-\beta}{(\beta-c)\rho-1+\beta}$, if the level of strategic behavior is moderate i.e., $\rho \in \left(\frac{1-\beta}{\beta-c}, 2\frac{1-\beta}{\beta-c}\right]$.

PM1 is beneficial compared to NA4 for $p_1 = \beta$ only if $(\beta - c)^2 > \beta(\beta - s)(1 - \frac{\beta-\rho s}{1-\rho\beta})$, which, by part (1.2) of Proposition 7, is equivalent to $w > 1$. This condition never holds for β close to c and, when it holds (for large ρ), PM1 is beneficial for large n.

There is an important qualitative difference between PM1 and PM2 that should not be neglected by retailers. The difference is that, under competition, PM1 may be detrimental compared to no-PM equilibria N(A)3 or 4. This property leads to the situation that can be called a *PM-trap for retailers*. Assume, for the following data, that retailers are not using PM and N(A)4 is realized.

Example 3. $n = 3, \rho = 0.5, c = 0.1, s = 0.05, p_1 = 0.7, \beta = 0.25.$

Figure 6(a) shows RESE types in the neighborhood of these inputs in a (ρ, p_1) cross-section of the feasible inputs for fixed $n = 3$, $c = 0.1$, $s = 0.05$ and $\beta = 0.25$. For these inputs, $r^{*,N(A)4} = 0.0165$, and PM can be beneficial since PM2 exists and $r^{*,PM2} = 0.06$, which is 3.6 times higher than $r^{*,N(A)4}$. However, the attractive comparison of $r^{*,PM2}$ with $r^{*,N(A)4}$ may work as a bait in a trap. For these inputs, there also exists PM1 with the profit $r^{*,PM1} = 0.0056$, which is approximately 1/3 of $r^{*,N(A)4}$. The example illustrates Theorems 1, 5 and Corollaries 2, 7 showing that, depending on customer expectations, PM can lead to losses when theoretically, for the same inputs, a beneficial equilibrium exists.

The differences in PM-trap profits may be even higher, e.g., for the same data except $p_1 = 0.85$ and $\beta = 0.15$ (the equilibria exist by the same conditions), namely, $r^{*,N(A)4} = 0.0096, r^{*,PM2} = 0.0375$ (3.9 times higher than $r^{*,N(A)4}$), and possible outcome $r^{*,PM1} = 0.00104$ is 9.2 times less than the initial profit $r^{*,N(A)4}$. The area of the PM-trap shrinks with n since CB_1 and CB_2 decrease to zero in n and both P_{12} and P_{22} go to $\frac{c}{\beta}$, reducing to zero the area of coexistence of PM1 and PM2.

If retailers are trapped in the detrimental PM1, it is profitable for the manufacturer to help them out by adding to the contract a "no-PM" condition since, by Proposition 8, $Y^{*,N4}$ always exceeds both $Y^{*,PM1}$ and $Y^{*,PM2}$. However, local policymakers may counteract manufacturer activity because the aggregate welfare W is greater for PM1 in both examples. Besides the PM-trap, the overlap of N(A)4 with PM1 and PM2 contains the inputs where $r^{*,PM1} > r^{*,N4}$. By part (1.2) of

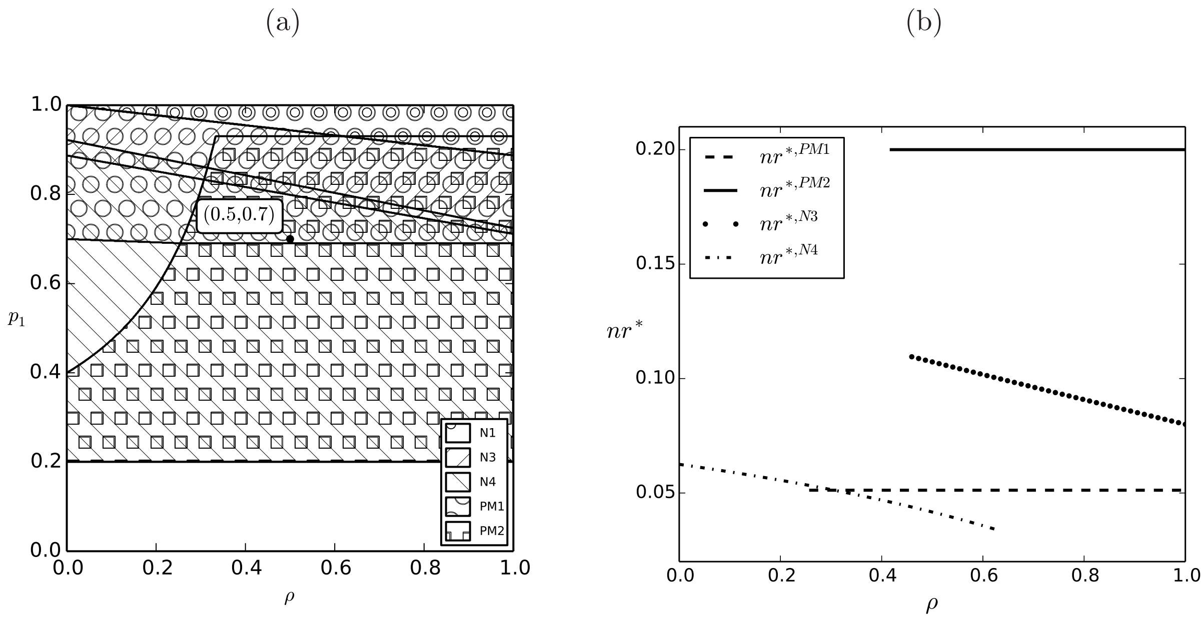


FIGURE 6. RESE in the neighborhoods of inputs of (a) Example 3: (ρ, p_1) cross-section for fixed $n = 3$, $c = 0.1$, $s = 0.05$ and $\beta = 0.25$; and (b) Example 4: Equilibrium profit as a function of ρ for fixed $p_1 = \beta = 0.5$, $n = 4$, $c = 0.1$, $s = 0$.

Proposition 7, these inputs correspond to larger ρ, n, and differences $\beta - c$. In this area, W is still greater for PM1, and the only part of the market suffering from PM is the manufacturer.

Example 3 quantifies the result of Proposition 7 showing the amount by which PM1-profit may be less than the least profit without PM. Compared to no-PM equilibria, PM1 can become detrimental when the level of strategic customer behavior is increasing even if it was beneficial for lower values of ρ. The following example shows the extent of the negative effect in this case.

Example 4. $n = 4, p_1 = \beta = 0.5, \rho^H = 0.65, \rho^L = 0.2, c = 0.1, s = 0.$

This example illustrates another PM-trap for retailers, which can be called a "regulator-facilitated PM-trap." The regulators may encourage retailers to switch to another equilibrium (with PM) at $\rho = 0.32$ (Figure 6(b)) since, under PM1, both retailers' profit and welfare are higher than under N4 ($W^{PM1} = 0.34 > W^{N4} = 0.26$). For larger ρ, there also exist equilibria PM2 and N3 with higher profits than under PM1, but the regulators may discourage retailer switching away from PM1 since W attains maximum under PM1 in this example. The manufacturer, who is also worse off under PM1 than under N3, could initiate the switch to no-PM for large ρ. However, the regulators may restrict manufacturer's interventions since, from their point of view, a socially-optimal outcome is realized. The negative performance of PM as a tool for enhancing the gain from increasing ρ, when ρ increases from 0.2 to 0.65, is $\eta(\text{NA4,NA3,PM1}) = -1.1$. That is, the loss from PM is greater than the gain from increased strategic behavior without PM. On the other hand, by Proposition 7, the performance of PM1 as a mitigating tool can be positive and, as the following example shows, PM1 may even lead to a notable gain.

Example 5. $n = 4, p_1 = 0.4, \beta = 0.65, \rho^H = 0.4, \rho^L = 0.3, c = 0.05, s = 0.$

NA4-profit decreases in ρ by 8.7% from $r^{*,NA4}|_{\rho=0.3} = 0.01258$ to $r^{*,NA4}|_{\rho=0.4} = 0.01149$. If PM becomes available and PM1 is realized at $\rho^H = 0.4$ (PM1 does not exist at $\rho^L = 0.3$), profit at ρ^H almost doubles to $r^{*,PM1}|_{\rho=0.4} = 0.02215$, which in terms of PM performance is $\eta(\text{NA4,NA4,PM1}) = 9.776$ — the increase in profit due to use of PM is almost ten times greater than the loss from increased strategic behavior under NA4. PM1 is beneficial for the aggregate welfare as well: $W^{PM1} = 0.346 > W^{NA4} = 0.277$.

6. Conclusions

The fundamental effects associated with PM in the markets described by the proposed model include a reduction in the total inventory and the corresponding increase in the clearance prices or even a complete elimination of the clearance sales. Moreover, since strategic customer behavior also tends to decrease equilibrium inventory, the PM can amplify this reduction in the presence of strategic customers. As a result, by using PM, retailers can mitigate losses from strategic customer behavior and even gain from an increase in its level. However, the gain is impossible for a monopolistic retailer. When MSRP is relatively high, the retailers cannot take advantage of it in a clearance PM-equilibrium (PM1) because of reimbursements. As a result, retailer profit in PM1 can be less than without PM for low levels of strategic behavior, competition, and durability as well as for high unit cost. In particular, there may exist a "PM-trap" for retailers — an input area leading to the worst "salvaging" equilibrium (N4) without PM. Depending on expectations, the PM equilibrium in this area can be either no-clearance PM2 (with a greater profit than in the "salvaging" N4) or the clearance PM1 (with the profit less than in N4 as in Example 3).

These combined effects of PM and strategic behavior lead to the following implications for *retailers*. (i) If the no-clearance PM-equilibrium exists, it is always efficient as a strategic-customer mitigating tool and always better than the clearance PM-equilibrium and no-PM equilibria for any given inputs. (ii) If both PM-equilibria exist for the same inputs, PM can hurt the retailers if the clearance PM-equilibrium is realized. (iii) PM effect can be smaller than the discontinuous profit gain due to the switch to a more profitable no-PM equilibrium (Example 4). This occurs if "salvaging" no-PM equilibrium N4 exists and, for a higher level of strategic behavior, there exist both the clearance PM-equilibrium and a no-PM equilibrium with a higher clearance price (N3).

There are several implications for a *manufacturer* that is able to influence the retailer PM policy. (i) Since PM is never beneficial for a current-profit-maximizing manufacturer, it may use contract terms to discourage retailers from this policy. (ii) A branded product manufacturer may support the no-clearance PM-equilibrium. This support is possible only for a sufficiently low first-period price when this equilibrium exists.

Since the aggregate welfare under PM can be either greater or less than without PM, the implications for *local policymakers* may vary. (i) A combination of PM with MSRP may indicate collusion among retailers or between retailers and a manufacturer (for a branded product). This case does not require interventions when PM is welfare-increasing. Otherwise, the retailers' PM policy may be targeted by regulations. (ii) The absence of PM for a non-branded product may indicate manufacturer interference. This case does not require any action when PM is welfare decreasing. Otherwise, the manufacturer activity may be conditioned in the corresponding way and retailers may be encouraged to use PM.

References

Aviv, Y., Y. Levin, M. Nediak. 2009. Counteracting strategic consumer behavior in dynamic pricing systems. S. Netessine, C. Tang, eds., *Consumer-Driven Demand and Operations Management Models*. Springer, 323–352.

Aviv, Y., G. Vulcano. 2010. Dynamic list pricing. O. Ozer, R. Phillips, eds., *Oxford Handbook of Pricing Management*. Oxford University Press, 522–584.

Bazhanov, A., Y. Levin, M. Nediak. 2015. Quantity competition in the presence of strategic consumers. *Working Paper* Available at SSRN: http://ssrn.com/abstract=2371728.

Buehler, S., D. Gärtner. 2013. Making sense of nonbinding retail-price recommendations. *American Economic Review* **103**(1) 335–359.

Butz, D. 1996. Does the per se rule deter vertical price-fixing? *Economic Inquiry* **34**(4) 770–780.

Cachon, G., R. Swinney. 2009. Purchasing, pricing, and quick response in the presence of strategic consumers. *Management Science* **55**(3) 497–511.

Coase, R. 1972. Durability and monopoly. *The Journal of Law and Economics* **15**(1) 143–149.

Cooper, T. 1986. Most-favored-customer pricing and tacit collusion. *RAND J. Econ.* **17**(3) 377–388.

Dixon, H. 2001. *Surfing Economics*. Palgrave Macmillan.

Flath, D. 2012. Are there any Cournot industries in Japan? *Japanese Economy* **39**(2) 3–36.

Holt, C., D. Scheffman. 1987. Facilitating practices: The effects of advance notice and best-price policies. *RAND J. Econom.* **18**(2) 187–197.

Jain, S., J. Srivastava. 2000. An experimental and theoretical analysis of price-matching refund policies. *J. Marketing Res.* **37**(3) 351–362.

Lai, G., L. Debo, K. Sycara. 2010. Buy now and match later: the impact of posterior price matching on profit with strategic consumers. *Manuf. Serv. Oper. Man.* **12**(1) 33–55.

Levin, Y., J. McGill, M. Nediak. 2010. Optimal dynamic pricing of perishable items by a monopolist facing strategic consumers. *Production and Operations Management* **19**(1) 40–60.

Liu, Q., G. van Ryzin. 2008. Strategic capacity rationing to induce early purchases. *Management Science* **54**(6) 1115–1131.

Nalca, A., T. Boyaci, S. Ray. 2013. Competitive price-matching guarantees: Equilibrium analysis of the availability verification clause under demand uncertainty. *Man. Sci.* **59**(4) 971–986.

Orbach, B. 2008. Antitrust vertical myopia: the allure of high prices. *Arizona Law Review* **50** 261–287.

Png, I. 1991. Most-favored-customer protection vs. price discrimination over time. *J. Pol. Econom.* **99**(5) 1011–1028.

Png, I., D. Hirshleifer. 1987. Price discrimination through offers to match price. *Journal of Businesses* **60**(3) 365–383.

Salop, S. 1986. Practices that (credibly) facilitate oligopoly coordination. J. Stiglitz, F. Mathewson, eds., *New developments in the analysis of market structure*. MIT Press, 265–290.

Shen, Z., X. Su. 2007. Customer behavior modeling in revenue management and auctions: A review and new research opportunities. *Production and Operations Management* **16**(6) 713–728.

Su, X. 2007. Inter-temporal pricing with strategic customer behavior. *Management Science* **53**(5) 726–741.

Su, X., F. Zhang. 2008. Strategic customer behavior, commitment, and supply chain performance. *Man. Sci.* **54**(10) 1759–1773.

Winter, R. 2008. Price-matching and meeting-competition guarantees. W. Collins, ed., *Issues in Competition Law and Policy*. American Bar Association, 1269–1296.

Zhao, X., D. Atkins. 2008. Newsvendors under simultaneous price and inventory competition. *Manuf. Serv. Oper. Man.* **10**(3) 539–546.

Symbol	Definition
p_1, p_2	first- and second-period price
c, s	unit cost and salvage value
$I = \{1, \ldots, n\}$	set of all retailers
y^i, q^i	retailer i inventory and sales in the first period
m^i	price matching (PM) decision: $m^i = 1$ — PM, $m^i = 0$ — no-PM
Y_k, Q_k	combined first-period inventories and sales of retailers with $m^i = k$
Y, Q	total first-period inventories and sales
v	first-period customer valuation of the product
β	factor of decrease of customer valuation by the second period
$\bar{\alpha}, \bar{p}_2$	customer expectations about product availability and price in the second period
ρ	the level of customer strategic behavior
$v_k^{\min}$	minimum valuation level of customers who, given a choice, would purchase from retailers with decision $m^i = k$ in the first period
y^{-i}, m^{-i}	vectors of inventories and PM decisions of all retailers except i
r^i	profit of retailer i
BR^i	best response of retailer i
$\hat{m}, \hat{Y}$	symmetric equilibrium PM and inventory decision
$(m^*, Y^*, \alpha^*, p_2^*)$	rational expectations symmetric equilibrium (RESE)
r^*, Σ, W	RESE-profit, total customer surplus, and aggregate welfare
CB_1, CB_2	c/β-bounds for RESE PM1 and PM2
P_{11}, P_{12}	p_1-low bounds for PM1
P_{21}, P_{22}	p_1-upper bounds for PM2
P_1^N	p_1-bound between RESE NA1 and NA3
P_2^N	p_1-bound between RESE NA2 and NA3
P_4^N	p_1-upper bound (necessary) for RESE NA4
η	performance of PM as a tool for mitigating the loss from customer strategic behavior
$a \vee b, a \wedge b$	$\max\{a, b\}$ and $\min\{a, b\}$ respectively

Table 1. Main abbreviations and notation

Appendix A. Proofs of the main text statements

A.1. **Proof of Theorem 1 (PM).** The proof uses the following technical lemma.

Lemma 14. *The roots $(p_1)_{1,2}(x)$ of the following equation exist for $x \geq c/\beta$:*

$$p_1^2 - (x + c)p_1 + \frac{\beta}{4}(x + c/\beta)^2 = 0. \tag{5}$$

Moreover, $(p_1)_2(x)$ is increasing in x if $x > c/\beta$, and $(p_1)_1(x) \leq \frac{1}{2}(x + c/\beta) \leq (p_1)_2(x) \leq x$ with strict inequalities if $x > c/\beta$ and, for the first two inequalities, if $\beta < 1$.

Assume that expectations are defined when all retailers use PM as well as when any retailer i deviates into no-PM, which, recall, we denote as $\bar{\alpha}(0), \bar{p}_2(0)$.

The proof of Theorem 1 is based on the lemma below that provides the necessary and sufficient conditions for PM response with positive inventory to consistent PM strategies of others. This consideration excludes the following trivial case where the PM best response can only have zero inventory: $Y^{-i} \geq 1 - c/\beta$ and $Y^{-i} \geq 1 - p_1$. Indeed, under these conditions the second-period sales are always below cost and it is impossible for retailer i to have positive sales in the first period only. The conditions in the form of p_1-bounds with $(p_1)_2$ in parts (a.2) and (b) of Lemma 15 guarantee that the maximum profit of a PM-retailer with the same product availability as other retailers (no second period sales or there are second-period sales) is not dominated by the profit of a deviator who also uses PM but has a different inventory policy: with second period sales in part (a.2) and without second-period sales in part (b). Condition (6) guarantees that the maximum profit of a PM-retailer without second-period sales is not dominated by the profit of a deviator into no-PM with sales in both periods.

Lemma 15. *There exists BR with PM and positive inventory to consistent PM strategies of others if and only if one of the following hold*

(a) *(no second-period sales)* $1 - Y_1^{-i} > p_1$ *and either of*

(a.1) $\frac{c}{\beta} \geq 1 - Y_1^{-i}$, *or*

(a.2) $\frac{c}{\beta} < 1 - Y_1^{-i}$, $v_0^{\min} \notin \left(\frac{c}{\beta}, 1 - Y_1^{-i}\right)$ *and* $p_1 \leq (p_1)_2$ *(where* $(p_1)_2 = (p_1)_2(x)|_{x=1-Y_1^{-i}} > \frac{c}{\beta}$*), or*

(a.3) $\frac{c}{\beta} < 1 - Y_1^{-i}$, $v_0^{\min} \in \left(\frac{c}{\beta}, 1 - Y_1^{-i}\right)$ *and*

$$p_1^2 - (v_0^{\min} + c)p_1 + \frac{\beta}{4}\left(v_0^{\min} + c/\beta\right)^2 \leq 0. \tag{6}$$

(b) *(there are second-period sales)* $\frac{c}{\beta} < 1 - Y_1^{-i}, v_0^{\min} \notin \left(\frac{c}{\beta}, 1 - Y_1^{-i}\right)$, *and* $p_1 \geq (p_1)_2$ *(where* $(p_1)_2 > \frac{c}{\beta}$*).*

The BR level of inventory y_1^i *is: in case (a)* $y_1^i = 1 - p_1 - Y_1^{-i}$, *and in case (b)* $y_1^i = \frac{1}{2}(1 - Y_1^{-i} - c/\beta)$; *the optimal inventory of a deviator into no-PM in case (a.3) is*

$$\tilde{y}_0^i = 1 - Y_1 - \frac{1}{2}\left(v_0^{\min} + c/\beta\right). \tag{7}$$

PM1. By part (b) of Lemma 15, the symmetric inventory of a retailer is $\frac{1}{n}Y_1^* = \frac{1}{2}(1 - \frac{n-1}{n}Y_1^* - c/\beta)$, resulting in $Y_1^* = \frac{n}{n+1}(1 - c/\beta)$ and $Y_1^{-i} = \frac{n-1}{n+1}(1 - c/\beta)$. Then condition $c/\beta < 1 - Y_1^{-i}$ is $\frac{n-1}{n+1}(1-c/\beta) < 1-c/\beta$, which always holds. Inequality $v_0^{\min} \leq c/\beta$, as a part of $v_0^{\min} \notin (c/\beta, 1-Y_1^{-i})$, is not relevant because $v_0^{\min} = v_0^{\min}(\bar{\alpha}(0), \bar{p}_2(0)) \leq c/\beta$ implies (see the proof of Lemma 15) that a possible deviator into no-PM selects $y_0^i = \breve{y}_0^i$ (no second-period sales). Then, by rationality, $\bar{\alpha} = 0$ and $v_0^{\min} = p_1 \leq c/\beta$, which cannot hold together with $p_1 \geq (p_1)_2 > c/\beta$.

Hence, the existence of PM1 is determined only by $v_0^{\min} \geq 1 - Y_1^{-i}$ and $p_1 \geq (p_1)_2$. Inequality $v_0^{\min} \geq 1 - Y_1^{-i}$ means that a possible deviator into no-PM has sales only in the second period with $r_0^i = [\beta(1 - Y_1 - y_0^i) - c]y_0^i$, the first-order condition $\frac{\partial r_0^i}{\partial y_0^i} = -2\beta y_0^i + \beta(1 - Y_1) - c = 0$, and the resulting profit-maximizing inventory

$$\tilde{y}_0^i = \frac{1}{2}(1 - Y_1 - c/\beta) = \tilde{y}_1^i, \tag{8}$$

giving rational $\bar{\alpha} = 1, \bar{p}_2 = p_2^* = \beta(1-Y_1^*)$, and $v_0^{\min} = \frac{p_1 - \rho\beta(1-Y_1^*)}{1-\rho\beta}$. Then inequality $v_0^{\min} \geq 1 - Y_1^{-i}$ is

$$\begin{aligned}
\frac{p_1 - \rho\beta[1 - \frac{n}{n+1}(1 - c/\beta)]}{1 - \rho\beta} &\geq 1 - \frac{n-1}{n+1}(1 - c/\beta) \Leftrightarrow \\
p_1 - \rho\beta + \frac{n\rho\beta}{n+1}(1 - c/\beta) &\geq 1 - \rho\beta - \frac{n-1}{n+1}(1 - c/\beta) + \rho\beta\frac{n-1}{n+1}(1 - c/\beta) \Leftrightarrow \\
p_1 &\geq 1 - \frac{n - 1 + \rho\beta}{n+1}(1 - c/\beta) = P_{11}.
\end{aligned} \tag{9}$$

P_{11} is less than $(p_1)_2(x)|_{x=1-\frac{n-1}{n+1}(1-c/\beta)} = P_{12}$ if after substitution of P_{11} for p_1 and $x = 1 - \frac{n-1}{n+1}(1-c/\beta)$ into (5) the LHS of (5) becomes negative. This condition takes the form of a quadratic inequality in c with the coefficient in front of c^2 equal to $\frac{(1-\rho\beta)^2 + (1-\beta)\rho[n-(1-\rho\beta)]}{\beta(n+1)^2} > 0$ and the roots

$\{\beta \cdot CB_1, \beta\}$, The first root is not greater than β (strictly less if $\beta < 1$). Therefore, $P_{11} \le P_{12}$ if $c/\beta \ge CB_1$. Then PM1 exists under condition $p_1 \ge P_{12} = \frac{1}{2}\left[x + c + \sqrt{(1-\beta)(x^2 - c^2/\beta)}\right]$ (part (1.2) of the Theorem).

Now we show that the substitution of P_{11} and $x = 1 - \frac{n-1}{n+1}(1 - c/\beta)$ into (5) results in positive LHS of (5) *only if* $P_{11} > P_{12}$. This conclusion results from the following chain of inequalities that is proved below:

$$P_{11} = 1 - \frac{n-1+\rho\beta}{n+1}(1 - c/\beta) > \frac{1}{2}(x + c/\beta) \ge (p_1)_1(x),$$

where $(p_1)_1(x)$ is the smaller root of (5) and $x = 1 - \frac{n-1}{n+1}(1 - c/\beta)$. Indeed, the first inequality is

$$\frac{2-\rho\beta}{n+1} + \frac{n-1+\rho\beta}{n+1}\frac{c}{\beta} > \frac{1}{n+1} + \frac{n}{n+1}\frac{c}{\beta} \Leftrightarrow \frac{1-\rho\beta}{n+1} > \frac{1-\rho\beta}{n+1}\frac{c}{\beta},$$

which always holds, and the second inequality $\frac{1}{2}(x+c/\beta) \ge (p_1)_1(x)$ holds by Lemma 14. Therefore, whenever $c/\beta < CB_1$, we have $P_{11} > P_{12}$ yielding part (1.1) of the Theorem. Moreover, $P_{11} = P_{12}$ when $c/\beta = CB_1$.

PM2. By part (a) of Lemma 15, the symmetric inventory in this case is $Y_1^* = 1 - p_1$ and condition $1 - Y_1^{-i} > p_1$ holds for $y^i > 0$. The condition of part (a.1), resulting in the existence of PM2, takes the form $c/\beta \ge 1 - \frac{n-1}{n}(1 - p_1) \Leftrightarrow p_1 \le 1 - \frac{n}{n-1}(1 - c/\beta) = c/\beta - \frac{1}{n-1}(1 - c/\beta) < c/\beta$. The complementary case to this inequality is covered by parts (a.2) and (a.3) of Lemma 15.

When $v_0^{\min} \le c/\beta$, a possible deviator into no-PM has no sales in the second period, and $v_0^{\min} = p_1 \le c/\beta$ implying $p_1 \le (p_1)_2$ by part (a.2) of Lemma 15. Therefore, PM2 exists for any feasible $p_1 \le c/\beta$.

Consider $v_0^{\min} \in (c/\beta, 1 - Y_1^{-i})$ (part (a.3) of Lemma 15). This condition in combination with (6) excludes $v_0^{\min} = p_1$ because (6) becomes $p_1^2 - (p_1+c)p_1 + \frac{\beta}{4}(p_1 + c/\beta)^2 \le 0 \Leftrightarrow \frac{\beta}{4}(p_1 - c/\beta)^2 \le 0$, which is impossible for $v_0^{\min} = p_1 > c/\beta$. The case $v_0^{\min} = 1$ is also irrelevant here because it contradicts $v_0^{\min} < 1 - Y_1^{-i}$. Since the range $v_0^{\min} \in (c/\beta, 1 - Y_1^{-i})$ means that a possible deviator into no-PM has sales in both periods (sales only in the first period yield profit that is not greater than under PM2), i.e., $\bar{\alpha} = 1$, the only relevant case for $v_0^{\min}$ is $v_0^{\min} = \frac{p_1 - \rho p_2}{1-\rho\beta}$, where $p_2 = \beta(1 - Y_1 - y_0^i)$, and, by (7), $y_0^i = \tilde{y}_0^i = 1 - Y_1 - \frac{1}{2}(v_0^{\min} + c/\beta)$, yielding

$$\begin{aligned} v_0^{\min} &= \frac{p_1 - \rho\beta\frac{1}{2}(v_0^{\min} + c/\beta)}{1-\rho\beta} \Leftrightarrow (1-\rho\beta)v_0^{\min} = p_1 - \rho\beta v_0^{\min}/2 - \rho\beta c/2 \Leftrightarrow \\ (1-\rho\beta/2)v_0^{\min} &= p_1 - \rho c/2 \Leftrightarrow v_0^{\min} = \frac{2p_1 - \rho c}{2-\rho\beta}. \end{aligned}$$

Then condition $v_0^{\min} < 1 - Y_1^{-i}$ becomes

$$\begin{aligned} \frac{2p_1 - \rho c}{2-\rho\beta} &< 1 - \frac{n-1}{n}(1-p_1) \Leftrightarrow 2p_1 - \rho c < (2-\rho\beta)\Big(\frac{1}{n} + \frac{n-1}{n}p_1\Big) \Leftrightarrow \\ \Big(2 - \frac{n-1}{n}(2-\rho\beta)p_1\Big)p_1 &< \frac{2-\rho\beta}{n} + \rho c \Leftrightarrow p_1 < \frac{2-\rho\beta+\rho cn}{2+(n-1)\rho\beta}, \end{aligned} \tag{10}$$

and condition $v_0^{\min} > c/\beta$ is $\frac{2p_1-\rho c}{2-\rho\beta} > c/\beta \Leftrightarrow 2p_1 - \rho c > 2c/\beta - \rho c \Leftrightarrow p_1 > c/\beta$. Under the combination of this inequality with (10), by part (a.3) of Lemma 15, an equilibrium with $Y_1^* = 1 - p_1$ exists if and only if inequality (6) holds. With the rational "symmetric" $v_0^{\min}$ this inequality becomes

$$p_1^2 - \Big(\frac{2p_1 - \rho c}{2-\rho\beta} + c\Big)p_1 + \frac{\beta}{4}\Big(\frac{2p_1 - \rho c}{2-\rho\beta} + \frac{c}{\beta}\Big)^2 \le 0. \tag{11}$$

The coefficient in front of p_1^2 is $a_2 = 1 - \frac{2}{2-\rho\beta} + \frac{\beta}{(2-\rho\beta)^2} = \frac{\beta(1-2\rho+\rho^2\beta)}{(2-\rho\beta)^2}$, which is positive if and only if $\rho < (1 - \sqrt{1-\beta}/\beta$ (the larger root of $1 - 2\rho + \rho^2\beta = 0$ is $(1+\sqrt{1-\beta})/\beta > 1$ – irrelevant here). If

$a_2 = 0$, (which means that $1-2\rho+\rho^2\beta = 0$) the inequality above becomes $p_1 \geq -a_0/a_1$, where $a_0 = \frac{\beta}{4}\Big(c/\beta - \frac{\rho c}{2-\rho\beta}\Big)^2$ and $a_1 = -(2-2\rho\beta-2\rho+\rho^2\beta+\rho^2\beta^2)/(2-\rho\beta)^2 = -[(1-\rho\beta)^2+1-2\rho+\rho^2\beta]/(2-\rho\beta)^2$ $= -(1-\rho\beta)^2/(2-\rho\beta)^2$, yielding $p_1 \geq c/\beta$, which always holds in this case.

If $a_2 > 0$, the reduced form of (11), after collecting the terms with p_1 and dividing by a_2, is

$$p_1^2 - \frac{c}{\beta}\Big(1 + \frac{(1-\rho\beta)^2}{1-2\rho+\rho^2\beta}\Big)p_1 + \Big(\frac{c}{\beta}\Big)^2\Big(\frac{(1-\rho\beta)^2}{1-2\rho+\rho^2\beta}\Big) \leq 0 \tag{12}$$

with the roots of the corresponding equation $(p_1)_{1,2} = \{\frac{c}{\beta}, \frac{c}{\beta}\frac{(1-\rho\beta)^2}{1-2\rho+\rho^2\beta}\}$, which can be seen by observing that $-[(p_1)_1+(p_1)_2]$ equals the coefficient in front of p_1 and $(p_1)_1(p_1)_2$ – the free coefficient of (12). The roots are distinct if and only if $\beta < 1$. Both (11) and (12) hold if p_1 is between the roots:

$$\frac{c}{\beta} \leq p_1 \leq \frac{c}{\beta}\frac{(1-\rho\beta)^2}{1-2\rho+\beta\rho^2} = P_{21}. \tag{13}$$

In the case $a_2 < 0$ (implying $1-2\rho+\rho^2\beta < 0$), inequality (12) is inverted and holds if p_1 does not exceed the smaller root, which is irrelevant since $P_{21} < 0$ in this case, or if p_1 is not less than the larger root: $p_1 \geq c/\beta$, which always holds in this case.

Hence, the case $v_0^{\min} \in (c/\beta, 1-Y_1^{-i})$ yields two upper bounds on p_1 that guarantee PM2 existence, namely, conditions (10) and (13). The bound on c/β below shows when P_{21} is less than the bound from (10).

$$P_{21} = \frac{c}{\beta}\frac{(1-\rho\beta)^2}{1-2\rho+\beta\rho^2} < \frac{2-\rho\beta+n\rho\beta\frac{c}{\beta}}{2-\rho\beta+n\rho\beta} \Leftrightarrow \frac{c}{\beta}\left[\frac{(1-\rho\beta)^2(2-\rho\beta+n\rho\beta)}{1-2\rho+\rho^2\beta} - n\rho\beta\right] < 2-\rho\beta \Leftrightarrow$$

$$\frac{c}{\beta}\frac{(1-\rho\beta)^2(2-\rho\beta)+n\rho\beta[(1-\rho\beta)^2-1+2\rho-\rho^2\beta]}{(2-\rho\beta)(1-2\rho+\rho^2\beta)} < 1 \Leftrightarrow \frac{c}{\beta}\frac{(1-\rho\beta)^2+n\rho^2\beta(1-\beta)}{1-2\rho+\rho^2\beta} < 1 \Leftrightarrow \frac{c}{\beta} < CB_2. \tag{14}$$

Consider $v_0^{\min} \geq 1-Y_1^{-i}$ (part (a.2) of Lemma 15). Since a possible deviator to no-PM has no first period sales, the optimal inventory, by (8), is $\tilde{y}_0^i = \frac{1}{2}(1-Y_1^{-i}-c/\beta)$, and inequality $v_0^{\min} \geq 1-Y_1^{-i}$ is

$$\begin{aligned}\frac{p_1-\rho\beta[1-Y_1^{-i}-\frac{1}{2}(1-Y_1^{-i}-c/\beta)]}{1-\rho\beta} &\geq 1-Y_1^{-i} \Leftrightarrow \\ p_1-\rho\beta(1-Y_1^{-i})+\rho\beta(1-Y_1^{-i}-c/\beta)/2 &\geq (1-Y_1^{-i})(1-\rho\beta) \Leftrightarrow p_1+\rho\beta(1-Y_1^{-i})/2-\rho c/2 \geq 1-Y_1^{-i},\end{aligned}$$

which with $1-Y_1^{-i} = \frac{1}{n}+\frac{n-1}{n}p_1$ is

$$p_1 + \frac{\rho\beta}{2}\Big(\frac{1}{n}+\frac{n-1}{n}p_1\Big) - \frac{\rho c}{2} \geq \frac{1}{n}+\frac{n-1}{n}p_1 \Leftrightarrow \Big(\frac{1}{n}+\frac{\rho\beta(n-1)}{2n}\Big)p_1 \geq \frac{\rho c}{2}+\frac{1}{n}\Big(1-\frac{\rho\beta}{2}\Big)$$

yielding $[2+\rho\beta(n-1)]p_1 \geq \rho cn+2-\rho\beta$, which gives inequality complementary to (10). $v_0^{\min} = 1$ is included in this condition since $v_0^{\min} = 1 \geq 1-Y_1^{-i}$ always holds; $v_0^{\min} = p_1$ is irrelevant here because $v_0^{\min} = p_1 \geq 1-Y_1^{-i}$ contradicts the necessary condition of part (a) Lemma 15.

Hence, when $v_0^{\min} \geq 1-Y_1^{-i} = x > c/\beta$, the existence of PM2 is guaranteed by condition

$$\frac{2-\rho\beta+\rho cn}{2+(n-1)\rho\beta} \leq p_1 \leq (p_1)_2(x)|_{x=\frac{1}{n}+\frac{n-1}{n}p_1}. \tag{15}$$

Similar to above, the resulting condition for c/β below is equivalent to the non-emptiness of this range. Consider inequality

$$p_1 \leq (p_1)_2(x)|_{x=\frac{1}{n}+\frac{n-1}{n}p_1}, \tag{16}$$

which, by (5), is equivalent to $2p_1 - (x+c) \le \sqrt{(x+c)^2 - \beta(x+c/\beta)^2}$. The LHS is $2p_1 - \Big(\frac{1}{n} + \frac{n-1}{n}p_1 + c\Big) = \frac{n+1}{n}p_1 - \frac{1+nc}{n}$, which is non-negative for $p_1 \ge (1+nc)/(n+1)$, implying that (16) is equivalent to

$$\left[p_1^2 - (x+c)p_1 + \beta(x+c/\beta)^2/4\right]\Big|_{x=\frac{1}{n}+\frac{n-1}{n}p_1} \le 0 \tag{17}$$

with the coefficient in front of p_1^2 equal to $1 - \frac{n-1}{n} + \beta\left(\frac{n-1}{n}\right)^2/4 > 0$. Moreover, for $p_1 = (1+nc)/(n+1)$, conditions (16) and (17) hold trivially. Therefore, $p_1 = (1+nc)/(n+1)$ is between the roots of (5) with $x = \frac{1}{n} + \frac{n-1}{n}p_1$.

Observe also that the LHS of range (15) is

$$\frac{2-\rho\beta+\rho cn}{2+(n-1)\rho\beta} = \frac{2-\rho\beta+n\rho\beta-n\rho\beta+n\rho\beta c/\beta}{2-\rho\beta+n\rho\beta} = 1 - \frac{n\rho\beta(1-c/\beta)}{2-\rho\beta+n\rho\beta} = 1 - \frac{n(1-c/\beta)}{2/\rho\beta - 1 + n},$$

which is decreasing in both ρ and β implying that $\frac{2-\rho\beta+\rho cn}{2+(n-1)\rho\beta} > \frac{1+nc}{n+1}$ for any feasible ρ and β. Therefore, $\frac{2-\rho\beta+\rho cn}{2+(n-1)\rho\beta}$ is greater than the smaller root of (5) with $x = \frac{1}{n} + \frac{n-1}{n}p_1$. Hence, the condition of non-emptiness of range (15) follows from inequality (17) with $p_1 = \frac{2-\rho\beta+\rho cn}{2+(n-1)\rho\beta}$. The resulting condition takes the form of a quadratic inequality in c

$$\Big(\frac{2-\rho\beta+\rho cn}{2+(n-1)\rho\beta}\Big)^2 - \Big(\frac{1}{n} + \frac{n-1}{n}\frac{2-\rho\beta+\rho cn}{2+(n-1)\rho\beta} + c\Big)\frac{2-\rho\beta+\rho cn}{2+(n-1)\rho\beta} + \frac{\beta}{4}\Big(\frac{1}{n} + \frac{n-1}{n}\frac{2-\rho\beta+\rho cn}{2+(n-1)\rho\beta} + \frac{c}{\beta}\Big)^2 \le 0$$

with the coefficient in front of c^2 equal to $\frac{\rho^2\beta n(1-\beta)+(1-\rho\beta)^2}{\beta[2+(n-1)\rho\beta]^2} > 0$ and the roots $\{\beta \cdot CB_2, \beta\}$, where $CB_2 = \frac{1-2\rho+\rho^2\beta}{(1-\rho\beta)^2+n\rho^2\beta(1-\beta)} \le 1$ implying that range (15) is not empty if and only if c is between these roots. The denominator of CB_2 is always positive. Therefore, if $1-2\rho+\rho^2\beta \le 0$, we have $CB_2 \le 0$ and range (15) is not empty for any feasible $c : 0 < c < \beta$.

If $1-2\rho+\rho^2\beta > 0$, range (15) is not empty for any $\frac{c}{\beta}$ such that $CB_2 \le \frac{c}{\beta} \le 1$. The LHS of this condition is complementary to (14), which means that when it does not hold, range (15) is empty and PM2 exists if $p_1 \le P_{21}$ (part (2.1) of the Theorem); and when it holds, PM2 exists if (16) is satisfied.

It remains to specify condition (16) by expressing the larger root of (5) with $x = \frac{1}{n} + \frac{n-1}{n}p_1$. After the substitution and collection of terms, (5) becomes

$$\left[\frac{1}{n} + \frac{\beta}{4}\left(\frac{n-1}{n}\right)^2\right]p_1^2 + \left[\frac{1}{2n}\left(\beta\frac{n-1}{n} + c(n-1)\right) - \frac{1}{n} - c\right]p_1 + \frac{\beta}{4}\left(\frac{1}{n} + \frac{c}{\beta}\right)^2 = 0,$$

which, multiplied by $4n^2$, is $\left[4n + \beta(n-1)^2\right]p_1^2 + 2\left[\beta(n-1) - cn(n+1) - 2n\right]p_1 + \beta\left(1 + nc/\beta\right)^2 = 0$. The larger root is

$$\frac{2[cn(n+1) + 2n - \beta(n-1)] + \sqrt{D}}{2[4n + \beta(n-1)^2]},$$

where $D = 4\Big\{\left[\beta(n-1) - cn(n+1) - 2n\right]^2 - \left[4n + \beta(n-1)^2\right]\beta\left(1 + nc/\beta\right)^2\Big\}$, where the squared bracket $[\cdot]^2$ is $\beta^2(n-1)^2 + c^2n^2(n+1)^2 + 4n^2 - 2\beta(n-1)cn(n+1) - 4\beta(n-1)n + 4cn^2(n+1)$, and the second term in the bracket $\{\cdot\}$ is

$$\begin{aligned}&-\left[4n + \beta(n-1)^2\right]\beta\left(1 + 2nc/\beta + n^2c^2/\beta^2\right)\\ &\qquad = -\left[4n\beta + 8n^2c + 4n^3c^2/\beta + \beta^2(n-1)^2 + 2\beta nc(n-1)^2 + (n-1)^2n^2c^2\right].\end{aligned}$$

After simplifications,

$$\begin{aligned} D &= 4\left\{4n^3c^2+4n^2-2\beta cn(n^2-1+n^2-2n+1)-4\beta n^2+4cn^2(n+1)-8n^2c-4n^3c^2/\beta\right\} \\ &= (4n)^2\left\{nc^2+1-\beta c(n-1)-\beta+c(n+1)-2c-nc^2/\beta\right\} \\ &= (4n)^2\left\{nc[c-\beta+1-c/\beta]+1-\beta+\beta c-c\right\}=(4n)^2(1-\beta)\left\{nc\,(1-c/\beta)+1-c\right\} \end{aligned}$$

and the expression for the larger root is

$$P_{22}=\frac{n(n+1)c+2n-\beta(n-1)+2n\sqrt{(1-\beta)\left[nc\Big(1-c/\beta\Big)+1-c\right]}}{4n+\beta(n-1)^2}. \tag{18}$$

The fact $P_{11},P_{12},P_{22}\to 1$ as $\frac{c}{\beta}\to 1$ can be shown by direct substitution of $\frac{c}{\beta}=1$ into the formulas for P_{11},P_{12}, and P_{22}.

A.2. **Proof of Corollary 2 (profit PM2 exceeds PM1).** *Part (1).* When $\frac{c}{\beta}<CB_1$, equilibria PM1 or PM2 exist (Theorem 1) if, respectively, $p_1\geq P_{11}$ or $p_1\leq P_{21}$. Therefore, PM-equilibria do not exist for $P_{21}<p_1<P_{11}$ if $P_{11}>P_{21}$. By the definition of P_{11} and P_{21}, inequality $P_{11}\geq P_{21}$, multiplied by $n+1$ is

$$\begin{aligned} n+1-n+1-\rho\beta+(n-1+\rho\beta)\frac{c}{\beta} &\geq \frac{c}{\beta}\frac{(1-\rho\beta)^2(n+1)}{1-2\rho+\beta\rho^2}\Leftrightarrow \\ 2-\rho\beta &\geq \frac{c}{\beta}\frac{(1-\rho\beta)^2(n+1)-(n-1+\rho\beta)(1-2\rho+\beta\rho^2)}{1-2\rho+\beta\rho^2}. \end{aligned}$$

Since $1-2\rho+\beta\rho^2=(1-\rho\beta)^2-\rho(1-\beta)(2-\rho\beta)$, the numerator of the second fraction can be written as $(1-\rho\beta)^2(2-\rho\beta)+(n-1+\rho\beta)\rho(1-\beta)(2-\rho\beta)$. Then, after dividing both sides by $2-\rho\beta$ and expressing $\frac{c}{\beta}$, the inequality becomes $\frac{c}{\beta}\leq\frac{1-2\rho+\rho^2\beta}{(1-\rho\beta)^2+(1-\beta)\rho[n-(1-\rho\beta)]}=CB_1$. Hence, $P_{11}\geq P_{21}$ is equivalent to $\frac{c}{\beta}\leq CB_1$. By the proof of part PM1 of Theorem 1, we also know that $P_{11}=P_{12}$ if $\frac{c}{\beta}=CB_1$.

Part (2). By Theorem 1, both P_{12} and P_{22} are greater than $\frac{c}{\beta}$ if $n<\infty$, and, by Lemma 15, both are the larger roots of (5) at different (for $n>1$) x, namely $x_{12}=1-\frac{n-1}{n+1}(1-c/\beta)$ and $x_{22}=\frac{1}{n}+\frac{n-1}{n}p_1$. For $n=1$, $x_{12}=x_{22}=1$, and the expression for $P_{12}=P_{22}=P_2$ results from direct substitution. For $n>1$, by Lemma 14, $x_{12}>\frac{c}{\beta}$ and $x_{22}>\frac{c}{\beta}$, and inequality $P_{12}<P_{22}$ follows from $x_{12}<x_{22}$ since the larger root of (5) increases in x. Inequality $x_{12}<x_{22}$ is

$$\frac{2}{n+1}+\frac{n-1}{n+1}\frac{c}{\beta}<\frac{1}{n}+\frac{n-1}{n}p_1\Leftrightarrow p_1>\frac{n}{n-1}\left[\frac{2}{n+1}-\frac{1}{n}+\frac{n-1}{n+1}\frac{c}{\beta}\right]=\frac{\beta+nc}{(n+1)\beta}.$$

This inequality holds for any p_1, corresponding to PM1 (including the overlap with PM2) if $\frac{\beta+nc}{(n+1)\beta}<P_{11}$, i.e., $\frac{\beta+nc}{(n+1)\beta}<1-\frac{n-1+\rho\beta}{n+1}\left(1-\frac{c}{\beta}\right)\Leftrightarrow 1+n\frac{c}{\beta}<n+1-n+1-\rho\beta+(n-1+\rho\beta)\frac{c}{\beta}$, which is equivalent to $(1-\rho\beta)c/\beta<1-\rho\beta$ and always true.

Assume, for $n\geq 1$, that $P_{12}\leq p_1\leq P_{22}$, which determines the overlap of PM1 and PM2 only if $\frac{c}{\beta}\geq CB_2$, i.e., $CB_2<1$ must hold ($\beta<1,\rho>0$). Inequality $r^{*,PM1}\leq r^{*,PM2}$ is equivalent to

$$p_1^2-(1+c)p_1+c+\frac{n(\beta-c)^2}{(n+1)^2\beta}\leq 0. \tag{19}$$

It can be shown that the LHS of (19) equals the LHS of (5) at $x=1$ for $n=1$ and strictly less for $n>1$. This property implies, first, by Lemma 14, that for any $\beta<1$ there are two distinct roots of the equation corresponding to (19), and, second, that, for $n>1$, the larger root is greater, and the smaller root is less than the corresponding roots of (5) at $x = 1$ (for $n = 1$ the equations coincide). Indeed, the free coefficient in the LHS of (19) can be written as

$$c + \frac{n(\beta-c)^2}{(n+1)^2\beta} - \frac{\beta}{4}\Big(1+\frac{c}{\beta}\Big)^2 + \frac{\beta}{4}\Big(1+\frac{c}{\beta}\Big)^2 = \frac{\beta}{4}\Big(1+\frac{c}{\beta}\Big)^2 + \frac{1}{4\beta}\left[\frac{4n(\beta-c)^2}{(n+1)^2} - (\beta+c)^2 + 4\beta c\right],$$

where the bracket $[\cdot] = \frac{4n(\beta-c)^2}{(n+1)^2} - (\beta-c)^2$ equals zero for $n = 1$ and decreases in n.

It can be shown also that range $[P_{12}, P_{22}]$ is strictly (for $n > 1$) between the roots of the equation, corresponding to (19). First, by Lemma 14, P_{22} is not greater than the greater root of (5) at $x = 1$ since $\frac{c}{\beta} < x_{22} \le 1$. For $n = 1$, P_{22} equals this root and equals P_{12}, implying that (19) holds as equality yielding $r^{*,PM1} = r^{*,PM2}$ if $p_1 = P_{12} = P_{22} = P_2$. Second, for $n > 1$, P_{12} is always greater than the smaller root of the equation, corresponding to (19), which follows from the chain of inequalities: first, by Lemma 14, $P_{12} = (p_1)_2(x)|_{x=x_{12}} \ge \frac{1}{2}(x + c/\beta)\big|_{x=x_{12}}$ and, second, $\frac{1}{2}(x + c/\beta)\big|_{x=x_{12}}$ is greater than the smaller root of the equation, corresponding to (19). The last inequality holds if the LHS of (19) becomes negative after the substitution of $\frac{1}{2}(x + c/\beta)\big|_{x=x_{12}} = \frac{1}{n+1} + \frac{n}{n+1}\frac{c}{\beta}$ for p_1. Indeed, this substitution yields

$$\left(\frac{1}{n+1} + \frac{n}{n+1}\frac{c}{\beta}\right)^2 - (1+c)\left(\frac{1}{n+1} + \frac{n}{n+1}\frac{c}{\beta}\right) + c + \frac{n(\beta-c)^2}{(n+1)^2\beta},$$

which, multiplied by $(n+1)^2\beta^2$ becomes $(\beta+nc)^2 - (1+c)(n+1)\left(\beta^2 + nc\beta\right) + c(n+1)^2\beta^2 + n\beta(\beta-c)^2$ $= n\{n[c^2 - (1+c)c\beta + c\beta^2] + c\beta - \beta^2 + \beta^3 - c\beta^2\}$, where $\{\cdot\}$ is $n[c^2(1-\beta) - c\beta(1-\beta)] + \beta(c-\beta) - \beta^2(c-\beta) = nc(1-\beta)(c-\beta) + \beta(c-\beta)(1-\beta) < 0$ for any $n > 1$. Hence, $p_1 \in [P_{12}, P_{22}]$ results in satisfaction of (19) as a strict inequality and $r^{*,PM1} < r^{*,PM2}$.

Part (3). This part is relevant only for $n > 1, \beta < 1$, and $0 < \rho < (1 - \sqrt{1-\beta})/\beta$ leading to $0 < CB_1 < CB_2$. By part (1), $P_{12} = P_{21}$ if $\frac{c}{\beta} = CB_1 > 0$. Then $P_{12} < P_{21}$ for $\frac{c}{\beta} > CB_1$ if $\frac{\partial P_{12}}{\partial(c/\beta)} < \frac{\partial P_{21}}{\partial(c/\beta)}$ for all $\frac{c}{\beta} > CB_1$. Denoting $x = 1 - \frac{n-1}{n+1}(1 - c/\beta)$, the derivatives $\frac{\partial P_{12}}{\partial(c/\beta)}$ and $\frac{\partial^2 P_{12}}{[\partial(c/\beta)]^2}$ are

$$\begin{aligned}
\frac{\partial P_{12}}{\partial(c/\beta)} &= \frac{1}{2}\left\{\frac{n-1}{n+1} + \beta + (1-\beta)\left(x\frac{n-1}{n+1} - \beta\frac{c}{\beta}\right)/\sqrt{(1-\beta)\left[x^2 - c^2/\beta\right]}\right\}, \\
\frac{\partial^2 P_{12}}{[\partial(c/\beta)]^2} &= \frac{1}{2}\left\{\left(\frac{n-1}{n+1}\right)^2 - \beta - \left(x\frac{n-1}{n+1} - c\right)^2/\left[x^2 - c^2/\beta\right]\right\}\sqrt{(1-\beta)/\left[x^2 - c^2/\beta\right]}.
\end{aligned}$$

Since P_{12} is a branch of a second-order curve, and such a branch is either convex or concave in its entire domain, the concavity of $P_{12}(c/\beta)$ can be shown at $c/\beta = 0$, where $x|_{c/\beta=0} = \frac{2}{n+1}$. Namely, $\frac{\partial^2 P_{12}}{[\partial(c/\beta)]^2}\Big|_{c/\beta=0} = -\frac{\beta}{4}(n+1)\sqrt{1-\beta} \le 0$. Since $P_{12}(c/\beta)$ is concave, $P_{12}|_{c/\beta=0} = \frac{1+\sqrt{1-\beta}}{n+1} > P_{21}|_{c/\beta=0} = 0$, and $P_{12} = P_{21}$ at $c/\beta = CB_1$, we have $\frac{\partial P_{12}}{\partial(c/\beta)} < \frac{\partial P_{21}}{\partial(c/\beta)}$ at $c/\beta = CB_1$. The last inequality combined with the concavity of $P_{12}(c/\beta)$ implies that $\frac{\partial P_{12}}{\partial(c/\beta)} < \frac{\partial P_{21}}{\partial(c/\beta)}$ for all $c/\beta > CB_1$.

Inequality $r^{*,PM1} < r^{*,PM2}$ for $P_{12} \le p_1 \le P_{21}$ follows from part (2) since $P_{21} < P_{22}$ for $c/\beta < CB_2$.

Part (4). $c/\beta \ge CB_1$ is $1 - 2\rho + \rho^2\beta \le [1 - 2\rho\beta + \rho(1-\beta)(n-1) + \rho^2\beta]c/\beta$ or $\rho^2(\beta - c) - \rho b + 1 - c/\beta \le 0$, where $b \triangleq 2 + [n(1-\beta) - 1]c/\beta - c$. The discriminant of the corresponding equation is $D = b^2 - 4(\beta-c)^2/\beta$, which is non-negative since $\frac{\partial D}{\partial\beta} = 2b(-nc+c)/\beta^2 - 4[2(\beta-c) - (\beta-c)^2]/\beta^2 < 0$ and $D|_{\beta=1} = 0$. The larger root is irrelevant since $\frac{b}{2(\beta-c)}\Big|_{n=1} \ge 1$ and b increases in n. Therefore, $c/\beta \ge CB_1 \Leftrightarrow \rho \ge \bar\rho$, where $\bar\rho \triangleq \frac{1}{2(\beta-c)}\left[b - \sqrt{b^2 - 4(\beta-c)^2/\beta}\right]$ is the smaller root, which goes to zero with $n \to \infty$.

Part (5). The monotonicity of P_{11} and P_{21} in c, ρ, β, and n follows directly from the definitions of these bounds given in Theorem 1. In particular,

$$\frac{\partial P_{21}}{\partial \rho} = \frac{c}{\beta(1-2\rho+\beta\rho^2)^2}\left[-2\beta(1-\rho\beta)(1-2\rho+\beta\rho^2)-(1-\rho\beta)^2(-2+2\rho\beta)\right],$$

where $[\cdot] = 2(1-\rho\beta)\{(1-\rho\beta)^2-(\beta-2\rho\beta+\rho^2\beta^2)\}$, where $\{\cdot\} = 1-\beta$, leading to $\frac{\partial P_{21}}{\partial \rho} = \frac{c}{\beta}\frac{2(1-\rho\beta)(1-\beta)}{(1-2\rho+\beta\rho^2)^2} \geq 0$.

Both P_{12} and P_{22} are decreasing in n since they are the larger roots of (5), which, by Lemma 14, are increasing in x for any $x > \frac{c}{\beta}$, and both $x_{12} = 1-\frac{n-1}{n+1}(1-c/\beta)$ and $x_{22} = \frac{1}{n}+\frac{n-1}{n}p_1$ are greater than c/β and decreasing in n.

$$\frac{\partial P_{12}}{\partial c} = \frac{1}{2}\left[\frac{\partial x}{\partial c}+1+\frac{1}{2}\sqrt{1-\beta}\cdot 2\left(x\frac{\partial x}{\partial c}-\frac{c}{\beta}\right)/\sqrt{x^2-c^2/\beta}\right],$$

where $\frac{\partial x}{\partial c} = \frac{n-1}{\beta(n+1)}$ is increasing in n, and the last fraction in $[\cdot]$ is also increasing in n either for any c and $n \leq 3$ or for $n > 3$ and $c \geq c^0 = \frac{1}{2}-\frac{1}{n-1}$. This monotonicity follows from the expression for $x\frac{\partial x}{\partial c} = \frac{(n-1)[2+c(n-1)]}{\beta(n+1)^2}$ and the derivative

$$\begin{aligned}\frac{\partial}{\partial n}\left(x\frac{\partial x}{\partial c}\right) &= \frac{2(n+1)}{\beta(n+1)^4}\left(\left[1+c(n-1)\right](n+1)-(n-1)[2+c(n-1)]\right)\\ &= \frac{2}{\beta(n+1)^3}(2-(n-1)+2c(n-1)) = \frac{2[2+(n-1)(2c-1)]}{\beta(n+1)^3},\end{aligned}$$

where the last bracket $[\cdot] \geq 0$ under the above conditions on c and n. Then, denoting $d(x) \triangleq x^2-c^2/\beta$,

$$\frac{\partial}{\partial n}\left(x\frac{\partial x}{\partial c}/\sqrt{d(x)}\right) = \frac{1}{d(x)}\left\{\frac{2[2+(n-1)(2c-1)]}{\beta(n+1)^3}\sqrt{d(x)}-x\frac{\partial x}{\partial n}\frac{(n-1)[2+c(n-1)]}{\beta(n+1)^2}/\sqrt{d(x)}\right\},$$

where $\frac{\partial x}{\partial n} = -\frac{2(1-c/\beta)}{(n+1)^2} < 0$. Consider $n > 3$ and $c < c^0$. Then $\{\cdot\}$, multiplied by $\frac{\beta(n+1)^3}{2}\sqrt{d(x)} > 0$, becomes $[2+(n-1)(2c-1)]d(x)+[2+c(n-1)]\frac{n-1}{n+1}(1-c/\beta)x$, where $\frac{n-1}{n+1}(1-c/\beta) = 1-x$. Collecting the terms with x, we have $x\{x[(n-1)(2c-1)-c(n-1)]+2+c(n-1)\}-c^2[2+(n-1)(2c-1)]/\beta$, where the last term is positive for $n > 3$ and $c < c^0$, and the bracket $\{\cdot\}$ in the first term is $\{\cdot\} = (n-1)[x(c-1)+c]+2$ is minimized at $c = 0$. Namely, $\{\cdot\}|_{c=0} = (n-1)\left[-1+\frac{n-1}{n+1}\right]+2 = 2\left[1-\frac{n-1}{n+1}\right] > 0$. Hence, since $\frac{\partial P_{12}}{\partial c}$ is increasing in n, it remains to show that it is positive at $n = 1$. Indeed, $\frac{\partial P_{12}}{\partial c}|_{n=1} = \frac{1}{2}\left(1-\frac{c}{\beta}\sqrt{\frac{1-\beta}{1-c^2/\beta}}\right)$, and since $\frac{c^2}{\beta} < \beta$, leading to $\sqrt{\frac{1-\beta}{1-c^2/\beta}} < 1$, we obtain $\frac{\partial P_{12}}{\partial c}|_{n=1} > \frac{1}{2}(1-c/\beta) > 0$.

Using (18) for P_{22}, we have

$$\frac{\partial P_{22}}{\partial c} = \frac{1}{4n+\beta(n-1)^2}\left[n(n+1)+\frac{n(1-\beta)\left(n\left(1-2c/\beta\right)-1\right)}{\sqrt{(1-\beta)\left(nc\left(1-c/\beta\right)+1-c\right)}}\right],$$

where $[\cdot] = n(n+1)+n\left(n\left(1-2c/\beta\right)-1\right)\sqrt{\frac{1-\beta}{nc(1-c/\beta)+1-c}}$ and $\sqrt{\frac{1-\beta}{nc(1-c/\beta)+1-c}} < 1$ since the LHS is decreasing in n and less than one for $n = 1$. Then $\frac{\partial P_{22}}{\partial c} > 0$ if $n(n+1)+n\left(n\left(1-2c/\beta\right)-1\right) > 0$. The last inequality is equivalent to $n^2\left(1+1-2c/\beta\right) > 0$, which always holds.

Consider $\beta < 1$ since $CB_1|_{\beta=1} = CB_2|_{\beta=1} = 1$ and both P_{12} and P_{22} are irrelevant — PM-equilibria are determined either by condition (1.1) or (2.1) of Theorem 1. Since P_{12} and P_{22} are the larger roots of (5), $\frac{\partial P_{12}}{\partial \beta}$ and $\frac{\partial P_{22}}{\partial \beta}$ can be found from the differentiation of (5):

$$2p_1\frac{\partial p_1}{\partial \beta} - (x+c)\frac{\partial p_1}{\partial \beta} - \frac{\partial x}{\partial \beta}p_1 + \frac{1}{4}\left[(x+c/\beta)^2 + 2\beta\,(x+c/\beta)\left(-c/\beta^2\right)\right] = 0,$$

which can be written as $\frac{\partial p_1}{\partial \beta}[2p_1 - (x+c)] = \frac{\partial x}{\partial \beta}p_1 - \frac{1}{4}\left(x^2 - c^2/\beta^2\right)$. The RHS is negative since both x_{12} and x_{22} are greater than $\frac{c}{\beta}$, $\frac{\partial x_{22}}{\partial \beta} = 0$, and $\frac{\partial x_{12}}{\partial \beta} = -\frac{n-1}{n+1}c/\beta^2 \leq 0$. The bracket $[\cdot]$ in the LHS is positive since, by Lemma 14, $2p_1 > x + \frac{c}{\beta} > x + c$ for $\beta < 1$. Therefore, $\frac{\partial P_{22}}{\partial \beta} = -\frac{1}{4}\frac{\left(x_{22}^2 - c^2/\beta^2\right)}{2P_{22}-(x_{22}+c)} < 0$ and $\frac{\partial P_{12}}{\partial \beta} = -\left[\frac{n-1}{n+1}cP_{12}/\beta^2 + \frac{1}{4}\left(x_{12}^2 - c^2/\beta^2\right)\right] / \ [2P_{12} - (x_{12}+c)] < 0$.

A.3. **Proof of Proposition 4 (p_1-bounds of PM and NA).** *Part (1)* follows from $\max_n P_2^N = \frac{c}{\beta} \leq \min\{P_{21}, P_{22}\}$. The inequality is shown in Theorem 1.

Part (2). $P_1^N \geq P_{11}$ is $1 - \frac{n}{n+1}\rho(\beta - c) \geq 1 - \frac{n-1+\rho\beta}{n+1}(1 - c/\beta) \Leftrightarrow n\rho\beta \leq n - 1 + \rho\beta \Leftrightarrow 1 - \rho\beta \leq n(1-\rho\beta)$, which is strict for $n > 1$. When $n = 1$, we have $P_1^N \equiv P_{11}$, and, for $\frac{c}{\beta} > CB_1$, the p_1-boundary of PM1 dominates P_1^N since $P_{12} > P_{11}$. For $n > 1$, $P_1^N \geq P_{12}$ for any $\rho \in [0,1)$ if $\inf_\rho P_1^N \geq P_{12}$, which is $1 - \frac{n}{n+1}(\beta - c) \geq \frac{1}{2}\left[x + c + \sqrt{(1-\beta)(x^2 - c^2/\beta)}\right]$. Substitution for $x = 1 - \frac{n-1}{n+1}(1 - c/\beta)$ leads to $\sqrt{\cdot} - 1 + c - \frac{(\beta-c)[n(1-2\beta)-1]}{\beta(n+1)} \leq 0$, where $\sqrt{\cdot}$ and the last term are decreasing in n. Indeed, considering n as a continuous variable, the derivative of the last term w.r.t. n is $\frac{-1}{\beta(n+1)^2}\{(\beta-c)(1-2\beta)\beta(n+1) - \beta(\beta-c)[n(1-2\beta)-1]\} = \frac{-(\beta-c)}{(n+1)^2}\{(1-2\beta)(n+1) - n(1-2\beta) + 1\} = \frac{-2(\beta-c)}{(n+1)^2}(1-\beta) \leq 0$. Therefore, inequality $\inf_\rho P_1^N \geq P_{12}$ holds if it holds for $n = 2$, which is

$$\sqrt{(1-\beta)(x^2 - c^2/\beta)} \leq 1 - c + (\beta - c)(1-4\beta)/(3\beta), \tag{20}$$

where $x|_{n=2} = (2\beta + c)/(3\beta)$ and $\left(x^2 - c^2/\beta\right)|_{n=2} = [(2\beta+c)^2 - 9\beta c^2]/(3\beta)^2$. Then (20), multiplied by 3β, can be written as $\sqrt{(1-\beta)[\cdot]} \leq 4\beta + \beta c - 4\beta^2 - c$ or $\sqrt{(1-\beta)[\cdot]} \leq (1-\beta)(4\beta - c)$, which holds as equality for $\beta = 1$. Consider $\beta < 1$. Then (20), squared, is $(2\beta+c)^2 - 9\beta c^2 \leq (1-\beta)(4\beta-c)^2$, which, divided by β, can be written as $(4\beta - c)^2 - 12\beta + 12c - 9c^2 \leq 0$ or $12\beta^2 - 12c^2 + 4(\beta-c)^2 - 12(\beta - c) \leq 0 \Leftrightarrow (\beta-c)(\beta+c-3) + (\beta-c)^2 \Leftrightarrow 2\beta - 3 \leq 0$, which holds strictly.

A.4. **Proof of Proposition 6 (NA4-profit less than NA3).** *Part (1).* NA4 exists only if $p_2^* = s$ or $Y^* > 1 - \frac{s}{\beta}$, which can be written as $c - s < \frac{n-1}{n}\frac{\beta(p_1-s)}{\beta-s}(1-v^*)$. The RHS is maximal at $\rho = 0$ ($v^* = p_1$ is minimal) and $p_1 = \frac{1+s}{2}$ yielding $c - s < \frac{n-1}{n}\frac{\beta(1-s)^2}{4(\beta-s)}$. NA4 can also exist (profit is positive) only if there are first-period sales, i.e., $v^{*,NA4} < 1 \Leftrightarrow p_1 - \rho s < 1 - \rho\beta \Leftrightarrow p_1 < 1 - \rho(\beta - s) = P_4^N$, which is less than $P_1^N = 1 - \frac{n}{n+1}\rho(\beta - c)$. These bounds can be written as $\rho < \frac{1-p_1}{\beta-s} = \rho_4^N$ and $\rho < \frac{n+1}{n}\frac{1-p_1}{\beta-c} = \rho_1^N$. respectively.

Part (2) follows from the observations: (a) the second-period sales under NA4 are always at loss with $p_2^{*,NA4} = s < p_2^{*,NA3}$ while $Y^{*,NA4} > Y^{*,NA3}$; and (b) since, by (2) and rationality of expectations in equilibrium, v^* is nonincreasing in p_2^*, we have $v^{*,NA4} \geq v^{*,NA3}$ implying that the first-period profit is less under NA4. A similar argument leads to $r^{*,NA4} < \frac{1}{n}(p_1 - c)(1-p_1)$ for any inputs where NA4 exists. The RHS of this inequality is the first-period profit in NA4 for $\rho = 0$, coinciding with the expression for profit under NA2 or PM2. Since $v^{*,NA4}$ is increasing in ρ, the first-period profit in NA4 is decreasing ρ. Consequently, because the second-period sales are at loss, the equilibrium profit is strictly less than the RHS.

A.5. **Proposition 16 (RESE N).** The conditions of N-equilibria existence require the comparison of retailer profit in various options, which leads to quadratic inequalities in $z \triangleq 1 - \frac{n-1}{n}Y_0^* - c/\beta$. When z is non-negative, one can interpret it as the level of inventory of a retailer deviating from a symmetric equilibrium that makes a clearance price equal to the unit cost. Thus, if $z \le 0$, the second-period price, by (3), cannot be above cost regardless of a one-retailer deviation from a symmetric strategy profile. Several thresholds on z result from quadratic inequalities and, below, we denote $z_1 \triangleq \frac{2}{\beta}\left[p_1 - c - \sqrt{(p_1-c)p_1(1-\beta)}\right]$, and $\tilde{z}_1 \triangleq$
$$\begin{cases} z_1, & \text{if } n=1, \text{ or } \rho=0, \text{ or } \beta=1, \text{or } p_1 = c/\beta, \\ \frac{(p_1-c)(2-\rho\beta)}{\beta(1-\rho\beta)}\left[1-\sqrt{1-\frac{4(\beta p_1 - c)(1-\rho\beta)}{(p_1-c)(2-\rho\beta)^2}}\right] & \text{otherwise,} \end{cases}$$
as the smaller roots of the corresponding equations, and $\hat{z}_2 \triangleq \hat{z}_2(\beta, c, n, s)$ — as the larger one.

Proposition 16. *If PM is available, N-equilibria with the following structure exist if and only if the respective conditions apply. The set of necessary and sufficient conditions is given in each case by the combination of the conditions in the corresponding NA-equilibrium and the additional conditions listed below.*

N1: *requires no additional conditions for $n > 1$ and, for $n = 1$, the condition $p_1 \ge P_2$; $\alpha^*(1) = 1$.*

N2: *requires no additional conditions and $\alpha^*(1) = 0$.*

N3: *under the following additional conditions, where Y^* is the larger root of the quadratic equation*

$$Y^2 - \frac{(\beta-c)n(1-\rho\beta) + \beta(1-p_1)n - (p_1-\beta)\rho\beta(n-1)}{\beta(n+1-\rho\beta)}Y - \frac{(p_1-\beta)(1-p_1)(n-1)}{\beta(n+1-\rho\beta)} = 0. \quad (21)$$

and $r^{*,N3}$ is defined by part NA3 of Theorem 3:

(3.1) *inequality $r^{*,N3} \ge \breve{r}_1^i = (p_1 - c)\left(1 - p_1 - \frac{n-1}{n}Y^*\right)$ holds and either $p_1 \le \frac{c}{\beta}$ and $\frac{n-1}{n}Y^* \le 1 - p_1$, or $p_1 > \frac{c}{\beta}$ and $\frac{n-1}{n}Y^* \le 1 - \frac{c}{\beta} - z_1$ with the corresponding rational expectations $\alpha^*(1) = 0$; or*

(3.2) *either $p_1 \le \frac{c}{\beta}$ and $\frac{n-1}{n}Y^* > 1 - p_1$, or $p_1 > \frac{c}{\beta}$ and either $\frac{n-1}{n}Y^* \ge 1 - \frac{c}{\beta}$, or $1 - \frac{c}{\beta} - \min\left\{\tilde{z}_1, 2\sqrt{r^{*,N3}/\beta}\right\} \le \frac{n-1}{n}Y^* < 1 - \frac{c}{\beta}$ with the corresponding rational expectations $\alpha^*(1) = 1$.*

N4: *under the following additional conditions, where $Y^* = \frac{n-1}{n}\frac{p_1-s}{c-s}\left(1 - \frac{p_1 - \rho s}{1-\rho\beta}\right)$:*

(4.1) *inequality $(1-p_1)\left[1 + \frac{c-s}{(n-1)^2(p_1-c)}\right]^{-1} \le \frac{n-1}{n}Y^*$ ($r^{*,N4} \ge \breve{r}_1^i$) holds and either $p_1 \le \frac{c}{\beta}$ and $\frac{n-1}{n}Y^* \le 1 - p_1$, or $p_1 > \frac{c}{\beta}$ and $\frac{n-1}{n}Y^* \le 1 - \frac{c}{\beta} - z_1$ with the corresponding rational expectations $\alpha^*(1) = 0$; or*

(4.2) *either $p_1 \le \frac{c}{\beta}$ and $\frac{n-1}{n}Y^* > 1 - p_1$, or $p_1 > \frac{c}{\beta}$ and either $\frac{n-1}{n}Y^* \ge 1 - \frac{c}{\beta}$, or $1 - \frac{c}{\beta} - \min\{\hat{z}_2, \tilde{z}_1\} \le \frac{n-1}{n}Y^* < 1 - \frac{c}{\beta}$ with the corresponding rational expectations $\alpha^*(1) = 1$.*

Equation (21) is derived in the proof of Theorem 3. This proof can be found in Bazhanov et al. (2015).

Proof of Proposition 16 is based on the following lemma, which uses the notations $z_0 \triangleq 2\left(p_1 - c/\beta\right)$.

Lemma 17. *Consider retailer i using PM and a profile of competitor strategies not using PM with combined inventory $Y_0 \ge 0$. There exist optimal inventory of retailer i and the corresponding rational expectations of customers in one of the forms given below. No other forms can exist.*

(a) *$\breve{y}_1^i = 1 - p_1 - Y_0 = z - \frac{z_0}{2}$ (no second-period sales) with positive profit $\breve{r}_1^i = \breve{y}_1^i(p_1 - c)$ and rational expectations $\bar{\alpha} = 0$ if and only if either of the two conditions holds: (a.1) $p_1 \le \frac{c}{\beta}$ and $z > \frac{z_0}{2}$ ($Y_0 < 1 - p_1 \Rightarrow p_1 < 1$), or (a.2) $p_1 > \frac{c}{\beta}$ and $z \ge z_1$ ($\breve{r}_1^i|_{\bar{\alpha}=0} \ge \tilde{r}_1^i$);*

(b) $\tilde{y}_1^i = \frac{z}{2}$ *(with second-period sales) with positive profit* $\tilde{r}_1^i = \beta(\tilde{y}_1^i)^2$ *and rational expectations* $\bar{\alpha} = 1, \bar{p}_2 = p_2 = \beta(1 - Y_0 - \tilde{y}_1^i)$ *if and only if* $p_1 > \frac{c}{\beta}, z > 0$ *and either of the two conditions holds: (b.1)* $p_1 = 1$*, or (b.2)* $p_1 < 1$*, and* $z \le \tilde{z}_1$ $(\breve{r}_1^i|_{\bar{\alpha}=1} \le \tilde{r}_1^i)$ *for* $Y_0 > 0$ *or* $z \le z_1$ *for* $Y_0 = 0$*;*

(c) *the optimal inventory and profit are zero if and only if* $z \le 0 \wedge \frac{z_0}{2}$*. Rational expectations are* $\bar{\alpha} = 0$ *if* $z = \frac{z_0}{2}$ *and* $\bar{\alpha} = 1, \bar{p}_2 = \beta(1 - Y_0)$ *if* $z < \frac{z_0}{2}$*.*

Moreover, under the conditions of part (a.2), $\frac{z_0}{2} < z_1 \le z_0$*; part (b),* $c < p_2 < \beta p_1$*; part (b.2),* $\tilde{z}_1 \le z_0$*,* $p_1 > \frac{c}{\beta} + \frac{z}{2}$*, and* $v_0^{\min}$ *is not decreasing in* ρ*; under the conditions of both (a.2) and (b.2),* $z_1 \le \tilde{z}_1$ *with strict inequality if* $\bar{\alpha} = 1, \beta < 1, p_1 > \frac{c}{\beta}$*, and* $\rho > 0$*; the condition of part (c) never holds for* $Y_0 = 0$ *(a monopoly). The profit value* $\tilde{r}_1^i$ *and the corresponding* $\tilde{y}_1^i$ *do not depend on the specific values of rational expectations and are identical in parts (a) and (b). The general expression for the optimal inventory of a retailer who limits the sales to the first period is* $\breve{y}_1^i = 1 - v_0^{\min} - Y_0$*.*

The three parts of lemma correspond to the following mutually exclusive cases. Parts (a) and (b) describe positive optimal inventory decisions corresponding to rational customer expectations of sales, respectively, only in the first period and in both periods. Part (c) describes a trivial (zero) optimal inventory decision and corresponding rational expectations. Necessary and sufficient conditions of parts (a) and (b) allow an overlap of input parameter regions when $p_1 > c/\beta$ and $z_1 \le z \le \tilde{z}_1$. In this case, a potential form of retailer i decision, (a) or (b), depends on customer expectations, i.e., both $\bar{\alpha} = 0$ and $\bar{\alpha} = 1$ can be rational.

When PM is possible, a RESE N with the corresponding expectations for a one-retailer deviations into PM, $\alpha^*(1)$, exists if and only if either of the two conditions hold: (i) both possible deviations of a retailer i into PM are trivial (part (c) of the lemma) or (ii) at least one of the deviations is not trivial (parts (a) or (b) of the lemma) and optimal deviator's profit does not exceed the equilibrium profit under the corresponding RESE NA. Since $Y_0 = \frac{n-1}{n} Y^*$ and $z = 1 - \frac{n-1}{n} Y^* - c/\beta$, case (i) is characterized by $\frac{n-1}{n} Y^* \ge (1-p_1) \vee (1 - c/\beta)$. Rational expectations under deviation are $\alpha^*(1) = 0$, when $\frac{n-1}{n} Y^* \le 1 - p_1$, and $\alpha^*(1) = 1$, otherwise.

N1. By part NA1 of Theorem 3, $r^{*,N1} = \frac{(\beta-c)^2}{(n+1)^2\beta}$ and $Y^* = \frac{n}{n+1}(1 - c/\beta)$, yielding $Y_0 = \frac{n-1}{n+1}(1 - c/\beta)$. Since $p_1 > c/\beta$, this situation is covered by case (ii). By Lemma 17, $\tilde{y}_1^i = \frac{1}{2}(1 - Y_0 - c/\beta) = \frac{z}{2} = \frac{1}{n+1}(1 - c/\beta)$, which is strictly positive (i.e., $z > 0$). The resulting total inventory would be the same as in NA1, therefore, the rational expectations under deviation into $\tilde{y}_1^i$ would lead to $v_0^{\min} = 1$ for $n > 1$. Under this scenario $\breve{y}_1^i = 1 - v_0^{\min} - Y_0 \le 0$ is infeasible. Since the optimal deviator's profit $\tilde{r}_1^i = \beta(\tilde{y}_1^i)^2 = \frac{(\beta-c)^2}{(n+1)^2\beta}$ coincides with $r^{*,N1}$, N1 with $\alpha^*(1) = 1$ exists without any additional conditions. For $n = 1$, $\breve{y}_1^i = 1 - p_1 \ge 0$ is feasible, and N1, by part (b) of Lemma 17 exists if and only if $z \le z_1$ $(Y_0 = 0)$. This inequality is $1 - c/\beta \le \frac{2}{\beta}\left[p_1 - c - \sqrt{(p_1 - c)p_1(1-\beta)}\right] \Leftrightarrow \sqrt{(p_1 - c)p_1(1-\beta)} \le p_1 - \frac{\beta+c}{2} \Leftrightarrow p_1^2 - p_1(1+c) + \frac{1}{4\beta}(\beta+c)^2 \ge 0$. The smaller root of the corresponding equation is irrelevant since $\frac{1+c}{2}$ is less than the p_1-lower bound in NA1: $\frac{1+c}{2} < 1 - \frac{\rho}{2}(\beta - c)$. This inequality holds for any $\rho < 1$ since the RHS is minimized at $\rho \to 1$ and $1 + c \le 2 - \beta + c$. Therefore, the additional condition of existence of N1 for $n = 1$ is $p_1 \ge (p_1)_2$, where $(p_1)_2$ is the larger root of the equation, corresponding to the quadratic inequality above, and $(p_1)_2 = \frac{1}{2}\left[1 + c + \sqrt{(1+c)^2 - \frac{1}{\beta}(\beta+c)^2}\right]$. The expression under the square root is $1 + 2c + c^2 - \beta - 2c - c^2/\beta = (1-\beta)\left(1 - c^2/\beta\right)$.

N1 cannot exist with $\alpha^*(1) = 0$ because the rationality of expectations would imply optimality of $\breve{y}_1^i$ (part (a) of Lemma 17). Under the conditions of the lemma, this means that $r^{*,N1} = \tilde{r}_1^i$ is dominated by $\breve{r}_1^i$.

N2. The equilibrium profit in NA2 is not dominated by a possible deviation into PM with $y_1^i = \breve{y}_1^i$ because the prospective deviator's profit coincides with the equilibrium one: $\breve{r}_1^i\big|_{v_0^{\min}=p_1} = r^{*,N2} =$

$\frac{1}{n}(p_1-c)(1-p_1)$. The equilibrium profit is also not dominated by a possible deviation into PM with $y_1^i=\tilde{y}_1^i$ because, by Lemma 17, a non-trivial $\tilde{y}_1^i$ is optimal only if $z\le\tilde{z}_1\le 2\left(p_1-c/\beta\right)$, which is incompatible with the condition $p_1\le\frac{nc}{n-1+\beta}$ of the existence of NA2. Indeed, in this case, $Y_0=\frac{n-1}{n}(1-p_1)$ implying $z=1-Y_0-c/\beta=\frac{1}{n}+\frac{n-1}{n}p_1-c/\beta$. Inequality $z\le 2\left(p_1-c/\beta\right)$ yields the following lower bound on p_1 :

$$\frac{1}{n}+\frac{n-1}{n}p_1-c/\beta\le 2\left(p_1-c/\beta\right)\Leftrightarrow\frac{n+1}{n}p_1\ge c/\beta+\frac{1}{n}\Leftrightarrow p_1\ge\frac{nc+\beta}{(n+1)\beta},$$

which exceeds the upper bound given in NA2:

$$\frac{nc+\beta}{(n+1)\beta}>\frac{nc}{n-1+\beta}\Leftrightarrow(nc+\beta)(n-1+\beta)>nc(n+1)\beta\Leftrightarrow n^2c(1-\beta)+n(\beta-c)-\beta(1-\beta)>0.$$

The last inequality always holds because the LHS is increasing in n and positive for $n=1$: $c(1-\beta)+\beta-c-\beta(1-\beta)=(\beta-c)-(\beta-c)(1-\beta)=\beta(\beta-c)>0$.

N3 and N4. By part (c) of Lemma 17, both possible deviations into PM are trivial if and only if $z\le 0\wedge\frac{z_0}{2}$. Therefore, N3 and N4 exist under this condition, part of which, $z=\frac{z_0}{2}\le 0$, corresponding to $\alpha^*(1)=0$, is included into the conditions of parts (3.1) and (4.1), and the remaining part, corresponding to $\alpha^*(1)=1$, — into the conditions of parts (3.2) and (4.2).

By part (a) of Lemma 17, a pair of optimal deviator's inventory $\breve{y}_1^i$ with the corresponding $\alpha^*(1)=0$ exists if and only if either $p_1\le c/\beta$ and $\frac{n-1}{n}Y^*<1-p_1$ ($z>\frac{z_0}{2}$), or $p_1>c/\beta$ and $z\ge z_1\Leftrightarrow\frac{n-1}{n}Y^*\le 1-c/\beta-z_1$. Inequality $z\ge z_1$ implies, by Lemma 17, $z>\frac{z_0}{2}$. In this case, N3 and 4 exist if the corresponding equilibrium profit is not dominated by the profit of potential deviator: $r^{*,N3}\ge\breve{r}_1^i=\left(1-p_1-\frac{n-1}{n}Y^*\right)(p_1-c)$ for N3 and $r^{*,N4}\ge\breve{r}_1^i$ for N4. By Theorem 5, $Y^*=\frac{n-1}{n}\frac{p_1-s}{c-s}(1-v^*)$ and $r^{*,N4}=\frac{c-s}{n(n-1)}Y^*$; therefore inequality $\breve{r}_1^i\le r^{*,N4}$ becomes

$$1-p_1-\frac{n-1}{n}Y^*\le\frac{c-s}{n(n-1)(p_1-c)}Y^*\Leftrightarrow(1-p_1)\left[1+\frac{c-s}{(n-1)^2(p_1-c)}\right]^{-1}\le\frac{n-1}{n}Y^*.\quad(22)$$

The combination of the conditions $p_1\le\frac{c}{\beta}$ ($z_0\le 0$) and $\frac{n-1}{n}Y^*=1-p_1$ ($z=\frac{z_0}{2}$), which is the part of the condition $z\le 0\wedge\frac{z_0}{2}$ that corresponds to $\alpha^*(1)=0$, can be trivially included into the conditions of parts (3.1) and (4.1) because $\breve{r}_1^i=0$ in this case while $r^{*,N3}$ and $r^{*,N4}$ are strictly positive.

When $\bar{\alpha}(1)=1$, the conditions of the existence of N3 and N4 do not include the comparison of the equilibrium profit with the profit of a deviator if both possible deviations into PM are trivial, which is covered by the remaining conditions of part (c), Lemma 17. Namely, if $z\le 0$ ($\frac{n-1}{n}Y^*\ge 1-\frac{c}{\beta}$) and $z<\frac{z_0}{2}$ ($\frac{n-1}{n}Y^*>1-p_1$). The case $p_1=\frac{c}{\beta}$ and $\frac{n-1}{n}Y^*=1-\frac{c}{\beta}$ is included into parts (3.1) and (4.1). Therefore, the remaining combination of the conditions, yielding the existence of N3 and 4 with trivial deviations into PM and $\bar{\alpha}(1)=1$, is $p_1\le c/\beta$ and $\frac{n-1}{n}Y^*>1-p_1$ or $p_1>c/\beta$ and $\frac{n-1}{n}Y^*\ge 1-c/\beta$.

Recall that $p_1<1$ under the conditions of both NA3 and NA4. Therefore, by part (b.2) of Lemma 17, the pair of positive optimal deviator's inventory $\tilde{y}_1^i=\frac{z}{2}$ with profit $\tilde{r}_1^i=\frac{\beta}{4}\left(1-c/\beta-\frac{n-1}{n}Y^*\right)^2$ and $\bar{\alpha}(1)=1$ exists if and only if $p_1>c/\beta$ and $0<z\le\tilde{z}_1$, where the last condition is equivalent to $1-c/\beta-\tilde{z}_1\le\frac{n-1}{n}Y^*<1-c/\beta$. Then N3 and 4 exist if the corresponding equilibrium profit is not dominated by the profit of the deviator: $r^{*,N3}\ge\tilde{r}_1^i=\frac{\beta}{4}z^2$, which can be written as $z\le 2\sqrt{r^{*,N3}/\beta}$, and $r^{*,N4}\ge\tilde{r}_1^i$. The last inequality is $\frac{c-s}{n(n-1)}Y^*-\frac{\beta}{4}\left(1-c/\beta-\frac{n-1}{n}Y^*\right)^2\ge 0$ or, in terms of z, $\frac{\beta}{4}z^2-\frac{c-s}{(n-1)^2}\left(1-z-c/\beta\right)\le 0\Leftrightarrow\frac{\beta}{4}z^2+\frac{c-s}{(n-1)^2}z-\frac{c-s}{(n-1)^2}\left(1-c/\beta\right)\le 0$, which is strict at $z=0$. Therefore, the condition of N4 existence in this case is $z\le\hat{z}_2$, where $\hat{z}_2$ is the larger root of the corresponding equation, namely,

$$\hat{z}_2 = \frac{2}{\beta}\left[-\frac{c-s}{(n-1)^2} + \sqrt{\frac{(c-s)^2}{(n-1)^4} + \frac{(c-s)(\beta-c)}{(n-1)^2}}\right] = \frac{2(c-s)}{\beta(n-1)^2}\left[\sqrt{1+\frac{(n-1)^2(\beta-c)}{c-s}} - 1\right].$$

The combination of the inequalities $z \le \hat{z}_2$ and $z \le \tilde{z}_1$ yields $1 - c/\beta - \tilde{z}_1 \wedge 2\sqrt{r^{*,N3}/\beta} \le \frac{n-1}{n}Y^*$.

A.5.1. *Analysis of additional conditions.* The additional conditions of N-equilibria existence non-trivially restrict the parameter regions corresponding to NA3 and NA4, i.e., the conditions may hold or not hold depending on the market situation. For example, conditions of parts (3.1) and (4.1) hold for $\rho = 0$. Indeed, the profit of a deviator from NA3 (Theorem 3) or NA4 (Theorem 5) to no-PM with sales only in the first period is $(p_1 - c)\left(1 - v^*(\emptyset, I) - \frac{n-1}{n}Y^*\right)$, which coincides with $\breve{r}_1^i$ in both parts if $\rho = 0$. Since equilibrium profits $r^{*,N3}$ and $r^{*,N4}$ are not dominated by deviator profit, inequalities $r^{*,N3} \ge \breve{r}_1^i$ and $r^{*,N4} \ge \breve{r}_1^i$ hold under the corresponding NA if $\rho = 0$.

Combination of $p_1 \le \frac{c}{\beta}$ with $\frac{n-1}{n}Y^* \le 1 - p_1$ in parts (3.1) and (4.1) hold for p_1 near the lower bound for both NA3 and NA4 (which is below $\frac{c}{\beta}$ for $n > 1$) since Y^*, by continuity, approaches $1 - p_1$. On the other hand, the following lemma illustrates that for large ρ and small n, inequality $r^{*,N3} \ge \breve{r}_1^i$ may not hold.

Lemma 18. *For $n = 1, c \to 0, \beta \to 1,$ and $\rho = \left(1 - \sqrt{1-\beta}\right)/\beta \to 1$, conditions $p_1 > \frac{c}{\beta}$ and $\frac{n-1}{n}Y^* \le 1 - \frac{c}{\beta} - z_1$ of part (3.1) of Proposition 16 hold, while $r^{*,N3} \ge \breve{r}_1^i$ does not hold.*

By parts (3.2) and (4.2), equilibria N3 and N4 exist under the combination of conditions $p_1 \le \frac{c}{\beta}$ and $\frac{n-1}{n}Y^* > 1 - p_1$. The first condition implies $p_2 \le c$, preventing deviation into PM with sales in both periods, and the second one prevents any deviations with sales only in the first period. The following lemma shows that this combination has a non-empty intersection with the input areas of NA3 and NA4.

Lemma 19. *Under NA3 or 4, there exist $\beta < 1$ and $N > 1$ such that the condition $\frac{nc}{\beta+n-1} \vee 1 - \frac{n-1}{n}Y^* < p_1 \le \frac{c}{\beta}$ may hold for any $n \ge N$.*

When condition $1 - \frac{n-1}{n}Y^* < p_1 \le \frac{c}{\beta}$ does not hold, the following lemma provides an example, where inequality $r^{*,N3} \ge \tilde{r}_1^i$ of part (3.2) is satisfied.

Lemma 20. *If $p_1 > \frac{1+c}{2}, n = 1, \rho = 0,$ and $\beta = 1$, inequalities $p_1 > \frac{c}{\beta}, 1 - \frac{c}{\beta} - \tilde{z}_1 \le \frac{n-1}{n}Y^* < 1 - \frac{c}{\beta}$, and $r^{*,N3} \ge \tilde{r}_1^i$ of part (3.2) hold.*

The opportunity to deviate into PM is stipulated by the combined inventory of other retailers, i.e., $Y^{-i} = \frac{n-1}{n}Y^*$. Namely, if Y^{-i} is such that, regardless of the inventory of retailer i, there are sales in the second period, or these sales are such that $p_2 \le c$, the corresponding non-trivial forms of deviation into PM cannot exist. Combination of these conditions for Y^{-i} with the necessary conditions of existence of some NA-equilibria yield simple sufficient conditions of existence of N-equilibria under the conditions of NA.

Lemma 21. *If PM is available, N4 exists under the conditions of NA4 if $n \ge \frac{\beta-s}{p_1\beta-s} \vee \frac{\beta-s}{c-s}$.*

A.6. **Proof of Proposition 8 (PM inventory is not greater than no-PM).** *Part (1)* follows directly from Theorem 1, and parts NA1 and NA2 of Theorem 3.

Part (2) follows from the facts that (a) under NA3, $Y^* > \frac{n}{n+1}(1 - c/\beta) \vee (1 - p_1)$ (Theorem 3), where, by Theorem 1, $\frac{n}{n+1}(1 - c/\beta)$ is the total equilibrium inventory under PM1 and $1 - p_1$ — under PM2; and (b) under N4, $p_2 = s$, which is always less than p_2 under N3. Therefore, by formula (3) for p_2, the total inventory under N4 is always greater than under N3, which, by the argument above, is always greater than under PM1 or PM2.

A.7. **Proof of Proposition 7 (profits of PM1, PM2 and N(A)3, N(A)4).** *Part (1.1).* The RHS of inequality $p_1 > 1 - \frac{n}{n+1}(\beta - c)$ is the p_1-boundary between N(A)3 and N(A)1 for $\rho \to 1$. Therefore, if this inequality holds under N(A)3 ($p_1 < 1$), there exists $\rho_1^N = \frac{n+1}{n}\frac{1-p_1}{\beta-c} \in (0,1)$ such that $p_1 = 1 - \frac{n}{n+1}\rho_1^N(\beta - c)$ and, for the same given inputs except ρ, N(A) can be realized as N(A)3 for $\rho < \rho_1^N$ and as N(A)1 for $\rho \geq \rho_1^N$. By Proposition 4 in Bazhanov et al. (2015), profit $r^{*,N3}$ is decreasing in ρ for all $p_1 \geq \beta - \frac{n}{2(n+1)}(\beta - c)$, and, by continuity of the profit without PM, $r^{*,N3} = r^{*,N1}$ at $\rho = \rho_1^N$. The last inequality is implied by $p_1 > 1 - \frac{n}{n+1}(\beta - c)$ if $1 - \frac{n}{n+1}(\beta - c) \geq \beta - \frac{n}{2(n+1)}(\beta - c)$, which is equivalent to $\frac{2(n+1)}{n}(1-\beta) \geq \beta - c$ or $c \geq 3\beta - 2\left(1 + \frac{1-\beta}{n}\right)$. Hence, since $r^{*,N1} \equiv r^{*,PM1}$, and $r^{*,PM1}$ is constant in ρ, we have, under the conditions of part (1.1), that $r^{*,PM1} < r^{*,N3}$ for any $\rho < \rho_1^N$. The lower bound on c above holds for any c and n if $\beta \leq \frac{2}{3}$ and never holds for $\beta > \frac{4+c}{5}$. Indeed, the lower bound on c yields an upper bound on n, which is less that one if $\beta > \frac{4+c}{5}$. The lower bound on p_1 can be written as a lower bound on n: $n > \frac{1-p_1}{\beta-c-(1-p_1)}$.

Part (1.2). By Theorems 1 and 5, inequality $r^{*,PM1} > r^{*,N4}$ is equivalent to

$$\frac{n^2(\beta - c)^2}{(n+1)^2\beta} > (p_1 - s)\left(1 - \frac{p_1 - \rho s}{1 - \rho\beta}\right) \Leftrightarrow \frac{n}{n+1} > \sqrt{\frac{\beta(p_1 - s)(1 - p_1 - \rho(\beta - s))}{(\beta - c)^2(1 - \rho\beta)}} = \frac{1}{w},$$

where the expression under the root is always positive. The last inequality implies that $r^{*,PM1} > r^{*,N4}$ can hold only if $w > 1$ and holds for any $n \geq 2$ if $w > \frac{3}{2}$. If $w \in (1, \frac{3}{2}]$, inequality $\frac{n}{n+1} > \frac{1}{w}$, which is equivalent to $r^{*,PM1} > r^{*,N4}$, can be written as $w > 1 + \frac{1}{n}$ or $n > \frac{1}{w-1}$. By Theorem 5, fraction $\frac{1-p_1-\rho(\beta-s)}{1-\rho\beta}$ is $1 - v^* > 0$, where v^* increases in ρ implying that w increases in ρ.

Part (2.1). By Proposition 10 in Bazhanov et al. (2015), $nr^{*,N3}$ decreases in n while $nr^{*,PM2}$ is constant. Therefore, $r^{*,PM2} \geq r^{*,N3}$ for any $n \geq 1$ and any other inputs that are in the area where PM2 overlaps with N(A)3 for $n = 1$ if, in this area, $r^{*,PM2} \geq r^{*,N3}$ for $n = 1$. Indeed, for $n = 1$, $r^{*,N3}$ coincides with the profit of the deviator to no-PM with sales in both periods, and, in the area of PM2 existence, this profit does not exceed the equilibrium profit $r^{*,PM2}$. By the proof of part PM2 of Theorem 1 for $n = 1$, inequality $r^{*,PM2} \geq r^{*,N3}$ is equivalent to $p_1 \leq \frac{c}{\beta}\frac{(1-\rho\beta)^2}{1-2\rho+\beta\rho^2} = P_{21}$, where P_{21} does not depend on n. Therefore, first, inequality $r^{*,PM2} \geq r^{*,N3}$ is strict for $n = 1$ and $p_1 < P_{21}$; second, since $nr^{*,N3}$ is decreasing in n while $nr^{*,PM2}$ is constant, $r^{*,PM2} > r^{*,N3}$ for $p_1 \leq P_{21}$ if $n > 1$; and third, the p_1-bound of the overlap P_{21}, which is relevant for $\frac{c}{\beta} < CB_2$, does not change with n. For $\frac{c}{\beta} \geq CB_2$, the p_1-upper bound for N(A)3 is P_1^N and, for PM2, — P_{22}. By Proposition 4, P_1^N is the p_1-bound of the overlap for $n = 1$ and $\frac{c}{\beta} \geq CB_2$ since $P_1^N < P_{22}$ for $n = 1$ and $\frac{c}{\beta} > CB_1 = CB_2$ ($P_1^N = P_{22}$ at $\frac{c}{\beta} = CB_2$). P_1^N is decreasing in n, resulting in shrinking of the overlap. For $n > 1$, the overlap area shrinks also due to conditions (a), (b) of part NA3 of Theorem 3 and additional conditions (3.1) and (3.2) for N3 existence (Proposition 16). For $n = 1$, all these conditions hold trivially.

The only bound that leads to the expansion of the overlap with n is p_1-lower bound for N(A)3 $P_2^N = \frac{nc}{\beta+n-1}$ that separates N(A)3 from N(A)2. This bound decreases from $\frac{c}{\beta}$ for $n = 1$ to c for $n \to \infty$. Recall that PM2 and N(A)2 exist only if $\beta < 1$. P_2^N is strictly less than $\frac{c}{\beta}$ for any $n > 1$ while the upper p_1-bounds for PM2, P_{21} and P_{22} are not less than $\frac{c}{\beta}$. Therefore, if $p_1 \leq \frac{c}{\beta}$, the equality $p_1 = \frac{nc}{\beta+n-1}$ yields $n_2 = \frac{p_1(1-\beta)}{p_1-c}$ such that, for the same given inputs except n, N(A) can be realized as N(A)2 for all $n \leq n_2$ and as N(A)3 for all $n > n_2$. Since profits are continuous under N(A), i.e., $r^{*,N3} = r^{*,N2}$ at $n = n_2$, and, by Proposition 10 in Bazhanov et al. (2015), $nr^{*,N3}$ is decreasing in n, while $nr^{*,N2} \equiv nr^{*,PM2}$ is constant, we have $r^{*,PM2} > r^{*,N3}$ for any $n > n_2$, i.e., for

$p_1 \in \left(P_2^N, c/\beta\right]$ and any other inputs in the overlap of N(A)3 and PM2, inequality $r^{*,PM2} > r^{*,N3}$ also holds.

Part (2.2) follows from the facts: (a) for N(A)4, v^* is increasing in ρ, i.e., the total first-period profit $nr^{*,N4}$ does not exceed $nr^{*,N4}|_{\rho=0} = (p_1 - c)(1 - p_1) = nr^{*,PM2}$, and (b) the second period is always at loss under N(A)4 since $s < c$.

A.8. **Proof of Proposition 9 (customer surplus with PM vs. no-PM).**

Lemma 22. *Under the conditions of the corresponding RESE, the total customer surplus is*

(1) *under PM1:* $\Sigma^{PM1} = \frac{(1-p_1)^2}{2} + (1 - p_1)(p_1 - p_2^*) + \frac{(\beta p_1 - p_2^*)^2}{2\beta}$;

(2) *under PM2 and N(A)2:* $\Sigma^{PM2} = \Sigma^{N2} = \frac{(1-p_1)^2}{2}$;

(3) *under N(A)1:* $\Sigma^{N1} = \frac{(\beta - p_2^*)^2}{2\beta}$;

(4) *under N(A)3 and 4, Σ has the same form:* $\Sigma = \frac{(1-p_1)^2}{2} - \frac{(v^* - p_1)^2}{2} + \frac{\left(\beta v^* - p_2^*\right)^2}{2\beta}$*, where* $p_2^{*,N4} = s < p_2^{*,N3}$ *and* $v^{*,N4} \geq v^{*,N3}$*, which is strict for any* $\rho \in (0, 1)$.

The value $\Delta\Sigma^{A,B} \triangleq \Sigma^A - \Sigma^B$ below denotes the change in the total surplus that results from the switch from equilibrium B to equilibrium A given the same inputs when both RESE are possible. Consider Σ^{N1} as $\Sigma^{N1} = \int_{p_2^*}^{\beta p_1} (\tilde{v} - p_2^*) \frac{d\tilde{v}}{\beta} + \int_{\beta p_1}^{\beta} (\tilde{v} - p_2^*) \frac{d\tilde{v}}{\beta}$ where the first integral is Σ_2^{PM1}. Then

$$\Delta\Sigma^{PM1,N1} = \Sigma_1^{PM1} - \int_{\beta p_1}^{\beta} (\tilde{v} - p_2^*) \frac{d\tilde{v}}{\beta} = \int_{p_1}^{1} (v - p_2^*) dv - \int_{p_1}^{1} (\beta v - p_2^*) dv = \int_{p_1}^{1} v(1 - \beta) dv = \frac{(1 - \beta)}{2}(1 - p_1^2),$$

which is positive for any $p_1 < 1$ and $\beta < 1$.

The result for $\Delta\Sigma^{PM1,PM2}$ follows directly from parts (1) and (2) of Lemma 22 after substitution for $p_2^* = p_2^{*,PM1} = c + \frac{\beta - c}{n+1}$ leading to $\Delta\Sigma^{PM1,PM2} = (1 - p_1)(p_1 - c - \frac{\beta - c}{n+1}) + \frac{1}{2\beta}(\beta p_1 - c - \frac{\beta - c}{n+1})^2 > 0$.

By Lemma 22, $\Delta\Sigma^{PM2,N1} = \frac{1}{2}\left[(1 - p_1)^2 - \frac{1}{\beta}(\beta - p_2^*)^2\right] = \frac{1}{2}\left\{(1 - p_1)^2 - \frac{1}{\beta}\left[\frac{n}{n+1}(\beta - c)\right]^2\right\}$, because, by part NA1 of Theorem 3, $\beta - p_2^* = (\beta - c)\frac{n}{n+1}$. Since $\Delta\Sigma^{PM2,N1}$ decreases in p_1, it is always negative if $\Delta\Sigma^{PM2,N1} < 0$ at p_1-lower bound, which, minimized at $\rho \to 1$, by Theorem 3, is $p_1^{LB} = 1 - \frac{n}{n+1}(\beta - c)$. Indeed, $\Delta\Sigma^{PM2,N1}|_{p_1 = p_1^{LB}} = \frac{1}{2}\left[\frac{n}{n+1}(\beta - c)\right]^2 (1 - \frac{1}{\beta}) < 0$ for any $\beta < 1$.

By Lemma 22, $\Delta\Sigma^{PM2,N3} = \frac{1}{2}\left[(v^* - p_1)^2 - \frac{1}{\beta}(\beta v^* - p_2^*)^2\right]$, where, by part NA3 of Theorem 3,

$$\begin{aligned} v^* - p_1 &= \frac{p_1 - \rho p_2^* - p_1 + p_1 \rho\beta}{1 - \rho\beta} = \frac{\rho\beta(p_1 - 1 + Y^*)}{1 - \rho\beta} \text{ and} \\ \beta v^* - p_2^* &= \frac{\beta p_1 - \beta\rho p_2^* - p_2^* + p_2^* \rho\beta}{1 - \rho\beta} = \frac{\beta(p_1 - 1 + Y^*)}{1 - \rho\beta}, \end{aligned}$$

yielding $\Delta\Sigma^{PM2,N3} = \frac{\beta}{2}\left[\frac{Y^{*,N3} - (1 - p_1)}{1 - \rho\beta}\right]^2 (\rho^2\beta - 1) < 0$.

The sign of $\Delta\Sigma^{PM2,N4}$ can be shown in the same way using $p_2^* = s$ and $v^* = v^{*,N4} = \frac{p_1 - \rho s}{1 - \rho\beta}$. Then $v^* - p_1 = \frac{\rho(p_1\beta - s)}{1 - \rho\beta}$ and $\beta v^* - s = \frac{p_1\beta - s}{1 - \rho\beta}$, yielding $\Delta\Sigma^{PM2,N4} = \frac{1}{2\beta}\left(\frac{p_1\beta - s}{1 - \rho\beta}\right)^2 (\rho^2\beta - 1) < 0$.

A.9. **Proof of Lemma 10 (p_1-bounds are equivalent to ρ-bounds).** By the proof of Theorem 1 for $n = 1$, the p_1-bounds P_{11} and P_{21} separate PM1 and PM2 respectively from NA3. These bounds can be written as bounds on ρ. Indeed, $p_1 \geq P_{11} \Leftrightarrow 1 - p_1 \leq \frac{\rho}{2}(\beta - c) \Leftrightarrow \rho \geq \rho^{PM1} = \frac{2(1-p_1)}{\beta - c}$, and $p_1 \leq P_{21} \Leftrightarrow p_1\left[(1 - \rho\beta)^2 - (1 - \beta)\right] \leq c(1 - \rho\beta)^2 \Leftrightarrow (1 - \rho\beta)^2 \leq \frac{p_1(1-\beta)}{p_1 - c} \Leftrightarrow \rho \geq \rho^{PM2} = \frac{1}{\beta}\left[1 - \sqrt{\frac{p_1(1-\beta)}{p_1 - c}}\right]$, where $\sqrt{\frac{p_1(1-\beta)}{p_1 - c}} < 1$ under NA3 since $p_1\beta > c$. Inequality $\rho^{PM2} \leq 1$ is equivalent to $1-\sqrt{\frac{p_1(1-\beta)}{p_1-c}} \leq \beta$, which holds as equality if $\beta=1$. Consider $\beta<1$. Then inequality $\rho^{PM2} \leq 1$ can be written as $1-\beta \leq \frac{p_1}{p_1-c}$ or $-c-\beta(p_1-c) \leq 0$, which is strict for any feasible c, p_1, and β. In the same way, $\rho^{PM2}>0$ is equivalent to $p_1(1-\beta)<p_1-c$, which always holds.

A.10. **Proof of Proposition 11 (benefit from PM, $n=1$).**

Lemma 23. *For $n=1$ and $\frac{c}{\beta}<p_1<1-\frac{\rho}{2}(\beta-c)$, NA3 exists and unique with $v^*=\frac{2p_1-\rho c}{2-\rho\beta}, Y^*=1-\frac{\beta p_1+c(1-\rho\beta)}{\beta(2-\rho\beta)}, p_2^*=c+\frac{\beta p_1-c}{2-\rho\beta}, r^{*,N3}=\frac{(p_1-c)[2(1-p_1)-\rho(\beta-c)]}{2-\rho\beta}+\frac{(\beta p_1-c)^2}{\beta(2-\rho\beta)^2}, \Sigma^*=\frac{(1-p_1)^2}{2}+\frac{1}{2}\left(\frac{\beta p_1-c}{2-\rho\beta}\right)^2\left(\frac{1}{\beta}-\rho^2\right)$.*

The proof of the Proposition follows from the properties of the boundaries between RESE, established in Theorems 1, 3, Corollary 2, Proposition 16, and the fact that, for $n=1$, a monopolist is indifferent between two RESE at the boundary. For $n=1$, the area where PM1 exists is inside the area where NA1 exists because, by Proposition 4, $P_1^N=P_{11}$ and $P_1^N<P_{12}$.

Part (1.1). By part PM2 of Theorem 1, PM2 exists if $\frac{c}{\beta} \geq CB$ and $p_1 \leq P_2$ (if $p_1=P_2$, the form of a *realized* RESE depends on the expectations: for $\alpha^*=1$ it is PM1, for $\alpha^*=0$ — PM2). By part NA1 of Theorem 3, NA1 exists if $p_1 \geq P_1^N$. The benefit from PM is $B^{PM2,NA1}=r^{*,PM2}-r^{*,NA1}=(p_1-c)(1-p_1)-\frac{(\beta-c)^2}{4\beta}$, which is increasing in c because $\frac{\partial B^{PM2,NA1}}{\partial c}=-(1-p_1)+\frac{1}{2\beta}(\beta-c)=p_1-\frac{1}{2\beta}(\beta+c)>0$. The last inequality holds since $p_1 \geq P_1^N=1-\frac{\rho}{2}(\beta-c)$ under NA1, and $\frac{1}{2\beta}(\beta+c)<1+\frac{\rho}{2}(\beta+c)-\rho\beta \Leftrightarrow \frac{1}{2}\left(1+\frac{c}{\beta}\right)(1-\rho\beta)<1-\rho\beta \Leftrightarrow \frac{\beta+c}{2}<\beta$ holds for any $c<\beta$. Then $B^{PM2,NA1} \geq 0$ for any c if $B^{PM2,NA1}|_{c=0} \geq 0$, which is $-p_1^2+p_1-\frac{\beta}{4} \geq 0$. This inequality holds between the roots $(p_1)_{1,2}=\frac{1}{2}\left[1 \mp \sqrt{1-\beta}\right]$. Inequality $p_1 \geq (p_1)_1$ always holds if $(p_1)_1 \leq P_1^N|_{c=0}=1-\frac{\rho\beta}{2}$, which is satisfied for any $\rho<1$ since $P_1^N|_{c=0}$ is decreasing in ρ and $(p_1)_1 \leq P_1^N|_{c=0}$ holds for $\rho=1$: $\frac{1}{2}\left[1-\sqrt{1-\beta}\right] \leq 1-\frac{\beta}{2} \Leftrightarrow \beta-\sqrt{1-\beta} \leq 1$ (always holds). Inequality $p_1 \leq (p_1)_2$ always holds if $(p_1)_2$ is not less than p_1-upper bound for PM2, which, for $c=0$, is P_2 because if $0=\frac{c}{\beta}<CB$ (part 2.1 of Theorem 1), inequality $p_1 \leq P_{21}=0$ never holds. Equality $P_2|_{c=0}=\frac{1}{2}\left[1+\sqrt{1-\beta}\right] \equiv (p_1)_2$ implies $B^{PM2,NA1}|_{c=0} \geq 0$. Benefit $B^{PM2,NA1}$ decreases in β since $\frac{\partial B^{PM2,NA1}}{\partial \beta}=-\frac{1}{4\beta^2}\left[2(\beta-c)\beta-(\beta-c)^2\right]=-\frac{\beta^2-c^2}{4\beta^2}<0$.

Part (1.2). By Lemma 23, NA3 exists. By part PM2 of Theorem 1, PM2 exists either if $\frac{c}{\beta} \geq CB$ (since $p_1<P_1^N<P_2$) or $\frac{c}{\beta}<CB$ and $p_1 \leq P_{21}$, which, by Lemma 10, is equivalent to $\rho \geq \rho^{PM2}$. The benefit from PM is $B^{PM2,NA3}=r^{*,PM2}-r^{*,NA3}=(p_1-c)(1-p_1)-(p_1-c)\frac{2(1-p_1)-\rho(\beta-c)}{2-\rho\beta}-\frac{(\beta p_1-c)^2}{\beta(2-\rho\beta)^2}$

$$
\begin{aligned}
&= \frac{1}{2-\rho\beta}\left[(p_1-c)\rho(\beta p_1-c)-\frac{(\beta p_1-c)^2}{\beta(2-\rho\beta)}\right]=\frac{\beta p_1-c}{2-\rho\beta}\left[(p_1-c)\rho-\frac{\beta p_1-c}{\beta(2-\rho\beta)}\right] \\
&= \frac{\beta p_1-c}{2-\rho\beta}\left[c(1-2\rho\beta+\rho^2\beta^2)-p_1(\beta-2\rho\beta+\rho^2\beta^2)\right]=\frac{\beta p_1-c}{2-\rho\beta}\left[p_1(1-\beta)-(1-\rho\beta)^2(p_1-c)\right],
\end{aligned}
$$

which is increasing in ρ. Since $\beta p_1-c>0$ under NA3 for $n=1$, $B^{PM2,NA3}>0$ if and only if $1-\rho\beta<\sqrt{\frac{p_1(1-\beta)}{p_1-c}}$, which is, indeed, equivalent to $\rho>\frac{1}{\beta}\left[1-\sqrt{\frac{p_1(1-\beta)}{p_1-c}}\right]=\rho^{PM2}$. By Lemmas 22, and 23, $\Delta\Sigma^{PM2,NA3}=\frac{1}{2}\left(\frac{\beta p_1-c}{2-\rho\beta}\right)^2\left(\rho^2-\frac{1}{\beta}\right)<0$.

Part (2). Since the profits in the pairs PM1 – N(A)1 and PM2 – N(A)2 are identical, retailer is indifferent between these equilibria in the correspondent areas where (2.1) PM1 exists: $\frac{c}{\beta}<CB$ and $p_1 \geq P_{11} \Leftrightarrow \rho \geq \rho^{PM1}$ or $\frac{c}{\beta} \geq CB$ and $p_1 \geq P_2$; (2.2) N(A)2 exists: $\beta<1$, any ρ and $\frac{c}{\beta}$, and $p_1 \leq \frac{c}{\beta}$. Part (2.3) follows from the proof of part (1.2) since $B^{PM2,NA3}=0$ at the boundary between N3 and PM2 where $\rho=\rho^{PM2}$.

Part (3). The remaining area with $\frac{c}{\beta} < CB, p_1 > \frac{c}{\beta}, p_1 < P_{11} = P_1^N$ ($\rho < \rho^{PM1}$), and $p_1 > P_{21}$ ($\rho < \rho^{PM2}$) corresponds to inputs where only price-discriminating N(A)3 exists and PM-equilibria do not exist because of a lower profit.

PM never leads to a gain from increased strategic behavior because (i) profit is constant in ρ for both PM1 and PM2; (ii) profit is continuous at the boundaries between equilibria. Moreover, N(A)3 is realized for any $\rho^L < \rho^{PM1} \wedge \rho^{PM2}$ and one of PM-equilibria (denote it as PM) is realized for any $\rho^H \geq \rho^{PM1} \wedge \rho^{PM2}$. Therefore, since $r^{*,NA3}$ is decreasing in ρ for $n = 1$ (Bazhanov et al. (2015)), inequality $r^{*,NA3}|_{\rho=\rho^L} > r^{*,PM}|_{\rho=\rho^H}$ always holds yielding $\eta(\text{NA3,NA3,PM}) = \frac{(r^{*,PM} - r^{*,NA3})|_{\rho=\rho^H}}{r^{*,NA3}|_{\rho=\rho^L} - r^{*,NA3}|_{\rho=\rho^H}} < 1$.

A.11. **Proof of Proposition 12 (gain from PM2).** Assume that N(A)4 and PM2 exist for the same inputs including $\rho^H > 0$, and N(A)4 exists for these inputs except $\rho^L < \rho^H$. The loss from increased strategic behavior without PM is $r^{*,NA4}|_{\rho=\rho^H} - r^{*,NA4}|_{\rho=\rho^L} < 0$ and the performance of PM2 is $\eta(\text{NA4,NA4,PM2}) = \frac{r^{*,PM2} - r^{*,NA4}|_{\rho=\rho^H}}{r^{*,NA4}|_{\rho=\rho^L} - r^{*,NA4}|_{\rho=\rho^H}} = 1 + \frac{r^{*,PM2} - r^{*,NA4}|_{\rho=\rho^L}}{r^{*,NA4}|_{\rho=\rho^L} - r^{*,NA4}|_{\rho=\rho^H}}$. Since $r^{*,NA4}$ is decreasing in ρ, profit $r^{*,PM2} = \frac{1}{n}(p_1 - c)(1 - p_1)$ does not depend on ρ, and, by Proposition 6, $\frac{1}{n}(p_1 - c)(1 - p_1) > r^{*,NA4}$ for any inputs in the area of NA4 existence (implying that $r^{*,PM2} > r^{*,NA4}|_{\rho=\rho^L}$), PM2 leads to a gain ($\eta > 1$). A lower bound of η in ρ^L is at $\rho^L = 0$ where the denominator in the expression for η attains maximum. Then, the substitution of the expressions for profits yields

$$\eta(\text{NA4,NA4,PM2}) \geq 1 + \frac{(1 - p_1)\,[n(p_1 - c) - (p_1 - s)]}{(p_1 - s)(v^{*,NA4} - p_1)} = 1 + \frac{(1 - \rho^H\beta)(1 - p_1)\,[n(p_1 - c) - (p_1 - s)]}{(p_1 - s)\rho^H(p_1\beta - s)}.$$

This measure is unbounded in n since $nr^{*,NA4}$ decreases in n to zero while $nr^{*,PM2}$ is constant.

A.12. **Equilibria existence in Example 2 (gain from PM2).** NA4 exists for both $\rho = 0.5$ and $\rho = 0$, and, when PM is available and used by retailers at $\rho = 0.5$, N4 exists for $\rho = 0$ and PM2 — for $\rho = 0.5$. Indeed, by Theorem 5 for $\rho = 0$, $v^{*,N4} = p_1 = \frac{1}{2}, Y^{*,N4} = \frac{1}{4}\frac{0.45}{0.05} = \frac{9}{4}$, and condition (a) of Theorem 5 holds: $\frac{n-1}{n}Y^{*,N4} = \frac{9}{8} > 1 \geq 1 - \frac{s}{\beta} > 1 - \frac{c}{\beta}$. The last inequality means that the additional condition (4.2) of Proposition 16 for the existence of N4 also holds since $p_1 = 0.5 > \frac{c}{\beta} = 0.2$. PM2, by Theorem 1, does not exist because, for $\rho = 0, CB_2 = 1 > \frac{c}{\beta}$ and $P_{21} = \frac{c}{\beta} = 0.2 < 0.5 = p_1$. For $\rho = 0.5, v^{*,N4} = \frac{1-0.05}{2(3/4)} = \frac{1.9}{3}$, and $Y^{*,N4} = \frac{1}{2}\frac{0.45}{0.05}\frac{1.1}{3} = \frac{3.3}{2}$. Condition (a) of Theorem 5 does not hold: $\frac{n-1}{n}Y^{*,N4} = \frac{3.3}{4} = 0.825 < 1 - \frac{s}{\beta} = 0.9$, but condition (b) holds: $\frac{n-1}{n}Y^{*,N4}\frac{\beta}{c+\beta v^* - 2s} = \frac{3.3}{4\cdot 2\cdot(0.1+1.9/6-0.1)} = \frac{9.9}{7.6} > 1$. PM2 exists since $CB_2 = \frac{1-1+1/8}{(1-1/4)^2 + \frac{1}{2}\cdot 2\cdot\frac{1}{8}} = \frac{2}{11} < \frac{c}{\beta} = \frac{2}{10}$ and $P_{22} = \frac{2\left(4.6-0.5+4\sqrt{53}/10\right)}{17} > \frac{8.2+8\cdot 0.7}{17} > p_1 = 0.5$.

A.13. **Proof of Proposition 13 (PM-profit exceeds NA3, $p_1 = \beta$).** The proof uses the following lemma where p_1-bounds between NA3, 2, and 1 are written as the bounds on $\frac{c}{\beta}$ with $CB_{N1} \triangleq 1 - \frac{n+1}{n\rho\beta}(1 - \beta)$ and $CB_{N2} \triangleq 1 - \frac{1-\beta}{n}$.

Lemma 24. *If $p_1 = \beta$, the forms of NA3 and NA4 simplify as follows:*

NA3 ($p_2^* > s$): $Y^* = \frac{[(1-c/\beta)(1-\rho\beta)+1-\beta]n}{n+1-\rho\beta}$ *and* $r^{*,NA3} = \beta(Y^*/n)^2$; *condition* $P_2^N < p_1 < P_1^N$ *is equivalent to* $\frac{c}{\beta} < CB_{N2}$ *and either* $\beta < 1$ *for* $\rho = 0$ *or, for* $\rho > 0, \frac{c}{\beta} > CB_{N1}$; *condition (a) becomes* $\frac{n-1}{n}\frac{\beta^2(1-v^*)Y^*}{(c-s)(\beta-s)} \leq 1$; *and condition* $Y^* < 1 - \frac{s}{\beta}$ *becomes either* $c - s \geq \beta(1 - \beta)$ *or* $c - s < \beta(1 - \beta)$ *and* $n < \frac{(1-\rho\beta)(1-s/\beta)}{(1-\rho\beta)(1-c/\beta)-\beta+s/\beta}$.

NA4 ($p_2^* = s$): *$Y^* = \frac{n-1}{n}\frac{\beta-s}{c-s}\left(1-\frac{\beta-\rho s}{1-\rho\beta}\right)$ and $r^{*,NA4} = \frac{\beta-s}{n^2}\left(1-\frac{\beta-\rho s}{1-\rho\beta}\right)$; condition (b) becomes $\beta(n-1)Y^* \geq n(c+\beta v^* - 2s)$.*

As to equilibria NA1 and NA2, both of them may exist and have overlaps with PM1 and PM2 for some feasible inputs when $p_1 = \beta$. NA1 exists if and only if $\frac{c}{\beta} \leq CB_{N1}$ for $\rho > 0$ or $\beta = 1$ for $\rho = 0$; and NA2 — if and only if $\frac{c}{\beta} \geq CB_{N2}$.

By Theorem 1 and Lemma 24, inequality $r^{*,PM1} > r^{*,N3}$ is

$$\frac{(\beta-c)^2}{(n+1)^2\beta} > \frac{[\beta(1-\beta)+(\beta-c)(1-\rho\beta)]^2}{\beta(n+1-\rho\beta)^2} \Leftrightarrow n\left[(\beta-c)\rho - 1 + \beta\right] > 1-\beta, \tag{23}$$

which holds for any $n \geq 1$ if $(\beta-c)\rho > 2(1-\beta)$ since, under this condition, it holds for $n = 1$ and the LHS is increasing in n. On the other hand, (23) may hold only if $[\cdot] > 0$, which is equivalent to $\rho > \frac{1-\beta}{\beta-c}$. Then, if $\rho \in \left(\frac{1-\beta}{\beta-c}, 2\frac{1-\beta}{\beta-c}\right]$, inequality $r^{*,PM1} > r^{*,N3}$ is equivalent to $n > \frac{1-\beta}{(\beta-c)\rho-1+\beta}$. Condition $\beta > \frac{1+c}{2}$ follows from inequality $\frac{1-\beta}{\beta-c} \geq 1$.

A.14. **Equilibria existence in Examples 3-5.** *Example 3.* Condition (a) of Theorem 5 holds: $Y^{*,NA4} = \frac{208}{105}$ and $\frac{n-1}{n}Y^{*,NA4} = \frac{416}{315} > 1 > 1 - \frac{s}{\beta}$, i.e., "salvaging" is forced on retailers and N(A)4, indeed, exists with $v^* = \frac{27}{35}$ and the profit $r^{*,NA4} = \frac{0.65}{9}(1-v^*) = \frac{26}{1575} = 0.0165$. PM2 exists since $\frac{c}{\beta} = \frac{4}{10} > \frac{4}{58} = CB_2$ and $p_1 < P_{22} = 0.93$ with $r^{*,PM2} = 0.06$. PM1 exists since $\frac{c}{\beta} = \frac{4}{10} > \frac{4}{100} = CB_1$ and $p_1 > P_{12} = 0.69$, with the profit $r^{*,PM1} = \frac{0.15^2}{4^2\cdot 0.25} = 0.0056$.

Example 4. Equilibrium profits in Figure 6 (b) are computed under the existence conditions of the corresponding RESE types. In particular, for $\rho = 0.2$ and $\rho = 0.65$ the existence can be demonstrated as follows. For $\rho = 0.65$, NA3 is realized in no PM game since, by Lemma 24, $Y^{*,NA3} = 0.89 < 1 = 1 - \frac{s}{\beta}$, condition (b) of Theorem 3 holds: $r^{*,NA3} = 0.0247 > \tilde{r}^i = 0.0188$ (which can be shown using the expression for $\tilde{r}^i = \left\{\sqrt{(p_1-s)(1-v^*)} - \sqrt{\frac{n-1}{n}Y^*(c-s)}\right\}^2$ given in Bazhanov et al. (2015)), and NA4 does not exist because the necessary condition $Y^{*,NA4} > 1 - \frac{s}{\beta}$ does not hold: $Y^{*,NA4} = \frac{105}{108} < 1$. For $\rho = 0.2$, the only existing equilibrium is NA4 without PM or N4 with PM. Indeed, $1 - v^{*,NA4} = \frac{4}{9}, Y^{*,NA4} = \frac{3\cdot 5}{4}\frac{4}{9} = \frac{5}{3}$ and $\frac{n-1}{n}Y^{*,NA4} = \frac{5}{4} > 1 = 1 - \frac{s}{\beta} > 1 - \frac{c}{\beta}$, which means that condition (a) of Theorem 5 holds and additional condition (4.2) of Proposition 16 for existence of N4 holds ($p_1 > \frac{c}{\beta} = 0.2$). At the same time, NA3 does not exist since $Y^{*,NA3}|_{\rho=0.2} = \frac{488}{490}$, condition (a) of part NA3 of Theorem 3 does not hold: $\frac{n-1}{n}\frac{\beta^2(1-v^*)Y^*}{(c-s)(\beta-s)} = \frac{3\cdot 4.5}{4}\frac{488}{490} > 1$, and condition (b) does not hold: $r^{*,NA3} = 0.152 < \tilde{r}^i = 0.158$. PM-equilibria also do not exist for $\rho = 0.2$. PM1: $CB_1 = \frac{31}{56} > \frac{20}{100} = \frac{c}{\beta}$ and $P_{11} = \frac{2.52}{5} > p_1$; PM2: $CB_2 = \frac{62}{85} > \frac{20}{100} = \frac{c}{\beta}$ and $P_{21} = \frac{8.1}{31} < \frac{15.5}{31} = p_1$. When $\rho = 0.65$ and PM is available, PM1 exists since $CB_1 < 0$ and $P_{12} = 0.487 < p_1$. The PM performance is $\eta(\text{NA4,NA3,PM1}) = \frac{\left(r^{*,PM1} - r^{*,NA3}\right)|_{\rho=0.65}}{r^{*,NA3}|_{\rho=0.65} - r^{*,NA4}|_{\rho=0.2}} = \frac{0.0128-0.0247}{0.0247-0.0139} = \frac{0.0119}{0.0108} = -1.102$.

Example 5. Condition (a) of Theorem 5 holds for both $\rho^H = 0.4$ and $\rho^L = 0.3$. PM1 exists at ρ^H since $CB_1 = \frac{304}{1004} > \frac{100}{1300} = \frac{c}{\beta}$ and $P_{11} = 0.398 < p_1$, and PM1 does not exist at ρ^L since $CB_1 = 0.466 > \frac{c}{\beta}$ and $P_{11} = 0.410 > p_1$.

Appendix B. Proofs of auxiliary statements

B.1. **Proof of Lemma 14 (roots of equation $\breve{r}^i = \tilde{r}^i$).** Equation (5), i.e. $p_1^2 - (x+c)p_1 + \frac{\beta}{4}\left(x + \frac{c}{\beta}\right)^2 = 0$, originates from comparing the expressions $\breve{r}^i = (p_1 - c)(x - p_1)$ and $\tilde{r}^i = \frac{\beta}{4}\left(x - \frac{c}{\beta}\right)^2$, and collecting the terms in the equation $\tilde{r}^i - \breve{r}^i = 0$. The roots of (5) exist since the discriminant $D = (x+c)^2 - \beta\big(x + \frac{c}{\beta}\big)^2 \geq 0$. Indeed,

$$D = x^2(1-\beta) + c^2\Big(1 - \frac{1}{\beta}\Big) = (1-\beta)\Big[x^2 - \frac{c^2}{\beta}\Big] \geq (1-\beta)\Big[\frac{c^2}{\beta^2} - \frac{c^2}{\beta}\Big] \geq 0.$$

$\frac{1}{2}\big(x + \frac{c}{\beta}\big)$ is between the roots with strict inequalities when $x > \frac{c}{\beta}$ and $\beta < 1$ because substituting $p_1 = \frac{1}{2}\big(x + \frac{c}{\beta}\big)$ into the LHS of (5) we obtain:

$$\begin{aligned} & \frac{1}{4}\Big(x + \frac{c}{\beta}\Big)^2 - \Big[\Big(x + \frac{c}{\beta}\Big) + c - \frac{c}{\beta}\Big]\frac{1}{2}\Big(x + \frac{c}{\beta}\Big) + \frac{\beta}{4}\Big(x + \frac{c}{\beta}\Big)^2 \\ = \;\; & \frac{1}{4}\Big\{-\Big(x + \frac{c}{\beta}\Big)^2 - \frac{2c}{\beta}(\beta - 1)\Big(x + \frac{c}{\beta}\Big) + \beta\Big(x + \frac{c}{\beta}\Big)^2\Big\} = \frac{1}{4}\Big(x + \frac{c}{\beta}\Big)(\beta - 1)\Big[x - \frac{c}{\beta}\Big] \leq 0. \end{aligned}$$

Inequality $(p_1)_2(x) \leq x$ follows from $\tilde{r}^i - \breve{r}^i\big|_{p_1 = x} \geq 0$, which is strict unless $x = \frac{c}{\beta}$.

The larger root is increasing in x if $x > \frac{c}{\beta}$, which is evident from the implicit differentiation of the equation with respect to x

$$[2p_1 - (x+c)]\frac{\partial p_1}{\partial x} = p_1 - \frac{\beta}{2}\Big(x + \frac{c}{\beta}\Big),$$

since, for the larger root $2p_1 > x + c$ and $p_1 - \frac{\beta}{2}\Big(x + \frac{c}{\beta}\Big) = p_1 - \frac{1}{2}(\beta x + c) \geq p_1 - \frac{1}{2}(x+c) > 0$ implying that $\frac{\partial p_1}{\partial x} > 0$.

B.2. **Proof of Lemma 15 (PM BR).** When all retailers use PM, the general expression for retailer i profit, by (4), is

$$r_1^i = \begin{cases} (p_1 - c)y_1^i, & \text{if } Y = Q, \\ (p_2 - c)y_1^i, & \text{if } Y > Q. \end{cases}$$

In this case, $Q = Y \wedge (1 - v_1^{\min})$, where, by (1), $v_1^{\min} = p_1$. Therefore, $Y = Q$ if $Y \leq 1 - p_1$ (sales only in the first period) and p_2 is not defined. Otherwise ($Y > 1 - p_1$), there are second-period sales and, by (3), $p_2 < p_1$. Thus, r_1^i has a discontinuity at $Y = Q$. Moreover, the profit at $Y = Q + 0$ is strictly less than at $Y = Q$.

Consider two principal cases: (a') the maximum-profit PM response *without* the second-period sales (i.e., $Y_1 \leq 1 - p_1$) is not dominated by any PM response with the second period sales (i.e., $Y_1 > 1 - p_1$), and (b') the maximum-profit PM response *with* the second period sales is not dominated by any PM response without the second period sales.

We start by describing the nontrivial BR candidates for the cases (a') and (b'). The profit $r_1^i = (p_1 - c)y_1^i$ in case (a') is strictly increasing in y_1^i, implying that, if BR exists in this region, retailer i sets y_1^i to $\breve{y}_1^i = 1 - p_1 - Y_1^{-i}$ resulting in $Y_1 = 1 - p_1$. The nontrivial BR of this form exists if and only if $\breve{y}_1^i > 0$, i.e., $1 - Y_1^{-i} > p_1$, and the corresponding profit $\breve{r}_1^i = (p_1 - c)(1 - p_1 - Y_1^{-i})$ is not dominated by that of case (b') or by the profit corresponding to the no-PM response.

The nontrivial responses for case (b') are constrained by $Y_1 > 1 - p_1$ or, equivalently, $y_1^i > 1 - p_1 - Y_1^{-i}$, and $y_1^i > 0$. The profit function in this case, $r_1^i = (p_2 - c)y_1^i = [\beta(1 - Y_1^{-i} - y^i) - c]y^i$, is strictly concave.

The profit-maximizing y_1^i must satisfy the first-order condition $\frac{\partial r_1^i}{\partial y_1^i} = \beta(1 - Y_1^{-i}) - c - 2\beta y_1^i = 0$, yielding $\tilde{y}_1^i \triangleq \frac{1}{2}(1 - \frac{c}{\beta} - Y_1^{-i})$. The profit corresponding to $\tilde{y}_1^i$ is

$$\tilde{r}_1^i = \Big\{\beta\Big[1 - Y_1^{-i} - \frac{1}{2}\Big(1 - \frac{c}{\beta} - Y_1^{-i}\Big)\Big] - c\Big\} \times \frac{1}{2}\Big(1 - \frac{c}{\beta} - Y_1^{-i}\Big) = \frac{\beta}{4}\Big(1 - \frac{c}{\beta} - Y_1^{-i}\Big)^2.$$

By feasibility constraints, this inventory level is a nontrivial BR candidate only if $\tilde{y}_1^i > 0$ and $\tilde{y}_1^i > 1 - p_1 - Y_1^{-i}$. The first inequality is equivalent to $1 - Y_1^{-i} > \frac{c}{\beta}$. The second one is equivalent to

$$\frac{1}{2}\Big(1 - \frac{c}{\beta} - Y_1^{-i}\Big) > 1 - p_1 - Y_1^{-i} \Leftrightarrow p_1 > \frac{1}{2}\Big(1 - Y_1^{-i} + \frac{c}{\beta}\Big) \triangleq (p_1)_0.$$

If either of these conditions is violated, the profit function is strictly decreasing in the entire region of case (b'). Thus, a nontrivial BR of this form exists if and only if $1 - Y_1^{-i} > \frac{c}{\beta}$, $p_1 > (p_1)_0$ and $\tilde{r}_1^i$ is not dominated by the profit $\breve{r}_1^i$ of case (a') and the one corresponding to a no-PM response.

We now establish conditions when the maximum profit within case (a') is not dominated by the maximum profit within case (b') and vice versa. In particular, when either $\frac{c}{\beta} \geq 1 - Y_1^{-i}$ or $p_1 \leq (p_1)_0$, $\tilde{y}_1^i$ is not within the feasible region of case (b') and $\breve{r}_1^i$ dominates the profit corresponding to any response within case (b') as long as $\breve{y}_1^i$ is feasible and nontrivial, i.e., $1 - Y_1^{-i} > p_1$.

If $\tilde{y}_1^i$ is feasible and nontrivial, i.e., $1 - Y_1^{-i} > \frac{c}{\beta}$ and $p_1 > (p_1)_0$, the response of case (a') is not dominated by that of case (b') if and only if $\breve{y}_1^i$ is feasible, nontrivial, and $\breve{r}_1^i \geq \tilde{r}_1^i$. This inequality is $(p_1 - c)(1 - p_1 - Y_1^{-i}) \geq \frac{\beta}{4}\left(1 - c/\beta - Y_1^{-i}\right)^2$, which is equivalent to

$$p_1^2 - (1 + c - Y_1^{-i})p_1 + \frac{\beta}{4}\Big(1 - Y_1^{-i} + c/\beta\Big)^2 \leq 0. \tag{24}$$

By Lemma 14, the roots $(p_1)_{1,2}$ of (5) exist with $x = 1 - Y_1^{-i} \geq \frac{c}{\beta}$. Then (24) holds if and only if $(p_1)_1 \leq p_1 \leq (p_1)_2$. By Lemma 14, $(p_1)_1 \leq (p_1)_0 \leq (p_1)_2 \leq 1 - Y_1^{-i}$ where the last inequality is strict unless $1 - Y_1^{-i} = \frac{c}{\beta}$.

Combining all situations where the maximum profit in case (a') is strictly positive and not dominated by responses in case (b'), we obtain the following conditions: $1 - Y_1^{-i} > p_1$ (i.e., $\breve{y}_1^i$ is feasible and nontrivial) and either (a'.1) $\frac{c}{\beta} \geq 1 - Y_1^{-i}$ (i.e., $\tilde{y}_1^i \leq 0$ because the second-period sales are always at $p_2 \leq c$) or (a'.2) $\frac{c}{\beta} < 1 - Y_1^{-i}$ (i.e., $\tilde{y}_1^i > 0$) and $p_1 \leq (p_1)_0$ (i.e., $\tilde{y}_1^i \leq \breve{y}_1^i$ because the profit function decreases for all $y_1^i > \breve{y}_1^i$) or $(p_1)_0 < p_1 \leq (p_1)_2$ (i.e., even though $\tilde{y}_1^i$ is feasible, $\breve{r}_1^i \geq \tilde{r}_1^i$). Since $\frac{c}{\beta} < 1 - Y_1^{-i}$ implies $(p_1)_0 \leq (p_1)_2$, the subcase (a'.2) can be compactly described by the pair of conditions $\frac{c}{\beta} < 1 - Y_1^{-i}$ and $p_1 \leq (p_1)_2$.

Symmetrically, combining all situations where the maximum profit in case (b') is strictly positive and not dominated by responses in case (a'), we obtain the following conditions: $1 - Y_1^{-i} > \frac{c}{\beta}$ (i.e., $\tilde{y}_1^i$ is nontrivial) and either (b'.1) $1 - Y_1^{-i} \leq p_1$ (i.e., $\breve{y}_1^i$ is infeasible or trivial) or (b'.2) $1 - Y_1^{-i} > p_1 \geq (p_1)_2$. Indeed, for case (b'.1), there is no need to compare $\tilde{r}_1^i$ with $\breve{r}_1^i$ and the feasibility condition $p_1 > (p_1)_0$ is implied by $p_1 \geq 1 - Y_1^{-i} > \frac{c}{\beta}$ since, then, $(p_1)_0 < 1 - Y_1^{-i}$. For case (b'.2), $\tilde{r}_1^i \geq \breve{r}_1^i$ if and only if $p_1 \geq (p_1)_2$ or $p_1 \leq (p_1)_1$, but the feasibility condition $p_1 > (p_1)_0$ cannot be satisfied together with $p_1 \leq (p_1)_1$ because $(p_1)_1 \leq (p_1)_0$. On the other hand, $p_1 > (p_1)_0$ is implied by $p_1 \geq (p_1)_2$ and $1 - Y_1^{-i} > c/\beta$ (recall that the latter implies $(p_1)_2 > (p_1)_0$).

We now determine when the maximum profit of case (a') is not dominated by responses without PM, implying that Y_1 is equivalent to Y_1^{-i} above. Recall that these responses correspond to expectations $\bar{\alpha}(0), \bar{p}_2(0)$. First, response that results in first-period sales only cannot lead to profits higher than $\breve{r}_1^i$ because the potential first-period demand under such response does not exceed the first-period demand under PM. Second, a response with sales only in the second period results in all stock sold at p_2 and profit no higher than $\tilde{r}_1^i \leq \breve{r}_1^i$. The third remaining case is a response with sales in both periods characterized by $v_0^{\min} = v_0^{\min}(\bar{\alpha}(0), \bar{p}_2(0)) < 1 - Y_1$ and $y_0^i > 1 - Y_1 - v_0^{\min}$. The profit in this case is concave quadratic of the form

$$r_0^i = (p_1 - c)(1 - Y_1 - v_0^{\min}) + (\beta(1 - Y_1 - y_0^i) - c)(y_0^i - [1 - Y_1 - v_0^{\min}])$$

with $\frac{\partial r_0^i}{\partial y_0^i} = -2\beta y_0^i + \beta[1-Y_1-v_0^{\min}] + \beta(1-Y_1) - c$, the unique solution to the first-order condition $\tilde{y}_0^i = 1 - Y_1 - \frac{1}{2}(v_0^{\min} + c/\beta)$, yielding total inventory $\tilde{y}_0^i + Y_1 = 1 - \frac{1}{2}(v_0^{\min} + c/\beta) < 1 - s/\beta$ (since $c > s$ and $v_0^{\min} \geq p_1 > \frac{s}{\beta}$) and profit

$$\tilde{r}_0^i = (p_1 - c)(1 - Y_1 - v_0^{\min}) + \frac{\beta}{4}(v_0^{\min} - c/\beta)^2. \tag{25}$$

If $v_0^{\min} \leq c/\beta$, then $\tilde{y}_0^i \leq 1 - Y_1 - v_0^{\min}$ and r_0^i is decreasing for all $y_0^i \geq 1 - Y_1 - v_0^{\min}$. Thus, the profit-maximizing level of inventory without PM is $\breve{y}_0^i = 1 - Y_1 - v_0^{\min}$ that results only in the first-period sales. In this case, we have already established that no-PM response does not dominate $\breve{r}_1^i$.

If $v_0^{\min} > c/\beta$, we need to check when $\breve{r}_1^i = (p_1 - c)(1 - p_1 - Y_1) \geq \tilde{r}_0^i$ which is equivalent to inequality $(p_1 - c)(v_0^{\min} - p_1) \geq \frac{\beta}{4}(v_0^{\min} - c/\beta)^2$ and, in turn, (6). The corresponding equation is (5) with $x = v_0^{\min}$, and, by Lemma 14, its roots exist for $v_0^{\min} \geq c/\beta$. Moreover, relation $v_0^{\min} < 1 - Y_1$ implies that the larger root is less than $(p_1)_2|_{x=1-Y_1^{-i}}$. Thus, when $c/\beta < v_0^{\min} < 1 - Y_1$, there is a non-empty interval of p_1 in which (6) holds and, for any p_1 in this interval, $p_1 \leq (p_1)_2$ holds.

Summarizing all conditions where no-PM responses cannot dominate $\breve{r}_1^i$ we obtain: either (i) $v_0^{\min} \geq 1 - Y_1$, or (ii) $v_0^{\min} < 1 - Y_1$ and $v_0^{\min} \leq c/\beta$, or (iii) $c/\beta < v_0^{\min} < 1 - Y_1$ and (6). Combining these conditions with those of case (a'), we obtain the conditions of case (a) in the lemma. Indeed, in case (a'.1) $c/\beta \geq 1 - Y_1$ implies that either (i) or (ii) holds. In the complementary case (a'.2), subcases $v_0^{\min} \geq 1 - Y_1$ or $v_0^{\min} \leq c/\beta$ require only additional condition $p_1 \leq (p_1)_2$ ($v_0^{\min} < 1 - Y_1$ is implied by $v_0^{\min} \leq c/\beta$ and $c/\beta < 1 - Y_1$. An additional useful observation is that since $(p_1)_0 > c/\beta$ in this case, we have $(p_1)_2 > c/\beta$. On the other hand, if $c/\beta < v_0^{\min} < 1 - Y_1$, condition $p_1 \leq (p_1)_2$ is superseded by a stronger condition (6).

We complete the proof by describing when the maximum profit of case (b') is not dominated by responses without PM. Two out of three possibilities are ruled out in a way almost identical to the reasoning for the case (a'). First, response with the first-period sales only cannot lead to profits higher than $\tilde{r}_1^i$ because the potential first-period demand under such response does not exceed the first-period demand under PM while the latter would result in $\breve{r}_1^i \leq \tilde{r}_1^i$. Second, a response with sales only in the second period results in all stock sold at p_2 and profit no higher than $\tilde{r}_1^i$. The remaining case is a response with sales in both periods characterized by $v_0^{\min} = v_0^{\min}(\bar{\alpha}(0), \bar{p}_2(0)) < 1 - Y_1$ and $y_0^i > 1 - Y_1 - v_0^{\min}$.

Similarly to a comparison with $\breve{r}_1^i$, no-PM response cannot dominate $\tilde{r}_1^i$ if either (i) $v_0^{\min} \geq 1 - Y_1$ or (ii) $v_0^{\min} < 1 - Y_1$ and $v_0^{\min} \leq c/\beta$. The condition $v_0^{\min} < 1 - Y_1$ in (ii) is always satisfied for (b') because $1 - Y_1 > c/\beta$. Examine $c/\beta < v_0^{\min} < 1 - Y_1$. The PM BR with inventory level $\tilde{y}_1^i$ exists in this case if and only if $\tilde{r}_1^i \geq \tilde{r}_0^i$:

$$\begin{aligned}
\frac{\beta}{4}(1 - c/\beta - Y_1)^2 &\geq (p_1 - c)(1 - Y_1 - v_0^{\min}) + \frac{\beta}{4}(v_0^{\min} - c/\beta)^2 \quad \Leftrightarrow \\
\frac{\beta}{4}(1 - Y_1 - v_0^{\min})(1 - 2c/\beta - Y_1 + v_0^{\min}) &\geq (p_1 - c)(1 - Y_1 - v_0^{\min}) \quad \Leftrightarrow \\
\frac{\beta}{4}(1 - 2c/\beta - Y_1 + v_0^{\min}) &\geq p_1 - c \quad \Leftrightarrow \quad \frac{\beta}{4}(1 + 2c/\beta - Y_1 + v_0^{\min}) \geq p_1
\end{aligned}$$

(recall that Y_1 in $\tilde{r}_0^i$ and Y_1^{-i} in $\tilde{r}_1^i$ are equivalent here). The left-hand-side of the last inequality does not exceed $(p_1)_2$ because $\frac{\beta}{4}(1 + 2c/\beta - Y_1 + v_0^{\min}) < \frac{\beta}{4}[2(1 - Y_1) + 2c/\beta] = \beta(p_1)_0 < (p_1)_2$. However, this implies that $p_1 < (p_1)_2$ which is incompatible with case (b') because it requires $p_1 \geq (p_1)_2$. Thus, there is a no-PM BR that dominates $\tilde{r}_1^i$ when $\frac{c}{\beta} < v_0^{\min} < 1 - Y_1$. Combining (i) and (ii) with conditions of case (b'), we get the statement of the lemma.

B.3. **Proof of Lemma 17 (deviation from no-PM RESE into PM).** The form of r_1^i follows from general formula (4) and the expressions for the first-period sales given in §2.2 with $Y_1 = y_1^i$. There are two cases: (1) $1 - y_1^i \geq v_0^{\min}$ with $Q_1 = y_1^i$ (PM-retailer i has no sales in the second period) and $Q_0 = (1 - y_1^i - v_0^{\min}) \wedge Y_0$, and (2) $1 - y_1^i < v_0^{\min}$ with $Q_1 = (1-p_1) \wedge y_1^i$ and $Q_0 = 0$.

(1.1) If $1 - y_1^i - v_0^{\min} \geq Y_0$, then $Q_0 = Y_0$ implying $Q = Y$ (sales in the first period only) with $r_1^i = (p_1 - c)y_1^i$.

(1.2) If $1 - y_1^i - v_0^{\min} < Y_0$, which is possible only if $Y_0 > 0$, we have $Q_0 = 1 - y_1^i - v_0^{\min}$ and $Y > 1 - v_0^{\min} = Q$. This subcase implies sales in the second period with $p_2 = s \vee [\beta(1-Y)]$, which exceeds p_1 if $\beta(1-Y) \geq p_1 \Leftrightarrow y_1^i \leq 1 - Y_0 - p_1/\beta$. This inequality may hold for a non-trivial y_1^i only if $Y_0 < 1 - p_1/\beta$, which, in turn, is possible in this subcase if $v_0^{\min} > p_1/\beta$. Then r_1^i is

$$r_1^i = \begin{cases} (p_1 - c)y_1^i, & \text{if } y_1^i \leq 1 - Y_0 - p_1/\beta, \\ (p_2 - c)y_1^i, & \text{if } y_1^i > 1 - Y_0 - p_1/\beta, \end{cases} \tag{26}$$

which is continuous in y_1^i (since $p_2 = \beta(1 - Y_0 - y_1^i) = p_1$ at $y_1^i = \breve{y}_1^i \triangleq 1 - Y_0 - p_1/\beta$) and concave. Since $(p_1 - c)y_1^i$ increases in y_1^i, a profit maximizing retailer would not consider inventory levels below $\breve{y}_1^i$ implying $p_2 \leq p_1$.

(2.1) If $1 - p_1 \geq y_1^i$, then $Q_1 = y_1^i$. If $Y_0 = 0$, there are no sales in the second period and $r_1^i = (p_1 - c)y_1^i$. If $Y_0 > 0$, profit r_1^i is the same as in (1.2).

(2.2) If $1 - p_1 < y_1^i$, then $Q_1 = 1 - p_1$, $Q_0 = 0$, and there are sales in the second period with $r_1^i = (p_2 - c)y_1^i$ and $p_2 < \beta p_1$.

Summarizing all cases, we can conclude that r_1^i is defined by (26) if $v_0^{\min} > \frac{p_1}{\beta}$ and, otherwise, by

$$r_1^i = \begin{cases} (p_1 - c)y_1^i, & \text{if } y_1^i \leq 1 - Y_0 - v_0^{\min}, \\ (p_2 - c)y_1^i, & \text{if } y_1^i > 1 - Y_0 - v_0^{\min}. \end{cases}$$

If $Y_0 = 0$ (retailer i is a monopolist), $v_0^{\min}$ in the formula above for r_1^i is substituted by p_1 because sales in the second period occur only when y_1^i exceeds $1 - p_1$ (unlike the case of $Y_0 > 0$ for which the second period sales occur whenever y_1^i is not less than $1 - v_0^{\min}$).

Throughout the proof, we use the following notation:

$$\breve{y}_1^i \triangleq 1 - Y_0 - v_0^{\min}, \qquad \breve{r}_1^i \triangleq (p_1 - c)\breve{y}_1^i,$$
$$\tilde{y}_1^i \triangleq (1 - Y_0 - c/\beta)/2 = z/2, \qquad \tilde{r}_1^i \triangleq \beta(\tilde{y}_1^i)^2.$$

Quantity $\breve{y}_1^i$ is the maximizer of $(p_1 - c)y_1^i$ on the interval $y_1^i \leq 1 - Y_0 - v_0^{\min}$, and $\tilde{y}_1^i$ is an unconstrained maximizer of $(p_2 - c)y_1^i = [\beta(1 - Y_0 - y_1^i) - c]y_1^i$.

We can rule out any $y_1^i \leq 1 - Y_0 - p_1$ as a candidate for the optimal solution under rational expectations leading to $v_0^{\min} > p_1$ (which may take place only if $\rho > 0$ and $Y_0 > 0$). Indeed, for such y_1^i, we have $1 - Y \geq p_1$, resulting in $p_2 \geq \beta p_1$ and rational $v_0^{\min} = \left(p_1 \wedge \frac{p_1 - \rho p_2}{1 - \rho\beta}\right) \vee 1 = p_1$, a contradiction. On the other hand, any $y_1^i > 1 - Y_0 - p_1$ would result in $p_2 < \beta p_1$.

Hence, under rational expectations, an optimal inventory level of retailer i that deviates into PM may lead to the following three principal cases:

(a) Retailer i has positive inventory but sales occur only in the first period and $\bar{\alpha}(1) = 0$, leading to $v_0^{\min}(\bar{\alpha}(1), \bar{p}_2(1)) = p_1$. The inventory and profit are $y_1^i = \breve{y}_1^i|_{\bar{\alpha}=0} = 1 - Y_0 - p_1 = z - \frac{z_0}{2}$ and $r_1^i = \breve{r}_1^i|_{\bar{\alpha}=0} = (p_1 - c)\breve{y}_1^i|_{\bar{\alpha}=0}$. This inventory level can be a candidate for optimum *only if* it is positive, i.e., $Y_0 < 1 - p_1$ or $z > \frac{z_0}{2}$. Since $\breve{y}_1^i|_{\bar{\alpha}=0} \geq \breve{y}_1^i|_{\bar{\alpha}=1} \vee \breve{y}_1^i$, the necessary and sufficient conditions for $\breve{y}_1^i$ to be the maximizer, include $z > \frac{z_0}{2}$ and either $\breve{y}_1^i|_{\bar{\alpha}=0} \geq \tilde{y}_1^i$ (i.e., $z - \frac{z_0}{2} \geq \frac{z}{2} \Leftrightarrow z \geq z_0$) or $\breve{y}_1^i|_{\bar{\alpha}=0} < \tilde{y}_1^i$ (i.e., $z < z_0$) and $\breve{r}_1^i|_{\bar{\alpha}=0} \geq \tilde{r}_1^i$.

(b) Retailer i has positive inventory while sales occur in both periods, $\bar{\alpha}(1) = 1$, and any $v_0^{\min}(\bar{\alpha}(1), \bar{p}_2(1))$ from the interval $[p_1, 1]$ is plausible *a priori*. Since, under rational expectations, $p_2 < \beta p_1 \leq p_1$, case (b) involves reimbursements and the general expression for the profit of retailer i is $r_1^i = (p_2 - c)y_1^i$ regardless of the specific value of $v_0^{\min}$. The maximum must be internal, can only be at $\tilde{y}_1^i$, and the corresponding profit is $\tilde{r}_1^i$. Rationality of expectations requires that $\tilde{y}_1^i > 1 - Y_0 - p_1$, which implies $p_2 < \beta p_1$, and can be written either as $p_1 > \frac{c}{\beta} + \frac{z}{2}$, or $z > 2(1 - Y_0 - p_1) = 2z - 2(p_1 - \frac{c}{\beta})$, or $z < z_0$. Since $v_0^{\min} \geq p_1$ and $p_1 \leq p_1/\beta$, inequality $z < z_0$ implies that $\tilde{y}_1^i$ does belong to the range of inventory levels where the profit function has the form $r_1^i = (p_2 - c)y_1^i$, i.e., $\tilde{y}_1^i > \breve{y}_1^i|_{\bar{\alpha}=1} = 1 - v_0^{\min} - Y_0$ and $\tilde{y}_1^i > \breve{y}_1^i$. Thus, the inventory $\tilde{y}_1^i$ is the maximizer under rational expectations if and only if

- (feasibility) $\tilde{y}_1^i > 0$ or, equivalently, $Y_0 < 1 - c/\beta \Leftrightarrow z > 0$;
- (rationality) $z < z_0$; and
- (optimality) either $\breve{y}_1^i|_{\bar{\alpha}=1} \leq (\tilde{y}_1^i)^+$ (i.e., profit function is continuous and concave, and there is no need to compare profits), or $\breve{y}_1^i|_{\bar{\alpha}=1} > (\tilde{y}_1^i)^+$ (profit is discontinuous at $\breve{y}_1^i|_{\bar{\alpha}=1}$) and $\tilde{r}_1^i \geq \breve{r}_1^i|_{\bar{\alpha}=1} = \breve{y}_1^i|_{\bar{\alpha}=1}(p_1 - c)$.

(c) PM retailer i chooses to exit the market by setting $y_1^i = 0$ if and only if *neither* $\breve{y}_1^i|_{\bar{\alpha}=0} > 0$ nor $\tilde{y}_1^i > 0$ can be a candidate for the optimal solution. This outcome is possible if and only if $z \leq \frac{z_0}{2}$ ($Y_0 \geq 1 - p_1$ — positive $\breve{y}_1^i|_{\bar{\alpha}=0}$ with sales only in the first period is impossible) *and* either $z \leq 0$ ($Y_0 \geq 1 - c/\beta$), or $z \geq z_0$, or both (if $p_1 \leq c/\beta$) hold — positive $\tilde{y}_1^i$ is impossible under rational expectations. For $\frac{z_0}{2} > 0$, $z_0 \leq z \leq \frac{z_0}{2}$ cannot hold, and only $z \leq 0$ is compatible with a weaker condition $z \leq \frac{z_0}{2}$. For $\frac{z_0}{2} \leq 0$, at least one of $z \leq 0$ or $z \geq z_0$ holds for any $z \leq \frac{z_0}{2}$. A combination of these two cases yields the condition of part (c). Any $z < \frac{z_0}{2}$ results in the second period sales and rational expectations $\bar{\alpha} = 1$. If $z = \frac{z_0}{2}$, sales take place in the first period only with $\bar{\alpha} = 0$. For $Y_0 = 0, y_1^i = 0$ is never optimal since $Y_0 \geq 1 - p_1$ may hold only for $p_1 = 1$ and then $\tilde{y}_1^i = \frac{1}{2}(1 - c/\beta) > 0$ satisfies $z < z_0$, which becomes $1 - c/\beta > 0$.

It remains to show the equivalence of the above necessary and sufficient conditions in parts (a) and (b) to the corresponding conditions in the statement of the lemma.

Part (a.1) If $z_0 \leq 0$ (i.e., $p_1 \leq c/\beta$), then $z > \frac{z_0}{2}$ implies $z \geq z_0$ and there is no need to compare profits.

Part (a.2). If $z_0 > 0$ (i.e., $p_1 > c/\beta$), it is still possible that $z \geq z_0$ and there is no need to compare profits. Consider $\frac{z_0}{2} < z < z_0$, where the profits need to compared. In this case, $\breve{y}_1^i|_{\bar{\alpha}=0}$ is not less profitable than $\tilde{y}_1^i$ if and only if $\breve{r}_1^i|_{\bar{\alpha}=0} \geq \tilde{r}_1^i$, which is a quadratic inequality in $z: \frac{\beta}{4}z^2 - z(p_1 - c) + \frac{z_0}{2}(p_1 - c) \leq 0$ with the discriminant $(p_1 - c)^2 - \beta(p_1 - c/\beta)(p_1 - c) = (p_1 - c)p_1(1 - \beta) \geq 0$ (strict inequality if $\beta < 1$), and the roots of the corresponding equation $z_{1,2} = \frac{2}{\beta}\left[p_1 - c \mp \sqrt{(p_1 - c)p_1(1 - \beta)}\right]$, implying that $\breve{r}_1^i|_{\bar{\alpha}=0} \geq \tilde{r}_1^i$ is equivalent to $z_1 \leq z \leq z_2$. The roots and z_0 are such that $\frac{z_0}{2} < z_1 \leq z_0 \leq z_2$. Indeed, the LHS of the quadratic inequality in z is $\frac{\beta}{4}z_0^2 > 0$ at $z = \frac{z_0}{2}$ and non-positive at $z = z_0$:

$$\beta(p_1 - c/\beta)^2 - 2(p_1 - c/\beta)(p_1 - c) + (p_1 - c/\beta)(p_1 - c) \leq 0 \Leftrightarrow p_1\beta - c \leq p_1 - c,$$

which always holds. Hence, since in case (a) the comparison of $\breve{r}_1^i|_{\bar{\alpha}=0}$ with $\tilde{r}_1^i$ is relevant only in the range $\frac{z_0}{2} < z \leq z_0$, we can conclude that $\breve{r}_1^i|_{\bar{\alpha}=0} \geq \tilde{r}_1^i$ if $z \geq z_1$. This inequality includes as a particular case the condition $z \geq z_0$ for $p_1 > c/\beta$, when $\breve{y}_1^i|_{\bar{\alpha}=0}$ is optimal without comparing the profits.

Part (b), possible values of $v_0^{\min}$. As shown above, feasibility of $\tilde{y}_1^i$ and rationality of expectations require z be in the range $0 < z < z_0$. It remains to specify the conditions of optimality of $\tilde{y}_1^i$. These conditions depend on $\breve{y}_1^i|_{\bar{\alpha}=1} = 1 - Y_0 - v_0^{\min}$, which equals $1 - p_1$ if $Y_0 = 0$. In this case, $\breve{y}_1^i|_{\bar{\alpha}=1} = \breve{y}_1^i|_{\bar{\alpha}=0}$ and, by part (a), the condition of optimality is $z \leq z_1$.

Consider $Y_0 > 0$. Denote $V(z) \triangleq \frac{p_1 - \rho c - \rho\beta z/2}{1 - \rho\beta}$. Then, in part (b),

$$v_0^{\min} = p_1 \vee \left(\frac{p_1 - \rho\beta(1 - Y_0 - z/2)}{1 - \rho\beta} \wedge 1\right) = p_1 \vee [V(z) \wedge 1]. \tag{27}$$

Given $0 < z < z_0$, the possible values of $v_0^{\min}$ include the following subcases.

$v_0^{\min} = p_1$ if $p_1 - \rho c - \rho\beta z/2 \le p_1 - \rho\beta p_1 \Leftrightarrow \rho\beta p_1 - \rho c \le \rho\beta z/2$, which holds either if $\rho = 0$ or $\rho > 0$ and $\rho\beta\,(p_1 - c/\beta) \le \rho\beta z/2$. The last inequality contradicts $z < z_0$, therefore $v_0^{\min} = p_1$ may hold only if $\rho = 0$. Thus, for $\rho = 0$, $\breve{y}_1^i|_{\bar\alpha=1} = \breve{y}_1^i|_{\bar\alpha=0}$, and, again, by part (a), the condition of optimality is $z \le z_1$.

Consider $\rho > 0$. Then $v_0^{\min} > p_1$ if $p_1 < 1$ and $v_0^{\min} = 1$ if and only if either $p_1 = 1$ or $p_1 < 1$ and

$$p_1 - \rho c - \rho\beta z/2 \ge 1 - \rho\beta \Leftrightarrow z \le 2(p_1 - \rho c + \rho\beta - 1)/(\rho\beta). \tag{28}$$

In this case, $\breve{y}_1^i|_{\bar\alpha=1} = -Y_0 < 0$.

Part (b), condition $z \le \tilde{z}_1$ *(*$\breve{r}_1^i|_{\bar\alpha=1} \le \tilde{r}_1^i$*).* Recall that in the range $0 < z < z_0$, inventory $\tilde{y}_1^i$ is optimal if and only if (I) there is no need to compare profits ($\breve{y}_1^i|_{\bar\alpha=1} \le (\breve{y}_1^i)^+$), or (II) $\breve{y}_1^i|_{\bar\alpha=1} > (\breve{y}_1^i)^+$ and $\breve{r}_1^i|_{\bar\alpha=1} \le \tilde{r}_1^i$.

(I). *Consider* $\breve{y}_1^i \le 0$. Condition $\breve{y}_1^i|_{\bar\alpha=1} \le 0$ trivially holds for $v_0^{\min} = p_1 = 1$.

For $p_1 < 1$, condition $\breve{y}_1^i|_{\bar\alpha=1} \le 0$ is equivalent to $v_0^{\min} \ge 1 - Y_0$, or, in terms of z, $v_0^{\min} - c/\beta \ge z$, which, for $V(z) \in (p_1, 1]$, becomes

$$p_1 - \rho c - \rho\beta z/2 \ge (z + c/\beta)(1 - \rho\beta) \Leftrightarrow z(1 - \rho\beta + \rho\beta/2) \le p_1 - c/\beta \Leftrightarrow z \le \frac{z_0}{2 - \rho\beta}.$$

Combining the last inequality with (28), we obtain that inequality $\breve{y}_1^i|_{\bar\alpha=1} \le 0$ is equivalent to $z \le \frac{2}{\rho\beta}(p_1 - \rho c + \rho\beta - 1) \vee \frac{z_0}{2-\rho\beta}$, where the RHS is the maximum from the two bounds because $z \ge 2(p_1 - \rho c + \rho\beta - 1)/(\rho\beta)$ is equivalent to $V(z) \in (p_1, 1]$. Both bounds are always strictly less than z_0. Indeed, $2 - \rho\beta > 1$, and

$$2[p_1 - \rho c + \rho\beta - 1]/(\rho\beta) < z_0 \Leftrightarrow p_1 - \rho c + \rho\beta - 1 < \rho\beta p_1 - \rho c \Leftrightarrow 1 - \rho\beta > p_1(1 - \rho\beta),$$

which holds for any $p_1 < 1$.

Consider $\breve{y}_1^i > 0$. Inequality $\breve{y}_1^i|_{\bar\alpha=1} \le \breve{y}_1^i$ is $1 - Y_0 - v_0^{\min} \le 1 - Y_0 - p_1/\beta \Leftrightarrow p_1/\beta \le v_0^{\min}$, which, for $\rho > 0$, may hold only if $p_1 \le \beta$. Under this condition, $p_1/\beta \le v_0^{\min}$ is equivalent to $p_1(1 - \rho\beta)/\beta \le p_1 - \rho c - \rho\beta z/2$ or $z \le 2\,(p_1 - \rho c + \rho p_1 - p_1/\beta)\,/(\rho\beta)$.

(II). This subcase contains two conditions: $z \in (0, z_0) \cap \left\{z : \breve{y}_1^i|_{\bar\alpha=1} > (\breve{y}_1^i)^+\right\}$ and $\breve{r}_1^i|_{\bar\alpha=1} \le \tilde{r}_1^i$. The last inequality, after the substitution of $\breve{y}_1^i|_{\bar\alpha=1} = z + c/\beta - v_0^{\min}$

$$= z + \frac{c}{\beta} - \frac{p_1 - \rho c - \rho\beta z/2}{1 - \rho\beta} = \frac{c}{\beta} - \frac{p_1 - \rho c}{1 - \rho\beta} + \frac{z}{2}\frac{2 - \rho\beta}{1 - \rho\beta}$$

into $\breve{r}_1^i|_{\bar\alpha=1} = \breve{y}_1^i|_{\bar\alpha=1}(p_1 - c)$, becomes $\left(\frac{z}{2}\frac{2-\rho\beta}{1-\rho\beta} + \frac{c/\beta - \rho c - p_1 + \rho c}{1-\rho\beta}\right)(p_1 - c) \le \frac{\beta}{4}z^2$ or

$$z^2 - \frac{2z(p_1 - c)(2 - \rho\beta)}{\beta(1 - \rho\beta)} + \frac{2z_0(p_1 - c)}{\beta(1 - \rho\beta)} \ge 0. \tag{29}$$

The LHS of (29) for $z = z_0$ is $z_0\left\{z_0 - 2(p_1 - c)/\beta\right\} = 2z_0\left\{\beta p_1 - c - p_1 + c\right\}/\beta \le 0$ with strict inequality if $\beta < 1$, i.e., the roots $\tilde{z}_1$ and $\tilde{z}_2$ of the corresponding equation always exist and $\tilde{z}_1 \le z_0 \le \tilde{z}_2$.

It is easy to show that $\frac{z_0}{2-\rho\beta} < \tilde{z}_1$. Indeed, the LHS of (29) with $z = \frac{z_0}{2-\rho\beta}$ becomes $\left(\frac{z_0}{2-\rho\beta}\right)^2 > 0$. The bound $\frac{2}{\rho\beta}[p_1 - \rho c + \rho\beta - 1]$ that corresponds to $v_0^{\min} = 1$ is also strictly below $\tilde{z}_1$ because, by (27) with $z = \tilde{z}_1$, we have $v_0^{\min} < 1$. Otherwise, by (28) with $z = \tilde{z}_1$, $v_0^{\min} = 1$ implying $\breve{r}_1^i|_{v_0^{\min}=1} < 0$, and equality $\breve{r}_1^i|_{v_0^{\min}=1} = \tilde{r}_1^i$ cannot hold.

The bound $\frac{2}{\rho\beta}\,(p_1 - \rho c + \rho p_1 - p_1/\beta)$, which results from $p_1/\beta \le v_0^{\min}$, also does not exceed $\tilde{z}_1$ since the LHS of inequality $\breve{r}_1^i|_{\bar\alpha=1} \le \tilde{r}_1^i$, i.e., $\left(z + c/\beta - v_0^{\min}\right)(p_1 - c)$, is decreasing in $v_0^{\min}$, and the inequality holds for $v_0^{\min} = p_1/\beta$. Indeed,

$$\left(z + \frac{c}{\beta} - \frac{p_1}{\beta}\right)(p_1 - c) \le \frac{\beta}{4}z^2 \Leftrightarrow \frac{\beta}{4}z^2 - z(p_1 - c) + \frac{(p_1 - c)^2}{\beta} \ge 0 \Leftrightarrow \left(\frac{z\sqrt{\beta}}{2} - \frac{p_1 - c}{\sqrt{\beta}}\right)^2 \ge 0.$$

Hence, we have $0 < \frac{2}{\rho\beta}[p_1 - \rho c + \rho\beta - 1] \vee \frac{z_0}{2-\rho\beta} \vee \frac{2}{\rho\beta}(p_1 - \rho c + \rho p_1 - p_1/\beta) < \tilde{z}_1 \le z_0 \le \tilde{z}_2$, and, combining all the conditions in case (b) for $\rho > 0$ and $Y_0 > 0$, $\tilde{y}_1^i$ is optimal if and only if $z > 0$ and either $p_1 = 1$ or $p_1 < 1$ and $z \le \tilde{z}_1$. Namely, for $p_1 < 1$, a positive $\tilde{y}_1^i$ is optimal in the subrange $z \le \bar{z}$, where

$$\bar{z} = \begin{cases} \frac{2}{\rho\beta}[p_1 - \rho c + \rho\beta - 1] \vee \frac{z_0}{2-\rho\beta}, & \text{if } p_1 > \beta, \\ \frac{2}{\rho\beta}[p_1 - \rho c + \rho\beta - 1] \vee \frac{z_0}{2-\rho\beta} \vee \frac{2}{\rho\beta}(p_1 - \rho c + \rho p_1 - p_1/\beta), & \text{if } p_1 \le \beta \end{cases}$$

because in this subrange a positive $\breve{y}_1^i|_{\bar{\alpha}=1}$ cannot be feasible and rational; and $\tilde{y}_1^i$ is optimal in the subrange $z \in (\bar{z}, \tilde{z}_1]$ because both positive $\tilde{y}_1^i$ and $\breve{y}_1^i|_{\bar{\alpha}=1}$ are feasible and rational, and $\breve{r}_1^i|_{\bar{\alpha}=1} \le \tilde{r}_1^i$.

When positive $\tilde{y}_1^i, \breve{y}_1^i|_{\bar{\alpha}=0}$, and $\breve{y}_1^i|_{\bar{\alpha}=1}$ are feasible and rational, it can be shown that $z_1 \le \tilde{z}_1$ with equality only if $\bar{\alpha} = 0, Y_0 = 0$, or $\rho = 0$. Indeed, if we assume that $z_1 > \tilde{z}_1$ for $\bar{\alpha} = 1, Y_0 > 0$, and $\rho > 0$, then, for any $z \in (\tilde{z}_1, z_1)$, we have $\breve{r}_1^i|_{\bar{\alpha}=1} > \tilde{r}_1^i$ and, at the same time, $\breve{r}_1^i|_{\bar{\alpha}=0} < \tilde{r}_1^i$, contradicting the fact that $\breve{r}_1^i|_{\bar{\alpha}=1}$ is decreasing in $v_0^{\min}$. Therefore, $\tilde{y}_1^i$ is optimal for any $z \in [z_1, \tilde{z}_1]$ when $\bar{\alpha} = 1$, and $\breve{y}_1^i|_{\bar{\alpha}=0}$ is optimal for $z \ge z_1 = \tilde{z}_1$ when $\bar{\alpha} = 0$.

The expression for $\tilde{z}_1$ (for $n > 1$) is

$$\begin{aligned} \tilde{z}_1 &= \frac{1}{2}\left[\frac{2(p_1 - c)(2 - \rho\beta)}{\beta(1 - \rho\beta)} - \sqrt{\frac{4(p_1 - c)^2(2 - \rho\beta)^2}{\beta^2(1 - \rho\beta)^2} - \frac{8z_0(p_1 - c)}{\beta(1 - \rho\beta)}}\right] \\ &= \frac{(p_1 - c)(2 - \rho\beta)}{\beta(1 - \rho\beta)}\left[1 - \sqrt{1 - \frac{2z_0\beta(1 - \rho\beta)}{(p_1 - c)(2 - \rho\beta)^2}}\right]. \end{aligned}$$

For $\beta = 1$, this formula yields $\tilde{z}_1 = z_1 = z_0 = z_2$. Indeed, $\tilde{z}_1|_{\beta=1} = \frac{(2-\rho)(p_1-c)}{1-\rho}\left[1 - \sqrt{1 - \frac{4(1-\rho)}{(2-\rho)^2}}\right]$, where the expression under the square root is $\rho^2/(2-\rho)^2$ resulting in $\tilde{z}_1|_{\beta=1} = \frac{p_1-c}{1-\rho}[2 - \rho - \rho] = 2(p_1 - c) = z_1|_{\beta=1} = z_0|_{\beta=1} = z_2|_{\beta=1}$. In the general case, $\tilde{z}_1|_{\beta=1} \ne \tilde{z}_2|_{\beta=1}$.

If $p_1 = c/\beta$, then $\tilde{z}_1 = z_1 = z_0 = 0$, where $\tilde{z}_1 = z_1 = 0$ since the free coefficient in both quadratic equations for $z_{1,2}$ and for $\tilde{z}_{1,2}$ contains z_0, which is zero in this case.

$v_0^{\min}$ from part (b) is not decreasing in ρ since, if $V(z) \in (p_1, 1]$,

$$\frac{\partial v_0^{\min}}{\partial \rho} = \frac{1}{(1 - \rho\beta)^2} \times \{(-c - \beta z/2)(1 - \rho\beta) + \beta(p_1 - \rho c - \rho\beta z/2)\},$$

where $\{\cdot\} = (c + \beta z/2)(\rho\beta - 1 - \rho\beta) + \beta p_1 = \beta(p_1 - z/2 - c/\beta)$, which, as shown above, is positive for $v_0^{\min} \ge p_1$. If $V(z) \ge 1$, $v_0^{\min}$ is constant in ρ and equals one.

B.4. **Proof of Lemma 18 (condition of N3.1 does not hold).** For $n = 1$, $\breve{r}_1^i \equiv r^{*,PM2}$ (Theorem 1) and $r^{*,N3}$ is equal to the profit of a deviator from PM2 into no-PM with sales in both periods (see (25) with $Y^{-i} = 0$ in the proof of Lemma 15). By Theorem 1, PM2 exists in the area that intersects with the area of NA3 existence, which, for $n = 1$, requires $p_1 > c/\beta$, and, by part (2.2) of Theorem 1, PM2 exists for $\beta \to 1 - 0$ and $\rho = (1 - \sqrt{1-\beta})/\beta \to 1$, implying $CB_2 = 0$, and $p_1 \le P_{22}$, where $P_{22} > c/\beta$, yielding a non-empty range $c/\beta < p_1 \le P_{22}$, where $P_{22}|_{n=1} = \frac{1}{2}\left[1 + c + \sqrt{(1-\beta)(1 - c^2/\beta)}\right]$ (Corollary 2), which, for $c \to 0$ and $\beta \to 1$ goes to $\frac{1}{2}$. For these inputs, inequality $\frac{n-1}{n}Y^* \le 1 - c/\beta - z_1$ is $z_1 \le 1$, which is equivalent to $p_1 \le P_{22} = \frac{1}{2}$. Indeed, $z_1 \le 1 \Leftrightarrow p_1 - c - \sqrt{(p_1 - c)p_1(1-\beta)} \le \frac{\beta}{2} \Leftrightarrow p_1 \le \frac{1}{2}$.

At the same time, by Lemma 14 with $x = v_0^{\min} > c/\beta$, inequality $r^{*,PM2} \geq \tilde{r}_0^i$ may be strict for $\beta < 1$, resulting, by continuity of $r^{*,N3}$ and $\breve{r}_1^i$, in violation of $r^{*,N3} \geq \breve{r}_1^i$ in the vicinity of the area with $\beta = \rho = 1$ and $c = 0$.

B.5. **Proof of Lemma 19 (condition of N3.2 and N4.2 holds).** Under NA3, by Theorem 3, $Y^* > 1 - p_1$ for any $n \geq 1$, and, under NA4, $Y^* \geq 1 - s/\beta > 1 - p_1$ for any $n > 1$ since $p_2 = s$ and $p_1 > s/\beta$. Therefore, there exists $N > 1$ such that $\frac{n-1}{n} Y^* > 1 - p$ for any $n \geq N$. If $\beta < 1$ and $n > 1$, the lower bound for p_1 in both NA3 and NA4, $\frac{nc}{\beta+n-1}$, is strictly less than c/β and approaches c with $n \to \infty$. Hence, for $\beta < 1$ and $n \geq N$, there exist a non-empty range for p_1 such that $\frac{nc}{\beta+n-1} \vee 1 - \frac{n-1}{n} Y^* < p_1 \leq c/\beta$.

B.6. **Proof of Lemma 20 (condition for profits of N3.2 holds).** For $n = 1, \rho = 0$, and $\beta = 1$, the NA3 p_1-range is $p_1 \in (c, 1)$, hence the condition $p_1 > c/\beta$ holds and inequality $\frac{n-1}{n} Y^* \geq 1 - c/\beta - \tilde{z}_1$ becomes $p_1 - c \geq \frac{1-c}{2}$ or $p_1 \geq \frac{1+c}{2}$. Since, in this case, $v^* = p_1$, the inventory, p_2^*, and the profit are $Y^* = \frac{1}{2}(2-c-p_1)$, $p_2^* = \frac{1}{2}(c+p_1)$, $r^{*,N3} = (p_1-c)(1-p_1)+(p_2^*-c)[p_1-\frac{1}{2}(c+p_1)]$. Then inequality $r^{*,N3} \geq \tilde{r}_1^i$ takes the form

$$(p_1 - c)(1 - p_1) + \frac{1}{4}(p_1 - c)^2 \geq \frac{1}{4}(1 - c)^2 \Leftrightarrow 4(p_1 - c)(1 - p_1) \geq (1 - p_1)(1 + p_1 - 2c)$$

or $p_1 \geq \frac{1+2c}{3}$, which holds for any $p_1 \geq \frac{1+c}{2}$ since $c < 1$.

B.7. **Proof of Lemma 21 (N4 existence, sufficient conditions).** Lemma 17 provides necessary conditions of existence of positive $\breve{y}_1^i$ and $\tilde{y}_1^i$ in the form of upper bounds on Y_0. Namely, $Y_0 < 1 - p_1$ — for $\breve{y}_1^i$, and $Y_0 < 1 - c/\beta$ $(z > 0)$ — for $\tilde{y}_1^i$. At the same time, by Theorem 5, $Y^* > 1 - s/\beta$, which is a lower bound: $Y_0 > \frac{n-1}{n}(1 - s/\beta)$. Hence, a deviation *from* NA4 *into* one of the forms of PM is impossible if both Y_0-ranges are empty, i.e., $\left(1 - \frac{1}{n}\right)(1 - s/\beta) \geq (1 - p_1) \vee (1 - c/\beta)$, which can be rewritten as $\frac{1}{n}(1 - s/\beta) \leq (p_1 - s/\beta) \wedge \frac{c-s}{\beta}$ or $n \geq \frac{\beta-s}{p_1\beta-s} \vee \frac{\beta-s}{c-s}$.

B.8. **Proof of Lemma 22 (Total equilibrium customer surplus).** The total surplus is $\Sigma = \Sigma_1 + \Sigma_2$, where $\Sigma_1 = \int_{v^*}^1 (v - p_1)dv$ without PM and $\Sigma_1 = \int_{p_1}^1 (v - p_2^*)dv$ with PM. When there are second-period sales, $\Sigma_2 = \int_{\frac{p_2^*}{\beta}}^{v^*} (\beta v - p_2^*)dv = \int_{p_2}^{\beta v^*} (\tilde{v} - p_2^*)\frac{d\tilde{v}}{\beta}$ without PM and $\Sigma_2 = \int_{p_2}^{\beta p_1} (\tilde{v} - p_2^*)\frac{d\tilde{v}}{\beta}$ with PM. Straightforward integration specifies Σ as follows.

Part (1). $\Sigma_1^{PM1} = \int_{p_1}^1 (v - p_2^*)dv = \left(v^2/2 - p_2^* v\right)\big|_{p_1}^1 = \frac{1}{2} - p_2^* - p_1^2/2 + p_2^* p_1 + p_1 - p_1 + p_1^2/2 - p_1^2/2$ $= (1 - 2p_1 + p_1^2)/2 + p_1(1 - p_1) - p_2^*(1 - p_1) = (1 - p_1)^2/2 + (1 - p_1)(p_1 - p_2^*)$, and $\Sigma_2^{PM1} = \int_{p_2^*}^{\beta p_1} (\tilde{v} - p_2^*)\frac{d\tilde{v}}{\beta} = \left(\tilde{v}^2/2 - p_2^* \tilde{v}\right)/\beta\big|_{p_2^*}^{\beta p_1} = \left[\beta^2 p_1^2/2 - \beta p_2^* p_1 - (p_2^*)^2/2 + (p_2^*)^2\right]/\beta = (\beta p_1 - p_2^*)^2/(2\beta)$.

Part (2). Since $v^* = p_1$ under N2, $\Sigma^{PM2} = \Sigma^{N2} = \Sigma_1^{PM2} = \int_{p_1}^1 (v - p_1)dv = 1/2 - p_1 - p_1^2/2 + p_1^2 = (1 - p_1)^2/2$.

Part (3). Since $v^* = 1$ under N1, $\Sigma^{N1} = \Sigma_2^{N1} = \int_{p_2^*}^{\beta} (\tilde{v} - p_2^*)\frac{d\tilde{v}}{\beta} = (\beta - p_2^*)^2/(2\beta)$.

Part (4). Under N3 and 4, $\Sigma_1 = \int_{v^*}^1 (v - p_1)dv = \frac{1}{2} - p_1 - (v^*)^2/2 + p_1 v^* + p_1^2/2 - p_1^2/2 = (1 - p_1)^2/2 - (v^* - p_1)^2/2$, and $\Sigma_2 = \int_{p_2^*}^{\beta v^*} (\tilde{v} - p_2^*)\frac{d\tilde{v}}{\beta} = (\beta v^* - p_2^*)^2/(2\beta)$. By construction of N3 and N4, inequality $p_2^{*,N4} = s < p_2^{*,N3}$ always hold. In both N3 and N4, $v^* = \frac{p_1 - \rho p_2^*}{1-\rho\beta}$, which is decreasing in p_2^* except the case $\rho = 0$ when $v^{*,N3} = v^{*,N4} = p_1$. Therefore, $v^{*,N4} > v^{*,N3}$ for any $\rho \in (0, 1)$.

B.9. **Proof of Lemma 23 (NA3, $n = 1$).** By part NA3 of Theorem 3 with $n = 1$, condition (a) always holds, and the equation (21) in Y reduces to $Y\left[Y - \frac{(\beta-c)(1-\rho\beta)+\beta(1-p_1)}{\beta(2-\rho\beta)}\right] = 0$ yielding $Y^* = \frac{(\beta-c)(1-\rho\beta)+\beta(1-p_1)}{\beta(2-\rho\beta)} = 1 - \frac{\beta p_1 + c(1-\rho\beta)}{\beta(2-\rho\beta)}$, $v^* = \frac{1}{1-\rho\beta}\left[p_1 - \rho\frac{\beta p_1 + c(1-\rho\beta)}{2-\rho\beta}\right] = \frac{1}{1-\rho\beta}\frac{2p_1 - p_1\rho\beta - \rho\beta p_1 - c\rho(1-\rho\beta)}{2-\rho\beta} =$

$\frac{2p_1-\rho c}{2-\rho\beta}$, and $p_2^*=\beta(1-Y^*)=\frac{\beta p_1+c(1-\rho\beta)}{2-\rho\beta}=c+\frac{\beta p_1-c}{2-\rho\beta}$. Substitution into the formula for $r^{*,N3}$ results in $r^{*,N3}=(p_1-c)\frac{2(1-p_1)-\rho(\beta-c)}{2-\rho\beta}+\frac{\beta p_1-c}{2-\rho\beta}\left(\frac{2p_1-\rho c}{2-\rho\beta}-\frac{\beta p_1+c(1-\rho\beta)}{\beta(2-\rho\beta)}\right)$, where the bracket in the last term is $\frac{2\beta p_1-\rho\beta c-\beta p_1-c(1-\rho\beta)}{\beta(2-\rho\beta)}=\frac{\beta p_1-c}{\beta(2-\rho\beta)}$, leading to the expression in the lemma. The expression for Σ^* results from direct substitution of v^* and p_2^* into the general formula (Lemma 22).

B.10. **Proof of Lemma 24 (NA3, NA4, $p_1=\beta$).** *NA3.* The equation in Y with $p_1=\beta$ yields the expression for Y^*. With this Y^* and $p_1=\beta$ we have $v^*=\beta\frac{n+1-\rho\beta-\rho[1-\rho\beta-n(1-c/\beta)(1-\rho\beta)+n\beta]}{(1-\rho\beta)(n+1-\rho\beta)}=\beta\frac{1-\rho+n[1+\rho(1-c/\beta)]}{n+1-\rho\beta}$. Then $r^{*,N3}$ is

$$r^{*,N3}|_{p_1=\beta}=\frac{1}{n}\left[(\beta-c)(1-v^*)+(\beta(1-Y^*)-c)(Y^*-1+v^*)\right]=\frac{Y^*}{n}\left[\beta-c-\beta(Y^*-1+v^*)\right],$$

which after substitutions for Y^* and $Y^*-1+v^*=\frac{1}{n+1-\rho\beta}\left\{n\left(1-c/\beta\right)-\left(1-\beta\right)\right\}$ becomes

$$r^{*,N3}|_{p_1=\beta}=\frac{\left(1-c/\beta\right)\left(1-\rho\beta\right)+1-\beta}{(n+1-\rho\beta)^2}\left\{(\beta-c)(n+1-\rho\beta)-\beta\left[n\left(1-c/\beta\right)-\left(1-\beta\right)\right]\right\},$$

where $\{\cdot\}=(1-\rho\beta)(\beta-c)+\beta(1-\beta)$, yielding the expression for $r^{*,N3}|_{p_1=\beta}$. Condition $c/\beta<CB_{N2}$ results from the p_1-lower bound $p_1>\frac{nc}{\beta+n-1}$; the upper bound is $\beta<1$ for $\rho=0$ and, for $\rho>0$, it can be written as $c/\beta>1-\frac{n+1}{n\rho\beta}(1-\beta)=CB_{N1}$. Condition (a) is specified for $p_1=\beta$. Using the expression for Y^*, inequality $Y^*<1-s/\beta$ is equivalent to $n\left[\left(1-c/\beta\right)\left(1-\rho\beta\right)-\beta+s/\beta\right]<\left(1-\rho\beta\right)\left(1-s/\beta\right)$, which always holds if $[\cdot]\le 0$ or $\beta^2-s-(\beta-c)(1-\rho\beta)\ge 0$. Since the LHS is increasing in ρ, this inequality holds for any $\rho\ge 0$ if $c-s\ge\beta(1-\beta)$. Otherwise, $[\cdot]>0$ and $Y^*<1-s/\beta$ for any $n<\left(1-\rho\beta\right)\left(1-s/\beta\right)/\left[\cdot\right]$.

NA4. The expressions for $Y^*,v^*,r^{*,N4}$ and condition (a) follow directly from Theorem 5 with $p_1=\beta$. Condition (b) is $\frac{n-1}{n}\frac{\beta Y^*}{c+\beta v^*-2s}\ge 1$, which, after substitution for Y^* and v^*, becomes $(c-s)\left(c-s+\frac{\beta^2-s}{1-\rho\beta}\right)\le\left(\frac{n-1}{n}\right)^2\beta(\beta-s)\left(1-\frac{\beta-\rho s}{1-\rho\beta}\right)$.

The requirement in condition (c) that conditions (a) and (b) do not hold and the deviator profit is strictly decreasing in the interval corresponding to $p_2>s$ is equivalent, as shown in Bazhanov et al. (2015), to the following: "there are no real roots of equation

$$2Y^3-\left(2-v^*-c/\beta+\frac{n-1}{n}Y^*\right)Y^2+\left(1-p_1/\beta\right)\left(1-v^*\right)\frac{n-1}{n}Y^*=0 \tag{30}$$

in the interval $(1-v^*,1-s/\beta)$." If $p_1=\beta$, the single root of (30) is $\tilde{Y}=\frac{1}{2}\left(2-v^*-c/\beta+\frac{n-1}{n}Y^*\right)$. This root is not in the interval $(1-v^*,1-s/\beta)$ if and only if either $\tilde{Y}\le 1-v^*$, which, using $\frac{n-1}{n}Y^*=\left(\frac{n-1}{n}\right)^2\frac{\beta-s}{c-s}(1-v^*)$, becomes $1-c/\beta\le\left(1-v^*\right)\left[1-\left(\frac{n-1}{n}\right)^2\frac{\beta-s}{c-s}\right]$, or $\tilde{Y}\ge 1-s/\beta$, which, in the same way, becomes $1+\frac{c-2s}{\beta}\le\left(1-v^*\right)\left[1+\left(\frac{n-1}{n}\right)^2\frac{\beta-s}{c-s}\right]$. When $\tilde{Y}\in(1-v^*,1-s/\beta)$, NA4 exists if $r^{*,N4}\ge\tilde{r}^i=\left(\tilde{Y}-\frac{n-1}{n}Y^*\right)\left[\beta\left(2-v^*-\tilde{Y}\right)-c+\frac{(p_1-\beta)(1-v^*)}{\tilde{Y}}\right]\Big|_{p_1=\beta}=\left(\tilde{Y}-\frac{n-1}{n}Y^*\right)\left[\beta(2-v^*-\tilde{Y})-c\right]$, where $\beta(2-v^*-\tilde{Y})-c=\frac{\beta}{2}\left(2-v^*-c/\beta-\frac{n-1}{n}Y^*\right)$ and $\tilde{Y}-\frac{n-1}{n}Y^*=\frac{1}{2}\left(2-v^*-c/\beta-\frac{n-1}{n}Y^*\right)=\frac{1}{2}\left[\left(1-v^*\right)\left(1-\left(\frac{n-1}{n}\right)^2\frac{\beta-s}{c-s}\right)+1-c/\beta\right]$. These expressions yield $\tilde{r}^i=\frac{\beta}{4}\left[\left(1-v^*\right)\left(1-\left(\frac{n-1}{n}\right)^2\frac{\beta-s}{c-s}\right)+1-c/\beta\right]^2$.

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